Total domination in hesitancy fuzzy graphs

[1]R.Shakthivel, [2] R.Vikramaprasad and [3]N.Vinothkumar

[1] Department of Mathematics,Sona College of Technology, Salem - 636 005, Tamil Nadu, India.Email: shakthivelr@sonatech.ac.in.

[2] Department of Mathematics,Government Arts College, Salem-636 007, Tamil Nadu, India. Email: vikramprasad20@gmail.com.

[3]Department of Mathematics,Bannari Amman Institute of Technology, Erode - 638 401, Tamil Nadu, India.Email:vinoth@bitsathy.ac.in.

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ABSTRACT: Hesitant fuzzy sets introduced by Torra in 2010. Pathinathan et.al. Introduced Hesitancy fuzzy graph in 2015 and discussed various properties . Hesitancy Fuzzy Graphs (HFGs) has been applied to capture the common A.Prasanna, M.A.Rifayathali and S.Ismail Mohideen intricacy that occur during a selection of membership degree of an element from some possible values that make one to hesitate.

In this paper we present the idea oftotal domination number in Hesitancy fuzzy graph and examine some properties of total domination number in hesitancy fuzzy graphs and total domination number in operations of hesitancy fuzzy graphs like union, join, direct product investigated..

I. INTRODUCTION

A Hesitancy fuzzy graph $G(\mathrm{V},\mathrm{E})$, where the vertex set V is a triplet fuzzy functions it is defined by $\mu_1 : V \to [0,1]$, $\nu_1 : V \to [0,1]$ and $\beta_1 : V \to [0,1]$,these functions are called as membership, non-membership and hesitancy of the vertex $v_i \in V$ respectively and $\mu_1(v_i) + \nu_1(v_i) + \beta_1(v_i) = 1$, $\beta_1(v_i) = 1 - [\mu_1(v_i) + \nu_1(v_i)]$. The edge set of $G(\mathrm{V},\mathrm{E})$ is a triplet fuzzy functions it is defined by $\mu_2 : V \times V \to [0,1]$, $\nu_2 : V \times V \to [0,1]$ and $\beta_2 : V \times V \to [0,1]$, such that

$\mu_2(\mathrm{uv}) \le \mu_1(\mathrm{u}) \wedge \mu_1(\mathrm{v})$,

$\nu_2(\mathrm{uv}) \le \nu_1(\mathrm{u}) \vee \nu_1(\mathrm{v})$

$\beta_2(\mathrm{uv}) \le \beta_1(\mathrm{u}) \wedge \beta_1(\mathrm{v})$

and $0 \le \mu_2(\mathrm{uv}) + \nu_2(\mathrm{uv}) + \beta_2(\mathrm{uv}) \le 1$ for every $\mathrm{uv} \in \mathrm{E}$.

In a hesitancy fuzzy graph $G(V, E)$ there is a strong edge between every pair of vertices, then $G(V, E)$ is said to be as Complete hesitancy fuzzy graph.

A set D of V is said to be dominating set of a hesitancy fuzzy graph $G(A, B)$ if every $v \in V - D$ there exits $u \in D$ such that u dominates v.A dominating set D of a hesitancy fuzzy graph $G(A, B)$ is called minimal dominating set of G, if every node $v \in D$, $D - \{v\}$ is not a dominating set. The dominating number $\gamma_{hf}(G)$ of the hesitancy fuzzy graph $G(A, B)$ is the minimum cardinality taken over all minimal dominating set of G.

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Let $G(A,B)$ be a hesitancy fuzzy graph. A subset $T \subseteq V$ is said to be a total dominating set of a hesitancy fuzzy graph $G(A,B)$ if every $v \in V$ is adjacent to a vertex in T.

A total dominating set T of a hesitancy fuzzy graph $G(A,B)$ is called minimal total dominating set of G, if every subset of T is not a total dominating set. i.e every node $u \in T, T-\{u\}$, is not atotal dominating set.

The total dominating number $\gamma_T(G)$ of the hesitancy fuzzy graph $G(A,B)$ is the minimum cardinality taken over all minimal total dominating set of G.

II. TOTAL DOMINATING SET.

In this section some results on total domination number in various hesitancy fuzzy graphs.

Theorem 3.1

Let $G(A,B)$ be a complete hesitancy fuzzy graph,then $\gamma_T(G)=|u|$, here $d_N(u)=\Delta_N(G).$

Proof: Assume $G(A,B)$ be a complete hesitancy fuzzy graph. This implies that there is astrong edge between every pair of vertices. Let $u \in V$ such that $d_N(u)=\Delta_N(G).$ Therefore every vertex in $v \in V-u$ is adjacent to $u \in V$. Cleary $u \in V$ dominates every vertex in $v \in V-u$. Let $d_N(u)=\Delta_N(G).$ This implies $|u| \le |v|,$ for every vertex $v \in V-u$. Hence $u \in V$ totally dominates the complete hesitancy fuzzy graph $G(A,B)$. Therefore we get $\gamma_T(G)=|u|$. Hence proved.

Example 3.1

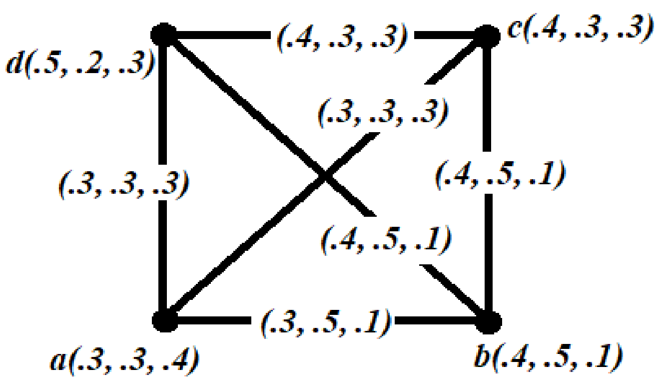

Figure 3.1: Complete hesitancy fuzzy graph $G(A,B)$

In the Complete hesitancy fuzzy graph $G(A,B)$, degree of the vertices are $d_N(a)=1.33,\ d_N(b)=1.47,$ $d_N(c)=1.33$ *and* $d_N(d)=1.47$ and $\Delta_N(G)=1.47$. The total domination number of the complete hesitancy fuzzy graph $G(A,B)$ is $\gamma_T(G)=|b|=0.33.$

Theorem 3.2

Let $G(A,B)$ be a connected hesitancy fuzzy graph with $N(u)=\{V-u\}$, then $\gamma_T(G)=O(G)-|u|$.

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Proof: Assume $G(A,B)$ be a connected hesitancy fuzzy graph. This implies there is no isolated vertex. Let $u \in V$ such that $N(u) = \{V-u\}$ Therefore every vertex in $v \in V-u$ is adjacent to $u \in V$. Cleary $u \in V$ dominates every vertex in $V-u$. Let $d_N(u) = \Delta_N(G).$ Hence the $u \in V$ totally dominates the hesitancy fuzzy graph $G(A,B)$. Therefore we get $\gamma_T(G) = |V-u| \Rightarrow O(G) - |u|$. Hence proved.

Theorem 3.3

Let $G(A,B)$ be a complete bipartite hesitancy fuzzy graph, then $\gamma_T(G) = |u| + |v|$, here $u\ and\ v$ having the minimum cardinality among the vertex sets in the set $V_1\ and\ V_2$ respectively.

Proof: Assume $G(A,B)$ be a complete bipartitehesitancy fuzzy graph. This implies there is a strong edge between every verticesin $V_1\ and\ V_2$. Let $u \in V_1$ and $v \in V_2$ such that $d_N(u) = \Delta_{V_1}(G)$ and $d_N(v) = \Delta_{V_2}(G)$ Therefore every vertex in $v_i \in V_2$ is adjacent to $u \in V_1$ and every vertex in $u_i \in V_1$ is adjacent to $v \in V_2$. Since $G(A,B)$ is a complete bipartite hesitancy fuzzy graph. Cleary $u\ and\ v$ dominates every vertex in $G(A,B)$. Assume $T = \{u,v\}$ is not a total dominating set of $G(A,B)$. This implies $w \in V - \{u,v\}$ is adjacent to more than a vertex i.evertex $w \in V - \{u,v\}$ is adjacent to $u\ and\ v$. This is contradict to our assumption $G(A,B)$ is a complete bipartite hesitancy fuzzy graph.Therefore we get $T = \{u,v\}$ is a total dominating set of $G(A,B)$. This implies $\gamma_T(G) = |T| = |u| + |v|$. Hence proved.

Example 3.2

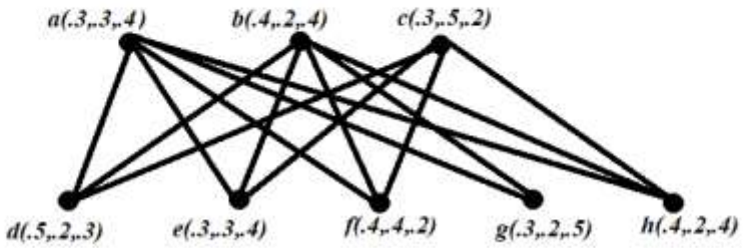


Figure 3.2: Complete Bipartite hesitancy fuzzy graph $G(A,B)$

In the Complete hesitancy fuzzy graph $G(A,B)$, degree of the vertices in V_1 and V_2 are $|a| = 0.47,\ |b| = 0.53,\ |c| = 0.33, |d| = 0.53,\ |e| = 0.47,\ |f| = 0.4,\ |g| = 0.33,\ |h| = 0.4$ .The total dominating set $T = \{c, g\}$ and the total domination number of the complete hesitancy fuzzy graph $G(A,B)$ is $\gamma_T(G) = |T| = |c| + |g| = 0.66.$

Theorem 3.4.

Let $G(V,E)$ be a hesitancy fuzzy graph and u is vertex having maximum degree of the graph $G(V,E)$, then $\gamma_T(G) \le O(G) - \Delta_N(G)$.

Proof: Let $G(V,E)$ be a hesitancy fuzzy graph and u is vertex having maximum degree of the graph $G(V,E)$, i.e $d_N(u) = \Delta_N(G).$ we prove $T = V - N(u)$ is a total dominating set of $G(V,E)$ .If $v \in T$ and $v \in N(\text{u})$ such that u totally dominates N(v). Note $v \notin N(u)$ therefore v totally dominates v and $w \in T - v$ are totally dominates itself. This implies we get $V-N(u)$ is a total dominating set of $G(V,E)$. Note $V-N(u)$ is not a minimal total dominating set. Therefore we get $\gamma_T(G)\le |V-N(u)| = O(G)-\Delta_N(G)$ Hence proved.

Definition 3.1. Let $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ are two hesitancy fuzzy graphs. The union of G_1 and G_2 is defined by

$$(\mu_{11}\cup\mu_{12})(u)=\begin{cases}\mu_{11}(u) & if\ u\in V_1\\ \mu_{12}(u) & if\ u\in V_2\end{cases}$$

$$(\upsilon_{11}\cup\upsilon_{12})(u)=\begin{cases}\upsilon_{11}(u) & if\ u\in V_1\\ \upsilon_{12}(u) & if\ u\in V_2\end{cases}$$

$$(\beta_{11}\cup\beta_{12})(u)=\begin{cases}\beta_{11}(u) & if\ u\in V_1\\ \beta_{12}(u) & if\ u\in V_2\end{cases}$$ and the edges of the form

$$(\mu_{12}\cup\mu_{22})(uv)=\begin{cases}\mu_{12}(uv)\ , & if\ uv\in E_1\\ \mu_{22}(uv)\ , & if\ uv\in E_2\end{cases}$$

$$(\upsilon_{12}\cup\upsilon_{22})(uv)=\begin{cases}\upsilon_{12}(uv)\ , & if\ uv\in E_1\\ \upsilon_{22}(uv)\ , & if\ uv\in E_2\end{cases}$$

$$(\beta_{12}\cup\beta_{22})(uv)=\begin{cases}\beta_{12}(uv)\ , & if\ uv\in E_1\\ \beta_{22}(uv)\ , & if\ uv\in E_2\end{cases}$$

Theorem 3.5

Let $G_1(A_1,B_1)\ and\ G_2(A_2,B_2)$ are two hesitancy fuzzy graphs . Let $T_1\ and\ T_2$ be the minimal total dominating sets of $G_1(A_1,B_1)\ and\ G_2(A_2,B_2)$ respectively. Then the total dominating number of $G_1\cup G_2$ is $\gamma_T(G_1\cup G_2)=|T_1|+|T_2|$.

Proof: Let $G_1(A_1,B_1)\ and\ G_2(A_2,B_2)$ are two hesitancy fuzzy graphs. Assume $T_1\ and\ T_2$ be the minimal total dominating sets of $G_1(A_1,B_1)\ and\ G_2(A_2,B_2)$ respectively. If every vertex $u\in G_1\cup G_2$ this implies $u\in G_1$ or $u\in G_2$ therefore there is a vertex $v\in T_1$ or $v\in T_2$ such that 'v' totally dominates $u\in G_1\cup G_2$. Since $T_1\ and\ T_2$ be the total dominating sets of $G_1(A_1,B_1)\ and\ G_2(A_2,B_2)$ respectively. The total dominating number of $G_1\cup G_2$ is $\gamma_T(G_1\cup G_2)=|T_1|+|T_2|$ .Hence proved.

Example 3.3

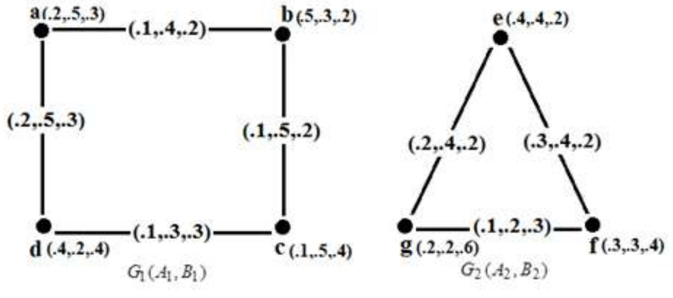
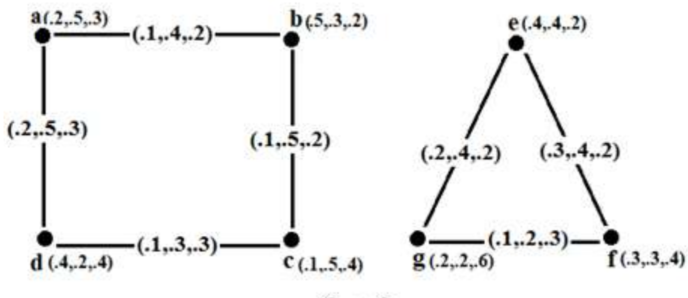


Figure3.3



$G_1 \cup G_2$

Figure3.4

In thehesitancy fuzzy graphs $G_1(A_1, B_1)$ *and* $G_2(A_2, B_2)$ .The total dominating sets are $T_1 = \{a, c\}$ *and* $T_2 = \{e\}$ .The total domination number of $G_1(A_1, B_1)$ *and* $G_2(A_2, B_2)$ are $\gamma_T(G_1) = |T_1| = |a| + |c| = 0.33 + 0.33 = 0.66$, $\gamma_T(G_2) = |T_2| = |e| = 0.4$. The total dominating set of $G_1 \cup G_2$ is $T = \{a, c, e\}$ and total dominating number $\gamma_T(G_1 \cup G_2) = |T| = 1.6$.

Definition 3.2 Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are two hesitancy fuzzy graphs. The join of G_1 and G_2 is defined by

$$(\mu_{11} + \mu_{12})(u) = \begin{cases} \mu_{11}(u) & if\ u \in V_1 \\ \mu_{12}(u) & if\ u \in V_2 \end{cases}$$

$$(\upsilon_{11} + \upsilon_{12})(u) = \begin{cases} \upsilon_{11}(u) & if\ u \in V_1 \\ \upsilon_{12}(u) & if\ u \in V_2 \end{cases}$$

$$(\beta_{11} + \beta_{12})(u) = \begin{cases} \beta_{11}(u) & if\ u \in V_1 \\ \beta_{12}(u) & if\ u \in V_2 \end{cases}$$

and Edge set E is defined by

$$(\mu_{12} + \mu_{22})(uv) = \begin{cases} \mu_{12}(uv)\ , & if\ uv \in E_1 \\ \mu_{22}(uv)\ , & if\ uv \in E_2 \\ \mu_{11}(u) \wedge \mu_{21}(v) & otherwise \end{cases}$$

$$(\upsilon_{12} + \upsilon_{22})(uv) = \begin{cases} \upsilon_{12}(uv)\ , & if\ uv \in E_1 \\ \upsilon_{22}(uv)\ , & if\ uv \in E_2 \\ \upsilon_{11}(u) \vee \upsilon_{21}(v) & otherwise \end{cases}$$

$$(\beta_{12} + \beta_{22})(uv) = \begin{cases} \beta_{12}(uv)\ , & if\ uv \in E_1 \\ \beta_{22}(uv)\ , & if\ uv \in E_2 \\ \beta_{11}(u) \wedge \beta_{21}(v) & otherwise \end{cases}$$

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Theorem3.6 The subsets $T_1 \subseteq V_1$ and $T_2 \subseteq V_2$ are the total dominating sets of the hesitancy fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. Let $T_1 = \{u\} or T_2 = \{u\}$ that is T_1 or T_2 is a singltonset. Then the total domination number $\gamma_T(G_1 + G_2) = |u|$.

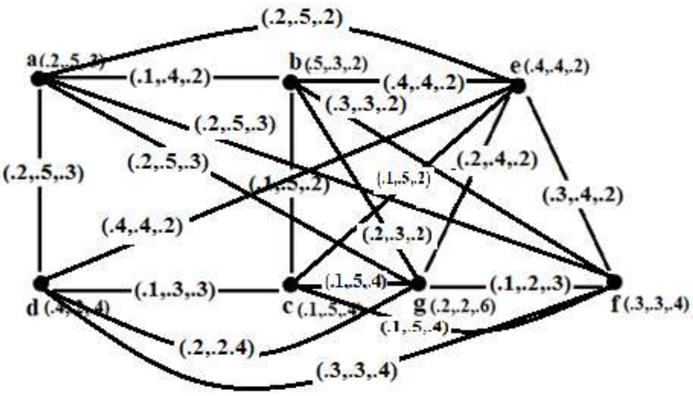
Proof: Let subsets $T_1 \subseteq V_1$ and $T_2 \subseteq V_2$ are the total dominating sets of the hesitancy fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. If $T_1 = \{u\}$ that is T_1 is a singltonset. The vertices in $G_1 + G_2$ are $V_1 \cup V_2$. Every vertices in $G_1(V_1, E_1)$ dominated by a vertex in T_1. Since T_1 is a singlton set. By the definition of $G_1 + G_2$ there is a strong edge between $T_1 = \{u\}$ and V_2. Therefore every vertices in $G_2(V_2, E_2)$ dominated by a vertex in T_1. This implies every vertex in $G_1 + G_2$ is dominated by a vertex in T_1. The total dominating number

$$\gamma_T(G_1 + G_2) = |T_1|.$$

$$\gamma_T(G_1 + G_2) = |u|$$

Similarly we prove If $T_2 = \{u\}$ that is T_2 is a singltonset.The total dominating number $\gamma_T(G_1 + G_2) = |u|$. Hence proved.

Example 3.4



$G_1 + G_2$

The total dominating set of $G_1 + G_2$ is $T = \{e\}$ and total dominating number $\gamma_T(G_1 + G_2) = |T| = 0.4$.

Definition 3.3 : The direct product of two Hesitancy Fuzzy Graphs G_1 and G_2 is defined as a Hesitancy Fuzzy Graphs $G = G_1 \cdot G_2$ where $V = V_1 \times V_2$ and with

$$E = \{(u_1u_2)(v_1v_2) | (u_1u_2) \in E_1 \ and \ (v_1v_2) \in E_2 \}$$

$$(\mu_{11} \bullet \mu_{21})(u_1u_2) = \mu_{11}(u_1) \wedge \mu_{21}(u_2)$$
$$(\upsilon_{11} \bullet \upsilon_{21})(u_1u_2) = \upsilon_{11}(u_1) \vee \upsilon_{21}(u_2)$$
$$(\beta_{11} \bullet \beta_{21})(u_1u_2) = 1 - [(\mu_{11}(u_1) \wedge \mu_{21}(u_2) + (\upsilon_{11}(u_1) \vee \upsilon_{21}(u_2)))]$$

and Edge set E is defined by if $(u_1u_2) \in E_1 \ and \ (v_1v_2) \in E_2$

$(\mu_{12} \bullet \mu_{22})(u_1u_2)(v_1v_2) = \mu_{12}(u_1u_2) \wedge \mu_{22}(v_1v_2)$
$(\upsilon_{12} \bullet \upsilon_{22})(u_1u_2)(v_1v_2) = v_{12}(u_1u_2) \vee \upsilon_{22}(v_1v_2)$
$(\beta_{12} \bullet \beta_{22})(u_1u_2)(v_1v_2) = \beta_{12}(u_1u_2) \wedge \beta_{22}(v_1v_2)$

Theorem 3.6

The subsets $T_1 \subseteq V_1$ and $T_2 \subseteq V_2$ are the total dominating setsof the hesitancy fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. Then $\gamma_T(G_1 \bullet G_2)$ is minum of $|V_1 \times T_2|$ *and* $|T_1 \times V_2|$ that is $\gamma_T(G_1 \bullet G_2) = \min\{|V_1 \times T_2|, |T_1 \times V_2|\}$.

Proof: Let $T_1 \subseteq V_1$ and $T_2 \subseteq V_2$ are the total dominating sets of the hesitancy fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. Therefore every vertex $v_1 \in V_1$ and $v_2 \in V_2$ are adjacent to a vertex in T_1 *and* T_2 respectively. Now we prove $V_1 \times T_2$ *and* $T_1 \times V_2$ are total dominating set of $G_1 \bullet G_2$. In $G_1 \bullet G_2$ the edges of the form $(u_1u_2)(v_1v_2) \in G_1 \times G_2$ if $(u_1u_2) \in E_1$ and $(v_1v_2) \in E_2$

If the edges $(u_1u_2) \in E_1$ and $(v_1v_2) \in E_2$ are strong edges implies

$(\mu_{12} \bullet \mu_{22})((u_1u_2)(v_1v_2)) = \mu_{12}(u_1v_2) \wedge \mu_{22}(u_1v_2)$
$= \mu_{11}(u_1) \wedge \mu_{11}(v_1) \wedge \mu_{21}(u_2) \wedge \mu_{21}(v_2)$
$= \mu_{11}(u_1) \wedge \mu_{21}(u_2) \wedge \mu_{11}(v_1) \wedge \mu_{21}(v_2)$
$= (\mu_{12} \bullet \mu_{22})(u_1u_2) \wedge (\mu_{12} \bullet \mu_{22})(v_1v_2)$

$(\upsilon_{12} \bullet \upsilon_{22})((u_1u_2)(v_1v_2)) = \upsilon_{12}(u_1v_2) \vee \upsilon_{22}(u_1v_2)$
$= \upsilon_{11}(u_1) \vee \upsilon_{11}(v_1) \vee \upsilon_{21}(u_2) \vee \upsilon_{21}(v_2)$
$= \upsilon_{11}(u_1) \vee \upsilon_{21}(u_2) \vee \upsilon_{11}(v_1) \vee \upsilon_{21}(v_2)$
$= (\upsilon_{12} \bullet \upsilon_{22})(u_1u_2) \vee (\upsilon_{12} \bullet \upsilon_{22})(v_1v_2)$

$(\beta_{12} \bullet \beta_{22})((u_1u_2)(v_1v_2)) = \beta_{12}(u_1v_2) \wedge \beta_{22}(u_1v_2)$
$= \beta_{11}(u_1) \wedge \beta_{11}(v_1) \wedge \beta_{21}(u_2) \wedge \beta_{21}(v_2)$
$= \beta_{11}(u_1) \wedge \beta_{21}(u_2) \wedge \beta_{11}(v_1) \wedge \beta_{21}(v_2)$
$= (\beta_{12} \bullet \beta_{22})(u_1u_2) \wedge (\beta_{12} \bullet \beta_{22})(v_1v_2)$

This implies that the edges $(u_1u_2)(v_1v_2) \in G_1 \times G_2$ is a strong edge such that $(u_1u_2) \in E_1$ and $(v_1v_2) \in E_2$ are strong edges in $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively.First we prove $V_1 \times T_2$ *and* $T_1 \times V_2$ are the total dominating set of $G_1 \bullet G_2$. The vertices in $G_1 \bullet G_2$ of the form $V_1 \times V_2$. Let $(u_1v_1) \in V_1 \times (V_2 - T_2)$, $T_2 \subseteq V_2$ is the total dominating setof the hesitancy fuzzy graph $G_2(V_2, E_2)$. Therefore there is a vertex $u_1w_1 \in V_1 \times T_2$ is adjacent to (u_1v_1) since $T_2 \subseteq V_2$ is the total dominating sets.This implies $V_1 \times T_2$ is a total dominating set of $G_1 \bullet G_2$.Similarly prove $T_1 \times V_2$ is a total dominating set of $G_1 \bullet G_2$. The total dominating number $\gamma_T(G_1 \bullet G_2)$ is minimunum of $|V_1 \times T_2|$ *and* $|T_1 \times V_2|$ that is $\gamma_T(G_1 \bullet G_2) = \min\{|V_1 \times T_2|, |T_1 \times V_2|\}$. Hence proved.

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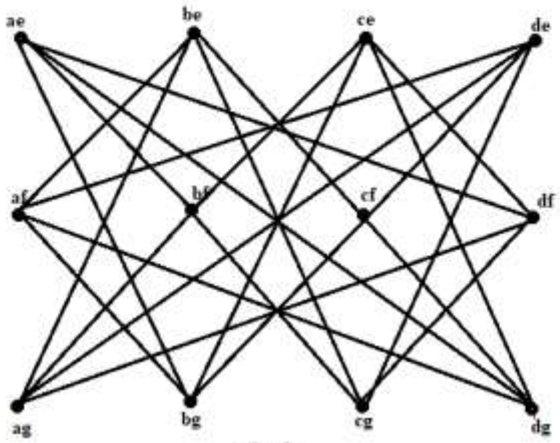
Example 3.5



$G_1 \bullet G_2$

The membership, non membership and hesitansy membership values of the vertices are ae(.2,.5,.3), be(.4,.4,2), ce(.1,.5,.4), de(.4,.4,.2) ,af(.2,.5,.3) bf(.3,.3,.4), cf(.1,.5,.4), df(.3,.3,.4), ag(.2,.5,.3), bg(.2,.3,.5), cg(.1,.5,.4) and dg(.2,.2,.6).The membership, non membership and hesitansy membership values of the edges are

Edge	value	Edge	Value	Edge	Value
(ae)(bf)	(.1,.4,.2)	(ag)(be)	(.1,.4,.2)	(bg)(ce)	(.1,.5,.2)
(ae)(bg)	(.1,.4,.2)	(ag)(bf)	(.1,.4,.2)	(bg)(cf)	(.1,.5,.2)
(ae)(df)	(.2,.5,.2)	(ag)(de)	(.2,.5,.2)	(ce)(df)	(.1,.4,.2)
(ae)(dg)	(.2,.5,.2)	(ag)(df)	(.1,.5,.3)	(ce)(dg)	(.1,.4,.2)
(af)(be)	(.1,.4,.2)	(be)(cf)	(.1,.5,.2)	(cf)(de)	(.1,.4,.2)
(af)(bg)	(.1,.4,.2)	(be)(cg)	(.1,.5,.2)	(cf)(dg)	(.1,.3,.3)
(af)(de)	(.2,.5,.2)	(bf)(ce)	(.1,.5,.2)	(cg)(de)	(.1,.4,.2)
(af)(dg)	(.1,.5,.3)	(bf)(cg)	(.1,.5,.2)	(cg)(df)	(.1,.3,.3)

In thehesitancy fuzzy graphs $G_1(A_1, B_1)$ *and* $G_2(A_2, B_2)$ .The total dominating sets are $T_1 = \{a, c\}$ *and* $T_2 = \{e\}$ .The total dominating set of $G_1 \bullet G_2$ are $T = \{ae, be, ce, de\}$ and The total dominating number $\gamma_T(G_1 \bullet G_2) = 1.46$.

Definition 3.4 : The Cartesian product of of two Hesitancy Fuzzy Graphs G_1 and G_2 is defined as a Hesitancy Fuzzy Graphs $G_1 \times G_2$ where $V = V_1 \times V_2$ and with

$$E = \{(u_1u_2)(v_1v_2) | (u_1u_2) \in E_1 \ and \ (v_1v_2) \in E_2 \}$$

$$(\mu_{11} \times \mu_{21})(u_1u_2) = \mu_{11}(u_1) \wedge \mu_{21}(u_2)$$
$$(\upsilon_{11} \times \upsilon_{21})(u_1u_2) = \upsilon_{11}(u_1) \vee \upsilon_{21}(u_2)$$
$$(\beta_{11} \times \beta_{21})(u_1u_2) = 1 - [(\mu_{11}(u_1) \wedge \mu_{21}(u_2) + (\upsilon_{11}(u_1) \vee \upsilon_{21}(u_2)))]$$

and Edge set E is defined by if $(u_1u_2) \in E_1$ *and* $(v_1v_2) \in E_2$

$$(\mu_{12}\times\mu_{22})((\mathrm{u}_1\mathrm{u}_2)(\mathrm{v}_1\,v_2))=\begin{cases}\mu_{11}(u_1)\wedge\mu_{22}(u_2v_2)\,if\ u_1=v_1\\ \mu_{21}(u_2)\wedge\mu_{12}(u_1v_1)\,if\ u_2=v_2\\ 0\quad otherwise\end{cases}$$

$$(\nu_{12}\times\nu_{22})((\mathrm{u}_1\mathrm{u}_2)(\mathrm{v}_1\,v_2))=\begin{cases}\nu_{11}(u_1)\vee\nu_{22}(u_2v_2)\,if\ u_1=v_1\\ \nu_{21}(u_2)\vee\nu_{12}(u_1v_1)\,if\ u_2=v_2\\ 0\quad otherwise\end{cases}$$

$$(\beta_{12}\times\beta_{22})((\mathrm{u}_1\mathrm{u}_2)(\mathrm{v}_1\,v_2))=\begin{cases}\beta_{11}(u_1)\wedge\beta_{22}(u_2v_2)\,if\ u_1=v_1\\ \beta_{21}(u_2)\wedge\beta_{12}(u_1v_1)\,if\ u_2=v_2\\ 0\quad otherwise\end{cases}$$

Theorem3.7. The subsets $T_1\subseteq V_1$ and $T_2\subseteq V_2$ are the total dominating sets of the hesitancy fuzzy graphs $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ respectively. Then the total dominating number is $\gamma_T(G_1\times G_2)=\min\{|V_1\times T_2|\,,|T_1\times V_2|\}$.

Proof: Let the sets $D_1\ and\ D_2$ are covering sets of the hesitancy fuzzy graphs $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ respectively. The edges $((u_1u_2)(\mathrm{v}_1\,v_2))\,if\ u_1=v_1\ and\,u_2v_2\in E_2$ in $G_1\times \mathrm{G}_2$

If $u_1=v_1\ and\,u_2v_2\in E_2$ is a strong edges. Therefore we get

$$\begin{aligned}(\mu_{12}\times\mu_{22})((\mathrm{u}_1\mathrm{u}_2)(\mathrm{v}_1\,v_2))&=\mu_{11}(u_1)\wedge\mu_{22}(u_2v_2)\\ &=\mu_{11}(u_1)\wedge\mu_{21}(u_2)\wedge\mu_{21}(v_2)\\ &=\mu_{11}(u_1)\wedge\mu_{21}(u_2)\wedge\mu_{11}(u_1)\wedge\mu_{21}(v_2)\end{aligned}$$

$$(\mu_{12}\times\mu_{22})((\mathrm{u}_1\mathrm{u}_2)(\mathrm{v}_1\,v_2))=(\mu_{11}\times\mu_{21})(u_1u_2)\wedge(\mu_{11}\times\mu_{21})(\mathrm{v}_1\,v_2)$$

$$\begin{aligned}(\nu_{12}\times\nu_{22})((\mathrm{u}_1\mathrm{u}_2)(\mathrm{v}_1\,v_2))&=\nu_{11}(u_1)\vee\nu_{22}(u_2v_2)\\ &=\nu_{11}(u_1)\vee\nu_{21}(u_2)\vee\nu_{21}(v_2)\\ &=\nu_{11}(u_1)\vee\nu_{21}(u_2)\vee\nu_{11}(u_1)\vee\nu_{21}(v_2)\end{aligned}$$

$$(\nu_{12}\times\nu_{22})((\mathrm{u}_1\mathrm{u}_2)(\mathrm{v}_1\,v_2))=(\nu_{11}\times\nu_{21})(u_1u_2)\vee(\nu_{11}\times\nu_{21})(\mathrm{v}_1\,v_2)$$

$$\begin{aligned}(\beta_{12}\times\beta_{22})((\mathrm{u}_1\mathrm{u}_2)(\mathrm{v}_1\,v_2))&=\beta_{11}(u_1)\wedge\beta_{22}(u_2v_2)\\ &=\beta_{11}(u_1)\wedge\beta_{21}(u_2)\wedge\beta_{21}(v_2)\\ &=\beta_{11}(u_1)\wedge\beta_{21}(u_2)\wedge\beta_{11}(u_1)\wedge\beta_{21}(v_2)\end{aligned}$$

$$(\beta_{12}\times\beta_{22})((\mathrm{u}_1\mathrm{u}_2)(\mathrm{v}_1\,v_2))=(\beta_{11}\times\beta_{21})(u_1u_2)\wedge(\beta_{11}\times\beta_{21})(\mathrm{v}_1\,v_2)$$

Therefore we get the edges $((u_1u_2)(\mathrm{v}_1\,v_2))\,if\ u_1=v_1\ and\,u_2v_2\in E_2$ are strong edges in $G_1\times \mathrm{G}_2$.Similarly the edges $((u_1u_2)(\mathrm{v}_1\,v_2))\,if\ u_2=v_2\ and\,u_1v_1\in E_1$ are strong edges in $G_1\times \mathrm{G}_2$ .First we prove $V_1\times T_2\ and\ T_1\times V_2$ are the total dominating set of $G_1\times \mathrm{G}_2$. The vertices in $G_1\times \mathrm{G}_2$ of the form $V_1\times V_2$. Let $(u_1v_1)\in V_1\times(V_2-T_2)$, $T_2\subseteq V_2$ is the total dominating set of the hesitancy fuzzy graph $G_2(V_2,E_2)$.

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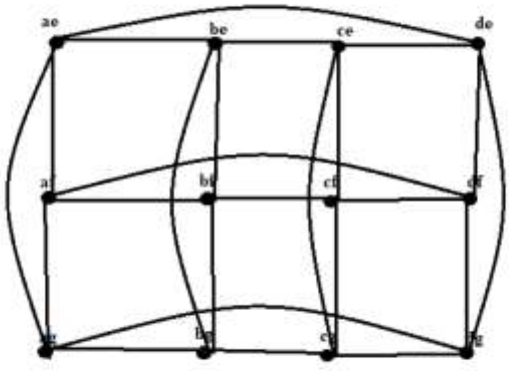
Therefore there is a vertex $u_1w_1 \in V_1 \times T_2$ is adjacent to (u_1v_1) since $T_2 \subseteq V_2$ is the total dominating sets.This implies $V_1 \times T_2$ is a total dominating set of $G_1 \bullet G_2$.Similarly prove $T_1 \times V_2$ is a total dominating set of $G_1 \times G_2$. The total dominating number $\gamma_T(G_1 \times G_2)$ is minimum of $|V_1 \times T_2|$ *and* $|T_1 \times V_2|$ that is $\gamma_T(G_1 \times G_2) = \min\{|V_1 \times T_2|, |T_1 \times V_2|\}$. Hence proved.

Example 3.6



$G_1 \times G_2$

The membership, non membership and hesitansy membership values of the vertices are ae(.2,.5,.3), be(.4,.4,2), ce(.1,.5,.4), de(.4,.4,.2) ,af(.2,.5,.3) bf(.3,.3,.4), cf(.1,.5,.4), df(.3,.3,.4), ag(.2,.5,.3), bg(.2,.3,.5), cg(.1,.5,.4) and dg(.2,.2,.6).The membership, non membership and hesitansy membership values of the edges are

Edge	value	Edge	Value	Edge	Value
(ae)(be)	(.1,.4,.2)	(ag)(bg)	(.1,.4,.2)	(be)(bg)	(.2,.4,.2)
(ae)(de)	(.2,.5,.2)	(ag)(dg)	(.2,.5,.3)	(bf)(bg)	(.1,.3,.2)
(be)(ce)	(.1,.5,.2)	(bg)(cg)	(.1,.5,.2)	(ce)(cf)	(.1,.5,.2)
(ce)(de)	(.1,.4,.2)	(cg)(dg)	(.1,.3,.3)	(ce)(cg)	(.1,.5,.2)
(af)(bf)	(.1,.4,.2)	(ae)(af)	(.2,.5,.2)	(cf)(cg)	(.1,.5,.3)
(af)(df)	(.2,.5,.3)	(ae)(ag)	(.2,.5,.2)	(de)(df)	(.3,.4,.2)
(bf)(cf)	(.1,.5,.2)	(af)(ag)	(.1,.5,.3)	(de)(dg)	(.2,.4,.2)
(cf)(df)	(.1,.3,.3)	(be)(bf)	(.3,.4,.2)	(df)(dg)	(.1,.2,.3)

In thehesitancy fuzzy graphs $G_1(A_1, B_1)$ *and* $G_2(A_2, B_2)$ .The total dominating sets are $T_1 = \{a, c\}$ *and* $T_2 = \{e\}$ .The total dominating set of $G_1 \times G_2$ are $T = \{ae, be, ce, de\}$ and The total dominating number $\gamma_T(G_1 \bullet G_2) = 1.46$.

III. Conclusion

Further we present the idea ofvarious domination number in Hesitancy fuzzy graph and examine some properties of these domination number in hesitancy fuzzy graphs and total domination number in operations of hesitancy fuzzy graphs like union, join, direct product investigated.

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