JOURNAL OF CRITICAL REVIEWS

ISSN- 2394-5125 VOL 7, ISSUE 04, 2020

SOME ALGORITHMS FOR EQUITABLE EDGE COLORING OF SOME SILICATE NETWORK GRAPHS

R VIKRAMA PRASAD [1], K SIVARAMAN [2]

[1] Department of Mathematics,
Government Arts College (Autonomous), Salem – 636007, Tamilnadu, India.
[2] Research Scholar, Department of Mathematics, Periyar University, Salem – 636011, Tamilnadu, India.

Received: 14 Feb 2020 Revised and Accepted: 25 March 2020

ABSTRACT: An Equitable Edge Coloring of G is a proper edge coloring of G such that the number of edges of one color class differs from the number of edges of any other color class by atmost one. In this Paper we discuss the Equitable edge coloring of some silicate Network Graphs.

KEYWORDS: Silicate graph, Flower Graph, Fan Graph, Equitable Edge Coloring.

I. INTRODUCTION

Let G be a finite simple graph. It is defined as a non-empty set of points V together with the set of all unordered pair of vertices E called edge set. A molecular graph is a graph representing the atoms of the molecule by vertices and the covalent bond between them by edges.

II. Preliminaries and Basic Definitions:

(i) Proper Edge Coloring: [10]

An Edge Coloring of a graph G is an assignment of colors to the edges of G. An Edge Coloring of G is called Proper K Coloring if no two adjacent edges have assigned same color.

(ii) Equitable Edge Coloring: [10]

A Proper Edge Coloring of G is called Equitable if $|l(s) - l(t)| \leq 1$wherel(s) and l(t) denote the number of edges in the color class s and t respectively.

(iii) Nearly Equitable Edge Coloring: [10]

A Proper Edge Coloring of G is called Nearly Equitable if $|l(s) - l(t)| \leq 2$wherel(s) and l(t) denote the number of edges in the color class s and

t respectively.

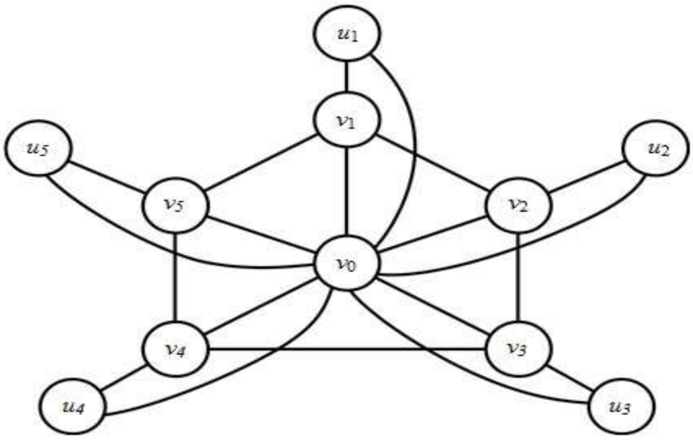
(iv) Molecular Graph: [10]

A Molecular graph is a graph G=(V,E), where V is the set of all nonhydrogen atoms in particular carbon items and E is the set of all covalent bonds between them.

(v) Flower Graph Fl_n: [3]

A flower graph denoted by Fl_n is defined as a graph obtained from the Helm graph by joining each pendant vertex to the central vertex of the helm.

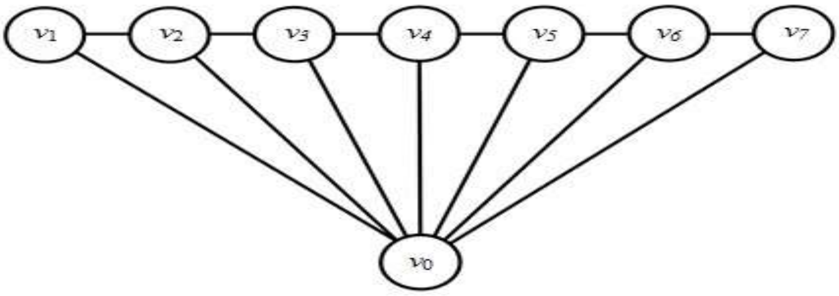
Example: Fl_5



(vi) Fan Graph f_n : [8]

A fan graph denoted by f_n is defined as the join of a single vertex called central vertex with the path graph P_n on n vertices.

Example : f_7



LEMMA 1[10]

Every simple undirected graph may be Edge coloured using a number of colours that is atmost one larger than the maximum degree Δ of the graph.

III. MAIN RESULTS

Algorithm 1:

Construction algorithm for flower- silicate network graph. [3] &[10]

Step 1: Consider a Cyclic graph C_n,n≥3 of length "n" whose vertices be designated as $v_1, v_2, \dots v_n$.

Step 2: Insert a new vertex say v_0 inside the cyclic graph C_n and join all the vertices of the cyclic graph C_n to the vertex v_0 graph to form the wheel graph W_n

Step 3: For each triangular face of the wheel graph insert a new vertex w_i, i=1,2,3,…n.

Then join the newly added vertex w_i to all the vertices in that particular face of the wheel graph. This is repeated for all the faces of the wheel graph W_n.So we get a cyclic Silicate network graph.

Step 4 : Add pendant edges to each vertex v_i ,i=1,2,3,….n and let the other end of these newly added pendant edges be u_i ,i=1,2,3,….n Then join the vertex u_i with the vertex w_i i=1,2,3…n

Observations1: In the above graph we have number of vertices =$|V|$=3n+1, number of edges = $|E|$= 7n and Sum of the degree of all the vertices of the graph G= 14n.Degree of the vertices , $v_2, v_3, \dots v_n v_1$ are equals to 6.

Degree of the central vertex V_0 is 2n. Degree of the vertices $W_1, W_2, \ldots W_n$ are equal to 4 and degree of the vertices u_i,i=1,2,3,….n is 2.So atleastΔColour needed for equitableedge coloring of this graph where Δ=4n.

Theorem 1 :

The flower silicate Network admits Equitable edge colouring and its equitable edge chromatic number is 2n
Proof:

Let G be a Flower Silicate network graph.

By Lemma 1[10] we need at least Δ colors for proper edge coloring of this graph. Now color the edges of by the mapping f: E(G) C where C={1,2,3,…2n}such that

$$f(uv=e)=\begin{cases} 2\,i \; for\; u=v_i \;\; and\;\; v=v_{i+1} \; i=1,2,3,..n-1 \\ 2n\;\; for\; u=v_n\; and\; v=v_1 \\ 2i-1 for\; u=v_0\; and\; v=v_i\; i=1,2,3\ldots n \\ 2j-1\; for\; u=w_j\; and\; v=v_{i+1,}i,j=1,2,3,\ldots n \\ 2j\; for\; u=v_0\; and\; v=w_j, j=1,2,3\ldots n \\ 2i+1\; for\;\; u=w_j\; and\; v=v_{j,}, j=1,2,3,\ldots n-1 \\ 1\; for\; u=w_n\; and\; v=v_{n,} \\ 2i+2\; for\; u=u_i\; and\; v=v_i\; i=1,2,3,\ldots n-1 \\ 2\;\; for\; u=u_n\; and\; v=v_{n,} \\ 2i+3\; or\; u=u_i\; and\; v=u_{i+1\,,}i=1,2,3\ldots n-2 \\ 1\; for\;\; or\; u=w_{n-1}\; and\; v=u_{n,} \\ 3\; for\; u=w_n\; and\; v=u_{1,} \end{cases}$$

From this mapping we get $|l(s)-l(t)|\leq 1$, where l(s) and l(t) denotes the number of edges in the color classes s and t respectively.Thus the graph G admits equitable edge coloring.Also its equitable edge chromatic number is χ_e(G)= 2n.

Algorithm 2:

Construction algorithm for Fan-Silicate Graph

Step 1: Draw the Fan graph with n+1 vertices $v_0, v_1, v_2, \ldots v_n$ where v_0 is the central vertex, n$\geq$ 3.

Step 2: Introduce n-1 new vertices u_i , I = 1,2,3,…n-1.

Step 3: Join the vertex v_0 to each of the vertex u_i,i=1,2,3,…n-1.

Step 4: Also to each vertex u_i, i=1,2,3,…n-1 join v_i &v_{i+1} ,i=1,2,3,…n-1.

Step 5: Thus we get a graph called Fan-Silicate Graph.

Observations 2:

In the above graph we have number of vertices =$|V|$=2n, number of edges =

$|E|$= 5n-4 and Sum of the degree of all the vertices of the graph G= 10n-8 Degree of the vertices v_1 and v_n are equal to 3.Degree of the vertices $v_2, v_3, \ldots v_{n-1}$ are equals to 5. Degree of the central vertex v_0 is 2n-1. Degree of the vertices $u_1, u_2, \ldots u_n$ are equal to 3 .So atleastΔ Color needed for equitable edge coloring of this graph where

Δ=4n.

Theorem 2 :

The Fan-Silicate graph G admits equitable edge coloring with equitable edge chromatic number χ_e (G) =2n-1.

Proof :

ISSN- 2394-5125 VOL 7, ISSUE 04, 2020

Let G be a Fan-Silicate graph. By Lemma 1[10] we need atleast Δ colors to coloring of the edges of G properly. So we need 2n-1 colours.

Now color the edges of G by the function

f: E(G)→ C where C={1,2,3,…2n} such that

$$f(uv)=\begin{cases} 2\,i-1 \;\; if \;\; u=v_0 \;\; and \;\; v=v_i, i=1,2,3,..n \\ 2i \;\; if \;\; u=v_0 \; and \; v=v_i, i=1,2,3,\ldots n-1 \\ 2i-1 \; if \;\; u=v_{i+1} \; and \; v=u_i, i=1,2,3\ldots n-1 \\ 2i+1 \; if \;\; u=v_i \; and \; v=u_i, i=1,2,3,\ldots n-1 \\ 2i \; for \; u=v_i \;\; and \; v=v_{i+1}, \; i=1,2,3\ldots n-1 \end{cases}$$

From this mapping we get $|l(s)-l(t)|\leq 1$ where l(s) and l(t) denote the number of edges in the color class s and t respectively and this proves that the fan-silicate graph admits the equitable edge coloring and its equitable edge chromatic number is χ_e' (G) =2n-1.

IV. CONCLUSION:

Thus in this paper, we have discussed algorithms for two silicate related graphs and also discussed about their equitable edge coloring.

V. REFERENCES:

[1] J. A Bondy and U.S.R. Murthy, Graph Theory with Applications, New York; The Macmillan Press Ltd,(1976).

[2] G Chartrand, L.Lesniak, Graphs & Digraphs, New York Chapman & Hall/CRC First CRC press reprint 2000.

[3] Eunice Mphako-Banda Some polynomials of Flower graphs, International Mathematical Forum 2, 2007, no. 51 pages 2511-2518.

[4] Frank Harary, Graph Theory Macmillan Press Ltd (1964)

[5] Gallain J. A Dynamic Survey of Graph Labeling The Electronic Journal of Combinatorics 22nd Edition December 15 2019, Classification: 05C78

[6] W.Meyer,Equitable Colouring, American Mathematical Monthly,80(1973),920-922.

[7] V.S. Shegegalli and R. Kanabur New version of degree-based topologocal indices of certain nanotube J.Math. Nanoscience 6(1) (2016) 29-42

[8] J . Vernold Vivion, M. Venkatachalam A note on b-coloring of Fan graphs , Journal of Discrete Mathematical Sciences and Cryptography , Volume 17 2014, Issue 5-6 pages 443-448.

[9] Vidya Ganesan, Indra Rajasingh Strong Chromatic index of certain anosheets, Journal of Mathematical Nano Science 7(1) (2017) 29-38

[10] R Vikrama Prasad, K. Sivaraman Algorithms For Equitable Edge Coloring of Cyclic Silicate Network Graphs,Advance in Mathematics Scientific Journal 9 (2020), no.3, 1281-1287.