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Review Article

EPQ MODEL FOR MULTIPLE MANUFACTURES SUPPLY CHAIN INVENTORY MODEL WITH ONE-PRICE BREAK

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Abstract

In this paper Mathematical EPQ model for multiple manufactures supply chain inventory model with one-price breaks. The model developed small and large scale producers with non coordination stage considered both small and large scale producers produce single product and allowed shortages only for small scale producers. To obtain optimum production quantity with price breaks which will optimize the total inventory cost, economic production quantity with one price breaks. Numerical example and sensitivity analysis are analysed with various decision variables.

Keywords: Production, Inventory, Quantity Discount, One-price Break.

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INTRODUCTION

In general products with fixed lifetime like pharmaceutical items, perishable items, and beverage, etc., are produced by large and small scale producers. Sometimes the small and large scale producers produce the same product. In this case, the small producer (Small scale producer) might decide to purchase the same product from the large scale producer (Large scale producer) instead of producing it himself. At that time, large scale producer usually offered discounts on Manufacturer1 if they bought a lot of goods.

Fujiwara et al. [2] developed a model using optimal policy for perishable products of the two-stage inventory system. Arora et al. [1] analyzed an integrated approach supply chain vendor inventory model. Kaj - Mikael Bjork [4] investigated many finite production quantity inventory problems. Kit Nam Francis Leung [3] developed a comprehensive production inventory problem in a multi-company supply chain for multiple situations. Mahdi Tajbakhsh et al. [6] analyzed an inventory model using a random discount. Khan et al. [9] considered integrated supply chain with errors production inventory to obtain minimum average cost. Muniappan et al. [7] developed a back-ordering product inventory model for with quantity discount. Huang et al. [10] analyzed the single vendor- single buyer inventory model with production. Muniappan et al. [5] obtained economic order quantity to minimize inventory costs with deteriorating items. Muniappan et al. [8] developed an incentive inventory model for exponential function of cost of deteriorating products. Saoussen Krichen et al. [11] discussed single and multiple suppliers, cooperative retailer's inventory models, a quantity discount. Umamaheswari et al. [13] analysed the perishable goods with fixed lifetime for an optimal inventory.

Yongrui et al. [12] discussed buyer-vendor inventory problems for various stages to analyze discount incentives for the product. In this model developed two-stage manufacturer produces the same product. This paper is discussed Mathematical Formulation, assumption and notation, and analyzed different manufacturers using the same product with two stages, numerical analysis procedure.

MODEL DEVELOPEMENT

In non-coordination stage, the following assumptions and notations are considered throughout the model

Assumptions

(1) Production rate is greater than constant demand (P >D)
(2) Shortages is allowed for only small scale producer
(3) Bothe producers produce single product

Notations

D_1,D_2Annual demand for small producer and large scale producers
P_1, P_2Both Producers rate of production per annum
b_1Range of small scale producer order quantity with one-price break
b_2Range of large scale producer order quantity with one-price break
LLife time of product
k_1, k_2Both Producers setup costs per annum
h_1, h_2Both Producers holding costs per unit per annum
p_1, p_2Both Producers price per unit per annum
$QQ = Q_1 + Q_2$, order size of small scale producer
Q_1 Order level which satisfies the shortage quantity
Q_2Order level which satisfies the demand units
Q_0Order quantity of large scale producer
mIn the absence of coordination order multiple of large scale producer
TC_{M1} Without coordination, overall cost of the small scale producer
$TC_{M2}(m)$Without coordination, overall cost of large scale producer

DEVELOPMENT OF MODEL WITHOUT COORDINATION

In this case small scale producer and large scale producer produce the same product and shortages are allowed for small scale producer. The model formula is:

The total yearly cost for the Small scale producer is

$$TC_{M1} = \frac{D_1 k_2}{Q} + \frac{1}{2Q}\left(\frac{P_1}{P_1 - D_1}\right)\left[h_2\left(Q\left(1 - \frac{D_1}{P_1}\right) - Q_2\right)^2 + Q_2^2 s_2\right] \quad (1)$$

Subject to One-Price Break

Quantity	Price per Unit per Dollar
$0 \leq Q < b_1$	h
$b_1 \leq Q_2$	h_2 (h_2<h)

Now $\frac{\partial TC_{M1}}{\partial Q} = 0$ and $\frac{\partial TC_{M1}}{\partial Q_2} = 0$ we get,

$$Q = \sqrt{\frac{2D_1 k_2 (h_2 + s_2)}{h_2 s_2}\left(\frac{P_1}{P_1 - D_1}\right)} \text{ and } Q_2 = Q\left(1 - \frac{D_1}{P_1}\right)\frac{h_2}{h_2 + s_2} \quad (2)$$

Without coordination strategy, the small scale producer order quantity is $Q = \sqrt{\frac{2D_1 k_2 (h_2 + s_2)}{h_2 s_2}\left(\frac{P_1}{P_1 - D_1}\right)}$ and optimum total cost $TC_{M1} = \sqrt{\frac{2D_1 k_2 h_2 s_2}{h_2 + s_2}\left(1 - \frac{D_1}{P_1}\right)}$.

Model Formulation for Large Scale Producer

Here, The annual average inventory of large scale producers is

$$= \frac{(m-1)Q_0 + (m-2)Q_0 + \dots + Q_0 + 0Q_0}{m} = \frac{(m-1)Q_0}{2}$$

Now, the total yearly cost for the large scale producers is

$$TC_{M2}(m) = \frac{D_2 k_1}{mQ_0} + \frac{(m-1)h_1 Q_0}{2}\left(1 - \frac{D_2}{P_2}\right)$$

$$= \frac{k_1}{m}\sqrt{\frac{D_2 h_1}{2k_1}\left(\frac{P_2 - D_2}{P_2}\right)} + (m-1)h_1\sqrt{\frac{D_2 k_1}{2h_1}\left(\frac{P_2 - D_2}{P_2}\right)}$$

Subject to the constraint One-Price Break

Quantity	Price per Unit per Dollar
$0 \leq Q < b_2$	h
$b_2 \leq Q_0$	h_1 (h_1<h)

The large scale producer order = mQ_0, where

$$Q_0 = \sqrt{\frac{2D_2 k_1}{h_1}\left(\frac{P_2}{P_2 - D_2}\right)}, \text{ with } t_0 = \sqrt{\frac{2k_1}{D_2 h_1}\left(\frac{P_2}{P_2 - D_2}\right)}.$$

Scenario 1:

Step 1: If the small scale producer order quantity Q_2^* lies in the prescribed range, $Q_2^* \leq b_1$, then Q_2^* is the optimum order quantity and optimum total inventory cost $TC_{M1}(Q_2^*)$.

Step 2: If Q_2^* is not equal to or is more than b_1, then calculate Q^* with price $C_1$and the corresponding total cost $TC_{M1}(Q^*)$. Identify total inventory cost of both $TC_{M1}(b_1)$ and $TC_{M1}(Q_2^*)$, if $TC_{M1}(Q_2^*) < TC_{M1}(b_1)$, then the optimum economic order quantity is equal to Q^* otherwise economic order quantity is b_1.

Scenario 2:

Step 1: If the small scale producer order quantity Q_0^* lies in the prescribed range, $Q_0^* \leq b_2$, then Q_0^* is the optimum order quantity and optimum total inventory cost $TC_{M1}(Q_0^*)$.

Step 2: If Q_0^* is not equal to or is more than b_2, then calculate Q^* with price C_3 and the corresponding total cost $TC_{M1}(Q^*)$. Identify total inventory cost of both $TC_{M1}(b_2)$ and $TC_{M1}(Q_0^*)$, if $TC_{M1}(Q_0^*) < TC_{M1}(b_2)$, then the optimum economic order quantity is equal to Q^* otherwise economic order quantity is b_2.

So without coordination, the large scale producers model , can develop as

$$\min TC_{M2}(m)$$
$$\text{s.t}\begin{cases} mt_0 \leq L, \\ m \geq 1, \end{cases} \quad (1)$$

here $mt_0 \leq L$ Indicates that the product has not expired before being sold by a small producer.

Theorem 1

Consider m^* be the optimum of (1), if $L^2 \geq \frac{2k_1}{D_2 h_1}\left(\frac{P_2}{P_2 - D_2}\right)$, then

$$m^* = min\left\{\left\lceil\sqrt{\frac{k_1 h_2}{k_2 h_1} + \frac{1}{4}} - \frac{1}{2}\right\rceil, \left\lceil\frac{L}{\sqrt{\frac{2k_1}{D_2 h_1}\left(\frac{P_2}{P_2 - D_2}\right)}}\right\rceil\right\}, \quad (3)$$

here $\lceil x \rceil$ is the least integer greater than or equal to x, $L^2 \geq \frac{2k_1}{D_2 h_1}\left(\frac{P_2}{P_2 - D_2}\right)$ is to ensure that $m^* \geq 1$

Proof

$\frac{d^2 TC_{M2}(m)}{dm^2} = \frac{k_1}{m^3}\sqrt{\frac{2D_2 h_1}{k_1}\left(\frac{P_2 - D_2}{P_2}\right)} > 0$, $TC_{M2}(m)$ is strictly convex in m.

Considered m_1^* be the optimum of $\min TC_{M2}(m)$, then

$$m_1^* = \max\{\min\{m / TC_{M2}(m) \leq TC_{M2}(m+1)\}, 1\}$$
$$= \max\{\min\{m / m(m+1)$$
$$\geq \frac{2D_2 k_1}{Q_0^2\left(1 - \frac{D_2}{P_2}\right)h_1}\}, 1\} = \left\lceil\sqrt{\frac{h_2 k_1}{h_1 k_2} + \frac{1}{4}} - \frac{1}{2}\right\rceil \geq 1$$

Put the value of t_0 into the constraints in (1), then we have

$$m\sqrt{\frac{2k_1}{D_2 h_1}\left(\frac{P_2}{P_2 - D_2}\right)} \leq L$$

Take $m_2^* = \frac{L}{\sqrt{\frac{2k_1}{D_2 h_1}\left(\frac{P_2}{P_2 - D_2}\right)}} \geq 1$, is true since $L^2 \geq \frac{2k_1}{D_2 h_1}\left(\frac{P_2}{P_2 - D_2}\right)$.

$m^* = m_1^*$ where $m_1^* \leq m_2^*$, otherwise $m^* = m_2^*$. Therefore $m^* = min\{m_1^*, m_2^*\}$, if $L^2 \geq \frac{2k_2}{D_2 h_2}\left(\frac{P_2}{P_2 - D_2}\right)$

Hence without coordination, the small scale producer optimum total cost is TC_{M1}, order size is $\sqrt{\frac{2D_1 k_2 (h_2 + s_2)}{h_2 s_2}\left(\frac{P_1}{P_1 - D_1}\right)}$ and the large scale producer optimum total cost is $TC_{M2}(m^*)$, order size is $m^*\sqrt{\frac{2D_2 k_1}{h_1}\left(\frac{P_2}{P_2 - D_2}\right)}$

NUMERICAL EXAMPLE

Example 1: The annual demand of small scale producer of his product D_1 = 10,000 units per year, the production rate P_1 = 15000 units per year,each unit price cost h = Rs.100, if the orders are placed in quantities b_1 = 200 units. For order of 200 and above, however the price h_2 = Rs.95. The annual setup cost k_2 = Rs.5 per order, shortage cost s_2 = Re.1, the annual inventory holding cost is 10% of the item value.

The optimum order quantity $Q_2^* = Q\left(1 - \frac{D_1}{P_1}\right)\frac{h_2}{h_2 + s_2} =$ units with $Q = \sqrt{\frac{2D_1 k_2 (h_2 + s_2)}{h_2 s_2}\left(\frac{P_1}{P_1 - D_1}\right)} =$ units

Since Q_2^* is not 200 or more than 200 units therefore economic order quantity $Q^* = 59$ units and optimum total inventory cost

$$TC(Q^*) = \frac{D_1 k_2}{Q} + \frac{1}{2Q}\left(\frac{P_1}{P_1 - D_1}\right)\left[h_2\left(Q\left(1 - \frac{D_1}{P_1}\right) - Q_2\right)^2 + Q_2^2 s_2\right]$$
$$= Rs.882.0016$$

Example 2: The annual demand of large scale producer of his product D_2 = 20,000 units per year, the production rate P_2 = 30000 units per year, each unit price cost h = Rs.100, if the orders are placed in quantities b_2 = 200 units. For order of 200 and above, however the price h_1 = Rs.95. The annual setup cost k_1 = Rs.5 per order, shortage cost s_2 = Re.1, the annual inventory holding cost is 10% of the item value.

The optimum order quantity

$$Q_0 = \sqrt{\frac{2D_2k_1}{h_1}\left(\frac{P_2}{P_2-D_2}\right)} = 2 \text{ units,}$$

Since Q_0^* is lie in the range 200 or more than 200 units therefore economic order quantity, $Q_0^* = 251$ units and optimum total inventory cost is

$$TC_{M2}(m) = \frac{D_2k_1}{mQ_0} + \frac{(m-1)h_1 Q_0}{2}\left(1-\frac{D_2}{P_2}\right)$$

$$= \frac{k_1}{m}\sqrt{\frac{D_2h_1}{2k_1}\left(\frac{P_2-D_2}{P_2}\right)} + (m-1)h_1\sqrt{\frac{D_2k_1}{2h_1}\left(\frac{P_2-D_2}{P_2}\right)}$$

$$= \text{Rs. } 596.6196$$

Sensitivity Analysis

We now check the effect of changing the values of the system parameters h1, h2, k_1, k_2, s_2 on the small scale producer and large scale producer minimum total relevant cost per unit time TC_{M1}^*, TC'^*_{M1}. Perform a sensitivity analysis by acquiring one parameter at a time and leaving the remaining parameters unchanged. Table 1 shows the results.

Table 1: Effect of changes in the parameters of the inventory

Parameters		TC_{M1}	TC'_{M1}
k_1	400	1250	1369.3
	450	1250	1369.3
	500	1250	1369.3
k_2	50	885	965.25
	100	125	1369.3
	150	1531	1677.1
s_2	35	1281	1369.3
	45	1299	1369.3
	55	1311	1369.3
h_1	12	1250	1369.3
	14	1250	1369.3
	16	1250	1369.3
h_2	6	1347	1500.0
	7	1432	1620.2
	8	1508	1732.1

The above table indicates that

- The optimum total cost of large scale producer with co-ordination is less than that without coordination .i.e., large scale producer is convincingly highly benefitted than small scale producer in spite of giving quantity discount.
- An increase in holding cost for large scale producer the optimal total cost of small scale producer and large scale producer remain same or increase.
- Reducing the cost of holding for small scale producer tends to reduce in total cost for small scale producer and large scale producer.
- The set up cost for small scale and large scale producer decrease, automatically the total cost of small scale producer and large scale producer gets decreased.

CONCLUSION

This paper develops Mathematical Analysis on multiple manufactures supply chain inventory model with one-price break. This model assumes non coordination system. In absence of coordination, small scale and large scale producer produce the similar product with one-price break, shortages allowed for small scale producer. This paper discussed to optimize the total inventory cost and optimum order quantity with one-price break, various decision variables. Numerical and sensitivity analysis of changes in decision variables is also discussed. Future models can be extended by considering multiple factors, such as multiple price discounts, providing random discounts, credit periods, varying holding cost, exponential demand etc.,

Numerical analysis and Sensitivity analysis on the decision variable changes is also discussed. The future model can be extended by considering various factors like multiple price break, random discount offering, credit periods, varying holding cost, exponential demand etc.,

REFERENCES

1. Arora, V., Chan, F.T.S., Tiwari, M.K., An integrated approach for logistic and vendor managed inventory in supply chain, Expert Syst. Appl., 37(2010), 39 – 44.
2. O. Fujiwara, H. Soewandi, D. Sedarage, An optimal and issuing policy for a two-stage inventory system for perishable products, European Journal of Operational Research, 99 (1997), 412-424.
3. Kit Nam Francis Leung, An integrated production inventory system in a multi - stage multi- firm supply chain, Transportation Research Part E, 46 (2010), 32-48.
4. Kaj - Mikael Bjork, A multi item fuzzy economic production quantity problem with a finite production rate, International Journal of Production Economics, 135 (2012), 702-707.
5. P. Muniappan, R. Uthayakumar, S. Ganesh, An EOQ model for deteriorating items with inflation and time value of money considering time-dependent deteriorating rate and delay payments, Systems Science & Control Engineering: An Open Access Journal, 3 (2015), 427–434.
6. M. Mahdi Tajbakhsh, Chi- Guhn Lee, Saeed Zolfaghari, An inventory model with random discount offerings, Omega, 39 (2011), 710-718.
7. P. Muniappan, R. Uthayakumar, S. Ganesh, A production inventory model for vendor–buyer coordination with quantity discount, backordering and rework for fixed life time products, Journal of Industrial and Production Engineering, 33 (2016), 355–362.
8. P. Muniappan, M. Ravithammal, S. Senthil, An incentive inventory model for exponential function of cost with maximum time of deteriorating products, International Journal of Recent Technology and Engineering, 7 (2019), 120 – 123.
9. Khan, M., Jaber, M.Y,Abdul-Rahim, A, An integrated supply chain model with errors in quality inspection and learning in production, Omega, 42(2014), 16 – 24.
10. Huang, C.K, An optimal policy for a single-vendor single buyer integrated production-inventory problem with process unreliability consideration, Ind, J. Prod. Econ., 91(2004), 91 – 98. Sharma Vikas, Chauhan Anand, Kumar Mukesh, EOQ Models with Optimal Replenishment Policy for Perishable Items taking Account of Time Value of Money, Indian Journal of Science and Technology, 9 (2016), DOI: 10.17485/ijst/2016/v9i25/46568.
11. Das, M., Pal, S., Ghosh, A.Association of metabolic syndrome with obesity measures, metabolic profiles, and intake of dietary fatty acids in people of Asian Indian origin(2010) Journal of Cardiovascular Disease Research, 1 (3), pp. 130-135.
    DOI: 10.4103/0975-3583.70911
12. Yongrui Duan, Jianwen Luo, Jiazhen Huo, Buyer-vendor inventory coordination with quantity discount incentive for fixed lifetime product, International Journal of Production Economics, 128 (2010), 351-357.
13. Umamaheswari, Chandrasekeran, Vijayalakshmi, Design and Analysis of an Optimal Inventory Model for Perishable Goods with Fixed Life Time, Indian Journal of Science and Technology, 9 (2016), DOI: 10.17485/ijst/2016/v9i25/85023.