THERMAL EFFECT OF WATER BASED CNT NANOFLUID DURING SPIN COATING

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Received: 20 Feb 2020 Revised and Accepted: 23 April 2020

Abstract

In this paper, we have discussed the thermal effect of thin film flow of nanoliquid over a rotating disk. In this work, we have discussed the flow and film thickness variation of water based CNT nanofluid over a rough rotating disk. Here we see the effect of two different kinds of carbon nanotubes with the base fluid water namely: "single wall carbon nanotubes (SWCNTs)" and "multi wall carbon nanotubes (MWCNTs)". Here the thermal effect of film development over a rotating disk is studied analytically.

Keywords: Volume fraction, Nanofluid, Spin coating, film thickness.

1 INTRODUCTION

"Thin film flow is characterized as a thin layer of fluid flowing over a surface beneath the activity of outside strengths such as gravity, shear stress etc. and having at least one free boundary". The flow of thin liquid film is imperative for the understanding and plan of different warm exchangers and chemical handling hardware. Applications include wire and fiber coating, polymer processing etc. We are living in the electronic age and using electronic gadgets frequently in our daily life. Many of these gadgets have the hard disk, screen, electronic circuit, etc. In manufacturing of these gadgets, the coating is required in many processing steps. Apart from this, coating of any surface or equipments is necessary to enhance its efficiency of work, durability, heat transfer performance/cooling performance, etc. All coating prepare requests a smooth gleaming wrap up to meet the necessities for best appearance and ideal execution such as moo contact, straightforwardness and quality. The rate of warm and mass exchange inside the lean fluid film features a coordinate bearing on the victory of the coating handle and chemical characteristics of the item. In the coating process, we are covering any surface or substrate by a thin uniform layer of liquid. "The thin layer of liquid can be developed over a surface by many ways e.g., liquid flow over a stretching surface, spin coating liquid flow over a rotating disk, spreading of a liquid, spray coating, blade coating, thin film flow over a incline plane etc. The mechanics of covering the surface of a horizontal rotating disk with the help of a very thin uniform film by applying the centrifugal force is known as spin coating". Nowadays, this method is broadly used in magnetic disk for data storage, small scale gadgets industry to fabricate the incorporated circuits, optical mirrors and colour television screens etc. Again nanoliquid is a liquid having nanoparticles that is nano meter sized particles. These liquids are designed colloidal suspensions of nanoparticles in a base liquid. These type of fluids are the heat transfer fluids containing metals, oxides etc. The term nanofluids first used by Choi and shows the anomalous increase of thermal conductivity of the conventional fluids. On a circular rotating disk the flow of a thin fluid film was first given by Emslie et al. [1]. Meyerhofer [2] gave us a model for the portrayal of thin films prepared in spin coating process. Flack, Soong, Bell and Hess [3] derived a mathematical model for non-newtonian fluid thin film formation. In asymptotic method the flow of unsteady fluid and film development was first considered by Higgins [4]. In 1991 a theoretical study was given for the unsteady non-Newtonian flow by Lawrence and Zhou [5]. Choi [6] first investigated the thermal conductivity of nanofluids. Then, Choi et al. [7] showed that the heat transfer of nanoliquids increase two times the thermal conductivity of the fluid. In 2001 Kwang and Jung [8] give us a numerical study of thick photoresist film formation on a spin coating process. Usha and Ravindran [9] gave us a numerical result for the fluid film which conduct heat. Charpin, Lombe and Myers [10] studied the flow of ellis fluid during spin coating process. Dandapat and Maity [11] gave us both numerical and analytical result for the flow of pure liquid in the presence of air interface and evaporation. Dandapat and singh[12] also gives us the result that when we increase the transverse magnetic field there is a decrease in thinning rate of two layer fluid film. The display investigation falls within the advancement of a thin film on the surface of a damp rotating disk from a fluid blob, which infers the inalienable presumption of a damp disk to encourage the utilize of the no-slip boundary condition. In this work we have discussed the flow and film thickness variation of water based CNT nanofluid over a rough rotating disk. Here we see the effect of two different kind of carbon nanotubes within the base fluid namely: "single wall carbon nanotubes (SWCNTs)" and "multi wall carbon nanotubes (MWCNTs)".

2. MATHEMATICAL ANALYSIS

ISSN- 2394-5125 VOL 7, ISSUE 04, 2020

Let us consider initial the film thickness be h_0. Here we consider full set of Navier - Stokes equations and the vitality condition for the investigation. And we are doing the spin coating process with the water based CNT nanofluid as the coating liquid. Here we consider the velocity components are (ψ_1, ψ_2, ψ_3) for the cylindrical coordinates (r, θ, z). The set of governing equations are given by as follows-

$$\frac{\partial \psi_1}{\partial r} + \frac{\psi_1}{r} + \frac{\partial \psi_3}{\partial z} = 0 \tag{1}$$

$$\rho_{nf}\left(\frac{\partial \psi_1}{\partial t} + \psi_1 \frac{\partial \psi_1}{\partial r} + \psi_3 \frac{\partial \psi_1}{\partial z} - \frac{{\psi_2}^2}{r}\right) = -p_r + \mu_{nf}\left[\frac{\partial^2 \psi_1}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{\psi_1}{r}\right) + \frac{\partial^2 \psi_1}{\partial z^2}\right] \tag{2}$$

$$\rho_{nf}\left(\frac{\partial \psi_2}{\partial t} + \psi_1 \frac{\partial \psi_2}{\partial r} + \psi_3 \frac{\partial \psi_2}{\partial z} + \frac{\psi_1 \psi_2}{r}\right) = \mu_{nf}\left[\frac{\partial^2 \psi_2}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{\psi_2}{r}\right) + \frac{\partial^2 \psi_2}{\partial z^2}\right] \tag{3}$$

$$\rho_{nf}\left(\frac{\partial \psi_3}{\partial t} + \psi_1 \frac{\partial \psi_3}{\partial r} + \psi_3 \frac{\partial \psi_3}{\partial z}\right) = -p_z + \mu_{nf}\left[\frac{\partial^2 \psi_3}{\partial r^2} + \frac{\frac{\partial \psi_3}{\partial r}}{r} + \frac{\partial^2 \psi_3}{\partial z^2}\right] \tag{4}$$

$$(\rho C_p)_{nf}\left(\frac{\partial \overline{T}}{\partial t} + \psi_1 \frac{\partial \overline{T}}{\partial r} + \psi_3 \frac{\partial \overline{T}}{\partial z}\right) = k_{nf}\left(\frac{\partial^2 \overline{T}}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{T}}{\partial r} + \frac{\partial^2 \overline{T}}{\partial z^2}\right) \tag{5}$$

Where "the pressure is represented by p, ρ_{nf} is the density, μ_{nf} is the dynamic viscosity, $(\rho C_p)_{nf}$) is thermal conductivity and k_{nf} is the heat capacity of the nanoliquid", which are defined as follows:

$$\rho_{nf} = (1-\phi)\rho_w + \phi\rho_{CNT}$$

$$\mu_{nf} = \frac{\mu_w}{(1-\phi)^{\frac{5}{2}}}$$

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_w + \phi(\rho C_p)_{CNT}$$

$$k_{nf} = k_w\left(\frac{(1-\phi) + \frac{2\phi k_{CNT}}{k_{CNT}-k_w}\log\left(\frac{k_{CNT}+k_w}{2k_w}\right)}{(1-\phi) + \frac{2\phi k_w}{k_{CNT}-k_w}\log\left(\frac{k_{CNT}+k_w}{2k_w}\right)}\right)$$

Table 1: Properties of different nanoparticles and base fluid

		ρ	C_p	K
Base fluid	Water	997	4179	0.613
Nanofluid	SWCNT	2600	425	6600
Nanofluid	MWCNT	1600	796	3000

On the surface of the disk no slip boundary condition i.e. at z=0

$$\psi_1 = 0,\ \psi_2 = r\omega,\ \psi_3 = 0,\ T = \theta(r) \tag{6}$$

At the free surface z=h(t)

$$-\bar{p}+\frac{2\mu_{nf}}{1+h_r^{\,2}}[h_r^{\,2}\psi_1 r-h_r(\psi_1 z+\psi_3 r)+\psi_3 z]=\frac{\sigma}{r}[\frac{h_r(1+h_r^{\,2})+rh_{rr}}{(1+h_r^{\,2})^{3/2}}] \quad (7)$$

$$\frac{\mu_{nf}}{(1+h_r^{\,2})^{1/2}}[2h_r(\psi_1 r-\psi_3 z)+(h_r^{\,2}-1)(\psi_3 r+\psi_1 z)]=-(\sigma_r+h_r\sigma_z) \quad (8)$$

$$\frac{\partial\psi_2}{\partial z}-rh_r(\frac{\psi_2}{r})_r=0 \quad (9)$$

$$h_t+\psi_1 h_r=\psi_3 \quad (10)$$

$$-k_{nf}[T_z-h_r T_r]=\alpha(T-T_0)\sqrt{1+h_r^{\,2}} \quad (11)$$

The initial condition at time t=0 are

$$\psi_1=\psi_2=\psi_3=0\,,\ h(r,0)=\delta r\,,\ T=T_0 \quad (12)$$

Using the dimensionless variables

$$\hat{t}=\frac{t}{t_c},\ \hat{r}=\frac{r}{R_0},\ \hat{z}=\frac{z}{h_0},\ \hat{\delta}=\frac{\delta}{h_0},\ \hat{h}=\frac{h}{h_0},\ \hat{\psi}_1=\frac{\psi_1}{\psi_{1_0}},\ \hat{\psi}_2=\frac{\psi_2}{U_0/\sqrt{\varepsilon\,\mathrm{Re}}},\ \hat{\psi}_3=\frac{\psi_3}{\varepsilon U_0},$$

$$\hat{p}=\frac{h_0^{\,2}t_c}{\nu_f R_0^{\,2}\rho_f}\bar{p},\ U_0=\frac{R_0}{t_c}=\frac{R_0\omega^2 H_0^{\,2}}{\nu_f},\ t_c=\frac{\nu_f}{(\omega H_0)^2},\ \hat{T}=\frac{\bar{T}-\bar{T_0}}{\bar{T_{d_0}}-\bar{T_0}}$$

Finally we get the non-dimensionalized governing set of equations as-

$$\frac{\partial\hat{\psi}_1}{\partial\hat{r}}+\frac{\hat{\psi}_1}{\hat{r}}+\frac{\partial\hat{\psi}_3}{\hat{\partial}\hat{z}}=0 \quad (13)$$

$$\varepsilon\,\mathrm{Re}\,\phi_1[\frac{\partial\hat{\psi}_1}{\partial\hat{r}}+\hat{\psi}_1\frac{\partial\hat{\psi}_1}{\partial\hat{r}}+\hat{\psi}_3\frac{\partial\hat{\psi}_1}{\hat{\partial}\hat{z}}]=-(1-\phi)^{2.5}\frac{\partial\hat{p}}{\partial\hat{r}}+\phi_1\frac{\hat{\psi}_2^{\,2}}{\hat{r}}+\varepsilon^2[\frac{\partial^2\hat{\psi}_1}{\hat{\partial}\hat{r}^2}+(\frac{\psi_1}{\hat{r}})_{\hat{r}}]+\frac{\partial^2\hat{\psi}_1}{\partial\hat{z}^2} \quad (14)$$

$$\varepsilon\,\mathrm{Re}\,\phi_1[\frac{\partial\hat{\psi}_2}{\partial\hat{r}}+\hat{\psi}_1\frac{\partial\hat{\psi}_2}{\partial\hat{r}}+\hat{\psi}_3\frac{\partial\hat{\psi}_2}{\hat{\partial}\hat{z}}+\frac{\psi_1\psi_2}{\hat{r}}]=[\varepsilon^2(\frac{\partial^2\hat{\psi}_2}{\hat{\partial}\hat{r}^2}+(\frac{\psi_2}{\hat{r}})_{\hat{r}})+\frac{\partial^2\hat{\psi}_2}{\partial\hat{z}^2}] \quad (15)$$

$$\varepsilon^3\,\mathrm{Re}\,\phi_1[\frac{\partial\hat{\psi}_3}{\partial\hat{r}}+\hat{\psi}_1\frac{\partial\hat{\psi}_3}{\partial\hat{r}}+\hat{\psi}_3\frac{\partial\hat{\psi}_3}{\hat{\partial}\hat{z}}]=-(1-\phi)^{2.5}\frac{\partial\hat{p}}{\partial\hat{z}}+\varepsilon^2[\varepsilon^2(\frac{\partial^2\hat{\psi}_3}{\hat{\partial}\hat{r}^2}+\frac{1}{r}\frac{\partial\hat{\psi}_3}{\partial\hat{r}})+\frac{\partial^2\hat{\psi}_3}{\partial\hat{z}^2}]-\varepsilon Fr\phi_1 \quad (16)$$

$$\varepsilon\,\mathrm{Re}\,r\phi_2[\frac{\partial\hat{T}}{\partial\hat{r}}+\hat{\psi}_1\frac{\partial\hat{T}}{\partial\hat{r}}+\hat{\psi}_3\frac{\partial\hat{T}}{\hat{\partial}\hat{z}}]=\frac{k_{nf}}{k_f}[\varepsilon^2(\frac{\partial^2\hat{T}}{\hat{\partial}\hat{r}^2}+\frac{1}{\hat{r}}\frac{\partial\hat{T}}{\partial\hat{r}})+\frac{\partial^2\hat{T}}{\partial\hat{z}^2}] \quad (17)$$

Where $\mathrm{Pr}=\frac{\nu_f}{\alpha_f}$ is the prandlt number and

$\phi_1=(1-\phi)^{\frac{5}{2}}[(1-\phi)+\phi\frac{\rho_{CNT}}{\rho_w}]$, $\phi_2=[(1-\phi)+\phi\frac{(\rho C_p)_{CNT}}{(\rho C_p)_w}]$ are dimensionless constants.

And the boundary conditions are transformed as follows:

At $\eta=0$

$$\hat{\psi}_1=0,\hat{\psi}_2=r,\hat{\psi}_3=0,\hat{T}=\theta(r) \quad (18)$$

At $\eta=H(\tau)$

$$-p+\frac{2\varepsilon^2(1-\phi)^{2.5}}{(1+\varepsilon^2\hat{h}_{\hat{r}}^{\ 2})}[\varepsilon^2\hat{h}_{\hat{r}}\hat{\psi}_{1\hat{r}}-\hat{h}_{\hat{r}}(\hat{\psi}_{1_{\hat{z}}}+\varepsilon^2\hat{\psi}_{3_{\hat{r}}})+\hat{\psi}_{3_{\hat{z}}}]=\frac{\varepsilon}{\hat{r}}[We-\varepsilon^2\alpha\hat{T}][\frac{\hat{h}_{\hat{r}}(1+\varepsilon^2\hat{h}_{\hat{r}})+\hat{r}\hat{h}_{\hat{r}\hat{r}}}{(1+\varepsilon^2\hat{h}_{\hat{r}}^{\ 2})^{3/2}}] \quad (19)$$

$$2\varepsilon^2\hat{h}_{\hat{r}}(\hat{\psi}_{3_{\hat{z}}}-\hat{\psi}_{1_{\hat{r}}})+(1-\varepsilon^2\hat{h}_{\hat{r}})(\varepsilon^2\hat{\psi}_{3_{\hat{r}}}+\hat{\psi}_{1_{\hat{z}}})=-(1-\phi)^{2.5}\varepsilon\alpha(\hat{T}_{\hat{r}}+\hat{h}_{\hat{r}}\hat{T}_{\hat{z}})\sqrt{1+\varepsilon^2\hat{h}_{\hat{r}}^{\ 2}} \quad (20)$$

$$\hat{\psi}_{2_{\hat{z}}}-\varepsilon^2\hat{r}\hat{h}_{\hat{r}}(\frac{\hat{\psi}_2}{\hat{r}})_{\hat{r}}=0 \quad (21)$$

$$\frac{k_{nf}}{k_w}(\hat{T}_{\hat{z}}-\varepsilon^2\hat{h}_{\hat{r}}\hat{T}_{\hat{r}})=-Bi\hat{T}\sqrt{1+\varepsilon^2\hat{h}_{\hat{r}}^{\ 2}} \quad (22$$

$$-\frac{\partial\hat{h}}{\partial\hat{t}}-\hat{\psi}_1\frac{\partial\hat{h}}{\partial\hat{r}}+\hat{\psi}_3=\frac{2}{3}E\sqrt{1+\varepsilon^2\hat{h}_{\hat{r}}^{\ 2}} \quad (23)$$

Where "the Reynolds number $\mathrm{Re}=\omega^2 L h_0^{\ 3}/\nu_0^{\ 2}$, Froude number $Fr=g/\omega^2 L$, the Prandlt number $\Pr=\nu_f/\alpha_f$, the Weber number $We=\sigma_0/\rho\omega^2 L^3$, and the Biot number $Bi=\beta h_0/\lambda$. $\beta,\lambda,\sigma,\alpha$ Signify the heat exchange coefficient, thermal conductivity, surface pressure and thermocapillary parameter". To get an asymptotic arrangement, all the subordinate factors are extended in powers of ε as taking after-

$$(\psi_1,\psi_2,\psi_3,p)=(\psi_{1_0},\psi_{2_0},\psi_{3_0},p_0)+\varepsilon(\psi_{1_1},\psi_{2_1},\psi_{3_1},p_1)+o(\varepsilon^2)$$

On putting the over frame for every dependent variable within the framework of conditions and comparing terms of like orders, new set of conditions are gotten as regular. We get-

$$\psi_1=\frac{1}{2}\phi_1(2h-z)rz+\varepsilon[\mathrm{Re}\,\phi_1^{\ 2}r(\frac{1}{2}h_t(\frac{z^3}{3}-h^2z)+\phi_1(\frac{z^6}{360}-\frac{hz^5}{60}+\frac{2}{9}h^3z^3-\frac{3}{5}h^5z+\frac{1}{6}rhh_r$$
$$(\frac{z^4}{4}-h^3z)))+Fr\phi_1(\frac{h_rz^2}{2}-hh_rz)-We(1-\phi)^{2.5}(\frac{1}{r}(rh_r)_r)_r(\frac{z^2}{2}-hz)+k\alpha(1-\phi)^{2.5}(\frac{\theta}{k+Bih})_r z]$$

$$\psi_2=r+\varepsilon[2\,\mathrm{Re}\,\phi_1^{\ 2}r(\frac{hz^3}{6}-\frac{z^4}{24}-\frac{h^3z}{3})]$$

$$\psi_3=\phi_1(\frac{z^3}{3}-hz^2)-\frac{1}{2}\phi_1rh_rz^2+\varepsilon[-2\,\mathrm{Re}\,\phi_1^{\ 2}[\frac{1}{2}h_t(\frac{z^4}{12}-\frac{h^2z^2}{2})+\phi_1[\frac{z^7}{2520}\frac{hz^6}{360}+\frac{1}{9}h^3z^2-\frac{3}{10}h^5z^2+$$
$$\frac{1}{6}rhh_r(\frac{z^5}{20}-\frac{h^3z^2}{2})]]-\mathrm{Re}\,\phi_1^{\ 2}r[1/2h_{t_r}(\frac{z^4}{12}-\frac{h^2z^2}{2})-hh_th_r\frac{z^2}{2}+\phi_1[\frac{-h_rz^6}{360}+1/6h^2h_rz^4-3/2h^4h_rz^2$$
$$+1/6(rhh_r)_r(\frac{z^5}{20}-\frac{h^3z^2}{2})-1/4h^3h_r^{\ 2}z^2]]-[Fr\phi_1h_{rr}-We(1-\phi)^{2.5}1/r(rh_r)_{rr}[1/r(rh_r)_{rr}(h_r+rh_{rr})$$
$$-\frac{1}{r^2}(rh_r)_r]](\frac{z^3}{6}-\frac{hz^2}{2})+[Fr\phi_1h_r-We(1-\phi)^{2.5}[1/r(rh_r)_r]][h_r\frac{z^2}{2}-1/r(\frac{z^3}{6}-\frac{hz^2}{2})]-k\alpha(1-\phi)^{2.5}$$
$$[(\frac{\theta}{k+Bih})_r\frac{z^2}{2r}+(\frac{\theta}{k+Bih})_{rr}[\frac{(k+Bih)\theta_r-Bih_r\theta}{(k+Bih)^2}]z]]$$

$$T=\frac{k+Bi(h-z)}{k+Bih}\theta(r)+\varepsilon[\mathrm{Re}\,\mathrm{Pr}\,\phi_2(\frac{Bi^2h_t\theta}{(k+Bih)^2}(\frac{z^3}{6k}-\frac{(3k+Bih)}{(k+Bih)}\frac{h^2}{6k}z)+\phi_1 r\theta_r(\frac{hz^3}{6k}-\frac{z^4}{24k}$$
$$-\frac{h^3z}{24k}\frac{8k+5Bih}{k+Bih})+\frac{Bi(h_t\theta-\theta_r)}{k+Bih}\phi_1 r(\frac{hz^4}{12k}-\frac{z^5}{40k}-\frac{25k+7Bih}{k+Bih}\frac{h^4z}{120k})-\frac{Bi\theta\phi_1}{k+Bih}(\frac{z^5}{60k}-\frac{hz^4}{12k}-\frac{rh_tz^4}{24k}$$
$$+\frac{30hk-20rh_tk+8h^2Bi+5Birh_rh}{120k}\frac{h^3}{k+Bih})))]$$

Now we integrate the equation (18) and the following equation we obtained the required equation

$$h_t+\psi_1 h_r=\psi_3 \tag{24}$$

We have

$$\frac{\partial h}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}\int(r\psi_1)dz=0 \tag{25}$$

From equation (30) and equation(33) we get the following expression

$$h_t=-\frac{1}{3r}\phi_1(r^2h^3)_r-\frac{\varepsilon}{r}[\mathrm{Re}\,\phi_1^{\,2}r^2(-\frac{5}{24}h_th^4+\phi_1(-\frac{311}{1260}h^7-\frac{3}{40}rh^6h_r))-\frac{1}{3}Fr\phi_1rh_rh^3$$
$$+\frac{1}{3}We(1-\phi)^{2.5}rh^3(\frac{1}{r}(rh_r)_r)_r+k\alpha r(1-\phi)^{2.5}(\frac{\theta}{k+Bih})_r\frac{h^2}{2}]_r \tag{26}$$

2 SOLUTION PROCESS

Now we expand the expression for film thickness in powers of ε as

$$h(r,t)=h_0(r,t)+\varepsilon h_1(r,t)+o(\varepsilon^2) \tag{27}$$

We get

$$h_{0_t}+\phi_1rh_0^{\,2}h_{0_r}=-\frac{2}{3}\phi_1h_0^{\,3} \tag{28}$$

$$h_{1_t}+\phi_1rh_0^{\,2}h_{1_r}=-2\phi_1(h_0^{\,2}+rh_0^{\,2}h_{0_r}h_1)-\frac{1}{r}[\mathrm{Re}\,\phi_1^{\,2}r^2(-\frac{5}{24}h_{0_t}h_0^{\,4}+\phi_1(-\frac{311}{1260}h_0^{\,7}-\frac{3}{40}rh_0^{\,6}h_{0_r}))$$
$$-\frac{1}{3}Fr\phi_1rh_{0_r}h_0^{\,3}+\frac{1}{3}We(1-\phi)^{2.5}rh_0^{\,3}(\frac{1}{r}(rh_{0_r})_r)_r+k\alpha r(1-\phi)^{2.5}(\frac{\theta}{k+Bih_0})_r\frac{h_0^{\,2}}{2}]_r$$

And

$$\frac{d}{dt}h_0[r(t),t]=-\frac{2}{3}\phi_1h_0^{\,3}[r(t),t]$$

Along the characteristics curve r(t) satisfying

$$\frac{d}{dt}r(t)=\phi_1r(t)h_0^{\,2}[r(t),t]$$

$$h_0[r(t),t]=C_0\chi^{-1/2} \tag{28}$$

Along with

$$r(t)=C_1\chi^{\frac{3}{4\phi_1}}$$

Where $\chi=1+(4/3)\phi_1C_0^{\,2}t$. It also follows that

$$r(t)h_0^{3/2}[r(t),t] = C_1 C_0^{3/2} = \text{Constant}$$

Similarly we get

$$h_1[r(t),t] = \frac{k\alpha}{2}(1-\phi)^{2.5} C_1^{-1}\chi^{-3/4}(\theta_r\varsigma^{-1} + \frac{2}{3}BiC_0C_1^{-1}\chi^{-5/4}\theta(r)\varsigma^{-2}) + \frac{62}{315}\mathrm{Re}\,\phi_1^{\,2}C_0^{\,5}\chi^{-5/2}$$
$$-\frac{2}{9}Fr\phi_1 C_0^{\,2}C_1^{\,2}\chi^{-5/2} + \frac{32}{81}We(1-\phi)^{2.5}C_0^{\,2}C_1^{-4}\chi^{-4} + C_2\chi^{-1/2} \quad (29)$$

Where $\varsigma = k + BiC_0\chi^{-1/2}$

3 RESULT AND DISCUSSION

In this article we described the flow of water based CNT nanofluids and also discuss the thermal effect of CNT nanofluids. Here we take the nanoparticles “SWCNT” and “MWCNT” with water. In the figure 1 we have shown the variation of film thickness for different nanoparticles SWCNT and MWCNT. From the figure 1 we have seen that in case of SWCNT the width of the film is high than the MWCNT. In figure 2 we have shown the discrepancy of film thinness for diverse values of ϕ for both SWCNT and MWCNT. From these figure we conclude that the width of the film is high for higher values of ϕ and we also observe that in case of MWCNT the film thinning rate is high as compared to SWCNT. Figure 3 describes the discrepancy of film thickness for different values of α for SWCNT. From this figure we can conclude that the width of film is low for higher values of α . In figure 4 we have shown the discrepancy of film thickness for diverse values of biot number. In figure 4 we observe that for increasing values of Biot number Bi the film thickness increases. The variety of the film thickness h and r given by equation (28) and (29), for temperature dissemination $\theta(r) = e^{(-r^2/2)}$ and times, is portrayed in figure 5. We can say from the 5th figure that the film diminishes quicker for temperature diminishing radially outwards than for a consistent distribution. Here we considered $\alpha = 1.0, \mathrm{Re} = 0.1, Fr = 0.5, \phi = 0.1$.

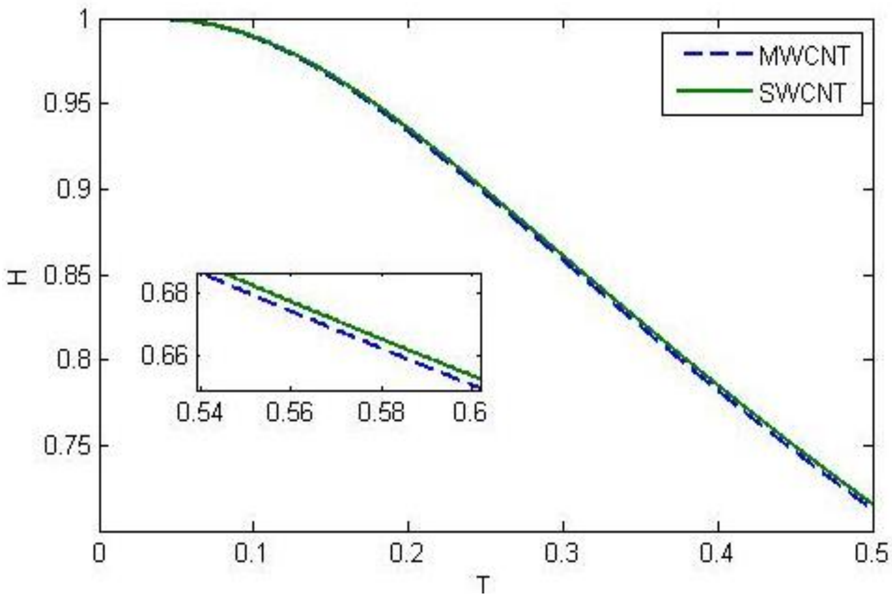

Figure 1: Film thickness variation for the nanoparticles SWCNT and MWCNT when Re=0.1, Pr=0.5

ISSN- 2394-5125 VOL 7, ISSUE 04, 2020

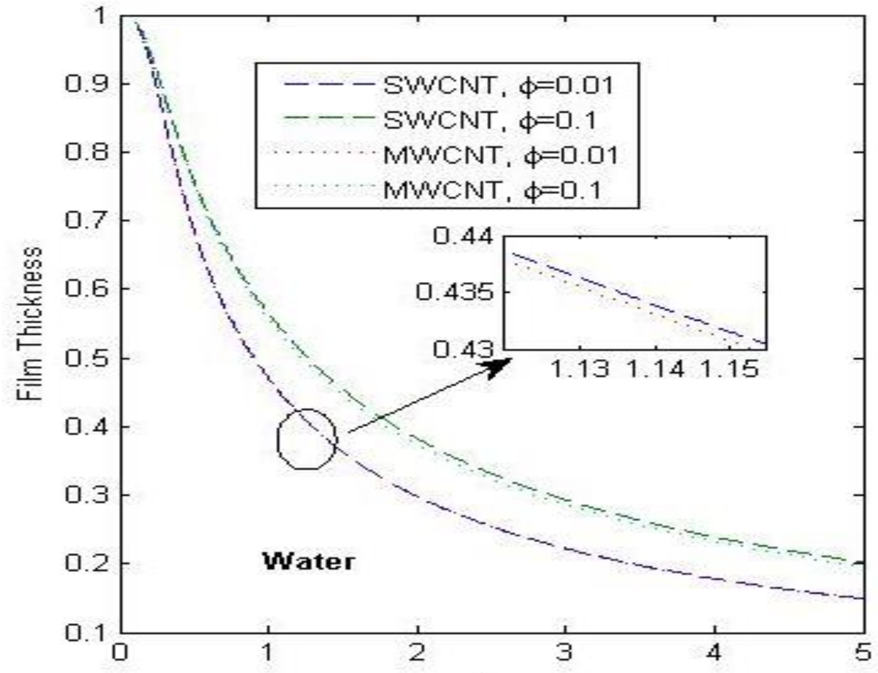

Figure 2: Film thickness variation for nanoparticles SWCNT and MWCNT for different values of ϕ (Re=0.1)

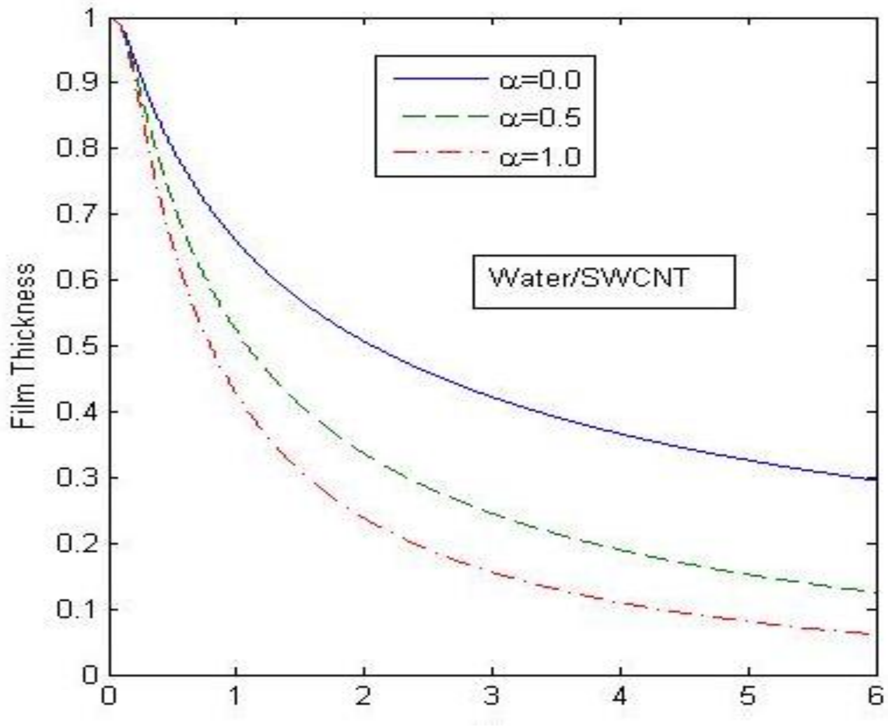

Figure 3: Film thickness variation for the nanoparticle SWCNT with the base fluid water for different values of α (Re=0.1)

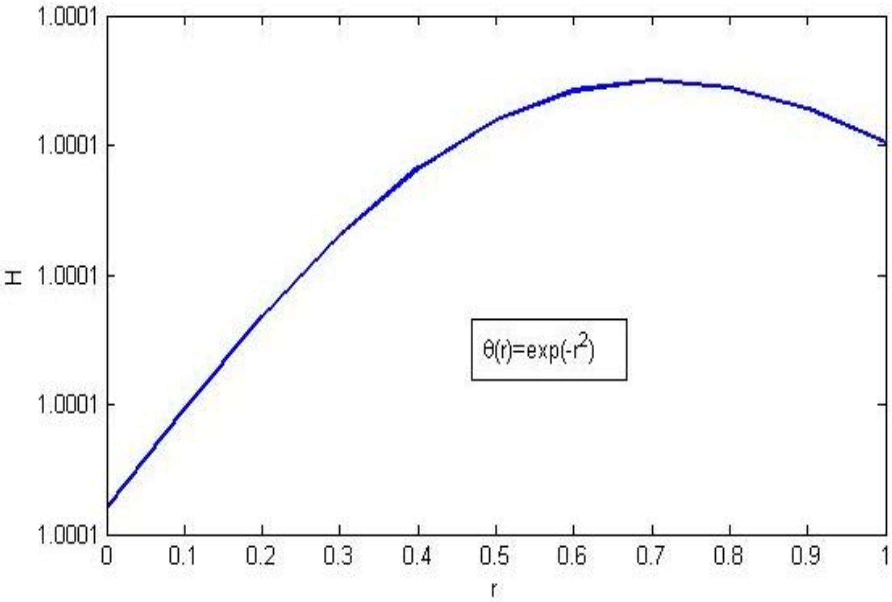

Figure 4: The variation of film thickness for different values of biot number

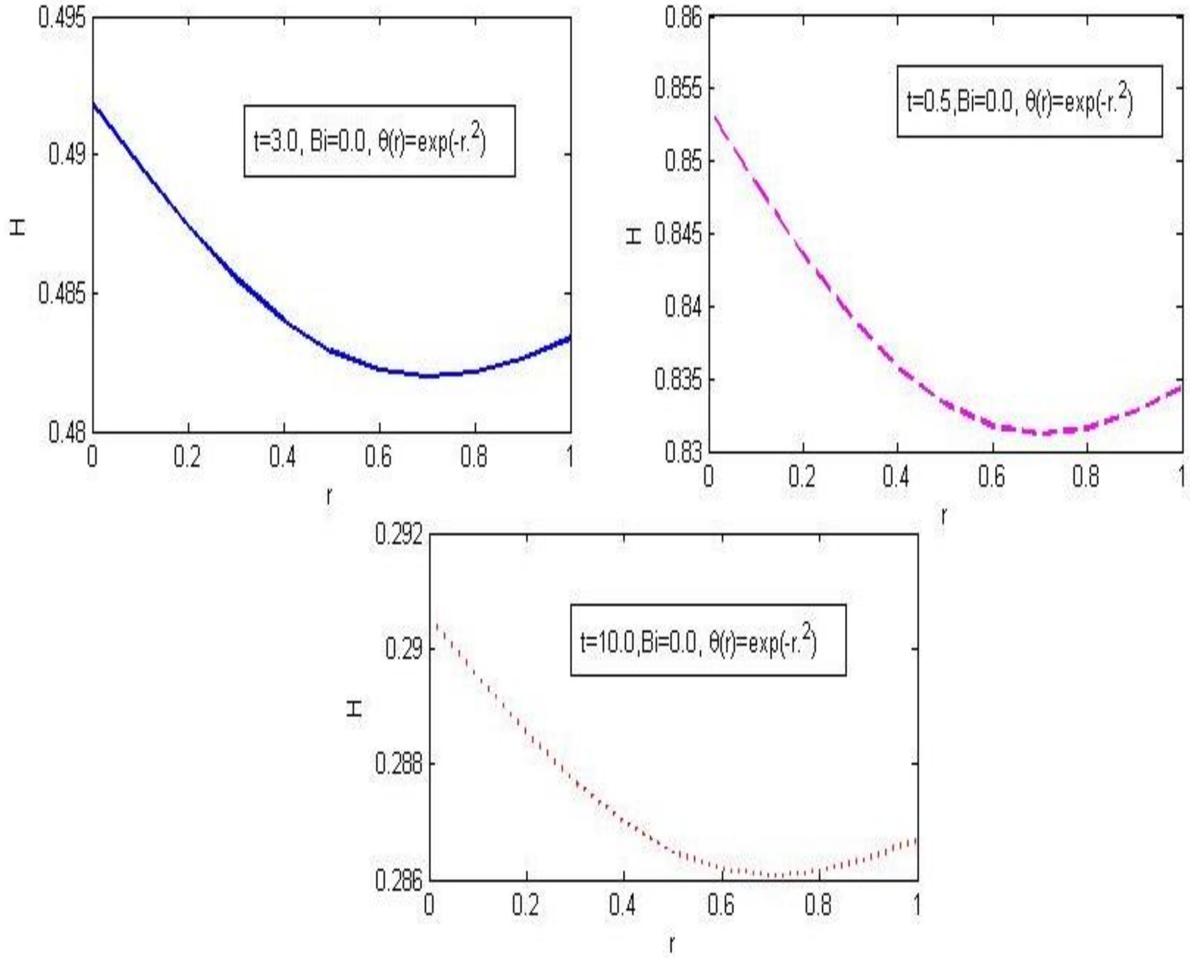

Figure 5: Variation of film thickness H with radial distance r from the axis of rotation

CONCLUSION

In this article we have researched the flow of SWCNT and MWCNT based water as nanoliquid on a rotating disk during spin coating process and we also observe the thermal effect of CNT nanofluid in film thinning process. Here we solve the problem by analytical method. The following conclusions are made from this article for CNT

ISSN- 2394-5125 VOL 7, ISSUE 04, 2020

nanofluid.

(i) “When we increase the value of volume fraction the film thinning rate decreases”.

(ii) “In case of SWCNT the thickness of the film is high as compared to MWCNT”.

(iii) “The film diminishes quicker for temperature diminishing radially outwards than for a consistent distribution”.

(iv) “When we increase the value of thermocapillary parameter the thickness of the film also increases”.

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