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Transportation Problem with Stochastic Demand and Supply

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Abstract

In the usual transportation model takes care of the defined supply and demand along with cost. In this model the researcher allows the supply and demand be stochastic. This model incorporates the production cost, transportation cost and selling price for each unit at various points. The optimum solution is obtained using the simulated demand and simulated supply by converting the transportation model into a linear programming model and the same is solved using TORA package.

Keywords: Transportation problem; Optimal Solution; Simulation; Mathematics Subject Classification: 90C08, 90C90

1. Introduction

1.1 Operations Research

In order to understand what OR is[P.Mariappan,2002], one must know its history and evolution. It is broadly concurred that the field began during World War II. Many strategic and technical problems linked with military effort were highly complicated. In response to these complex problems, groups of scientists with varied educational backgrounds assembled as special units within the armed forces.

Since the scientists are talented men, pressure sues to war time necessity and the synergism generated from the interactions of different works, they were remarkably successful in improving the effectiveness of complex military operations. By 1941, each of the three wings of the British Armed Forces was utilizing such scientist teams. Due to the immediate success of the idea, other allied nations chose the same approach and organized their own teams.Because the problems assigned to these groups were in the nature of military operations, their work was called Operational Research in U.K. and in other nations called Operational Research. Even though the Americans put their effort at a later date, produced many mathematical techniques for analyzing military problems.

After the World War II, many soldiers belong to the military OR groups turned their attention to the possibilities of applying similar approaches to day to day problems in the society.Initially in a large-scale profit-making organization like petroleum companies, and so on have introduced this OR techniques. Then slowly the researchers designed the model to suit even the small industries. Nowadays, one can apply this OR technique even for the day problems of whatever size and any class.Today OR is having more applications in Engineering, Business Management, Agriculture, Transporting system and hence on.

It is very much interesting to note that the modern perception of Operations Research as a body of well-built models and techniques. A modification of this kind is to be required in any emerging area of scientific research.

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1.2 Model Building using Linear Programming

Assumptions

The model of linear programming is based on linearity (which contains proportionality and additivity) certainty and divisibility assumptions.

1.2.1 Linearity:

It demands that the objective function and the constraints of linear programming problems lead themselves to a mathematical formulation in linear form.The term proportionality means that the contribution of each decision variable to the value of the objective part is proportional to the value chosen by the decision variable. Also, additive implies that the overall contributions of the objective are obtained by the algebraic sum of the individual contributions.

1.2.2 Certainty:

It implies that the parameters $c_j, b_j \; and \; a_{ij} (for \; i = 1.2, \dots, m; j = 1,2, \dots, n)$ of a linear programming model are fixed, i.e. known with certainty.

1.2.3 Divisibility:

It demands that the decision variables are continuous, such that they can hold whatever value (fractional or integer), based on the limitations imposed on the decision variables.

1.3 Phase of Operations Research Methodology

The O.R phases can be depicted as follows:

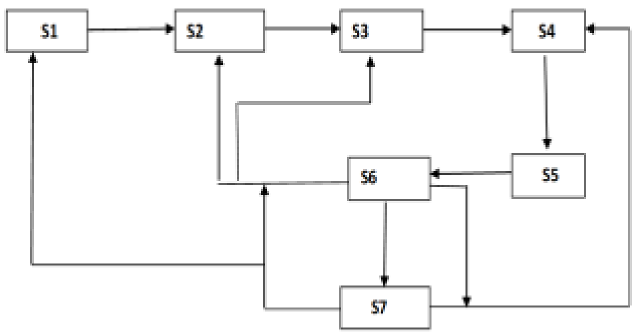

Figure 1. Phases of OR

S1	:	Formulate the problem
S2	:	Observe the system Mathematical model
S3	:	Construct the Mathematical model
S4	:	Verify the model and use the model for prediction
S5	:	Choose the best alternative
S6	:	Implement and evaluate the recommendations

1.4 General Model of the Linear Programming Problem

Maximize /Minimize $\mathrm{Z} = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

Subject to the constraints

$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1r}x_r + \cdots + a_{1n}x_n\{\leq, =, \geq\}b_1$
$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2r}x_r + \cdots + a_{2n}x_n\{\leq, =, \geq\}b_2$

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$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mr}x_r + \cdots + a_{mn}x_n\{\leq, =, \geq\}b_m$

$x_1, x_2 \dots x_n \geq 0.$

The above problem can be expressed using mathematical symbols as follows:

$$\text{Maximize /Minimize } Z = \sum_{j=1}^{n} c_j\, x_j$$

Subject to

$\sum_{j=1}^{n} a_{ij}\, x_j\{\leq, =, \geq\}b_i \quad (for\ i = 1,2, \dots m);\ x_j \geq 0(for\ j = 1,2, \dots n)$

Note: Where $cj's$; $bi's\ and\ aij's$ are constants for all $[\,j\ =\ 1\ to\ n]\ and\ [\,I\ =\ 1\ to\ m]$. The decision variables are referred by xj; $for\ all\ [\,j\ =\ 1\ to\ n]$.

1.5 Characteristics of an L.P.P

a) Regarding the symbol used in the General Model each constraint can take either$\{\leq, =, \geq\}$

b) The decision Variables $x_j's$ should take non-negative values only.

c) The values $c_j; b_j; a_{ij}(for\ i = 1,2, \dots m);\ \ (for\ j = 1,2, \dots n)$ can be got from the given data. These values are called parameters and assumed to be fixed constants.

1.6 Transportation Problem

The term "transportation" is somewhat deceptive it appears to be limited solely to transportation arrangements, but the font is different. In fact, many of the resource allocation problems arising in production systems can be treated as transportation problems. Typical examples are production scheduling, transportation scheduling, and so on. This problem was first introduced by Hitchcock (1941) and Koopmans (1947), and was solved by Dantzig (1947) using revised simplex method. Again in 1951 Dantzig [3] formulated the transportation problem as a linear programming problem and also provided the solution method. Straight off a day's transportation problem has become a standard application for industrial systems, having several manufacturing units, warehouses and distribution centers.

2. Methodology

In the usual transportation model takes care of the defined supply and demand along with cost. In this example the researcher allows the supply and demand be stochastic.This model contains the production price, transportation cost and selling price for each unit at various periods.By considering each supply point with varied supply units; simulation method is used to watch the simulated supply.In the same fashion the different market demand with varied demand units; simulation method is employed to see the simulated demand. The optimum solution is obtained using the simulated demand and simulated supply. The optimum solution is obtained by converting the transportation model into a linear programming model and the same is solved using TORA package.

Assumptions

- The supply at several factories are stochastic and known completely.
- The requirement at several factories are stochastic and known completely.
- The production cost at different factories should be known.
- The selling price at different market places should be known.
- The cost of transportation of all routes should be known.

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Notations

$$Fi \ - \ ith\ factory; \quad [\, i \ = \ 1, 2, \ldots, m]$$

$$Mj - \ jth\ market; \ [\, j \ = \ 1, 2, \ldots, n]$$

p_j – The unit cost of production in the j^{th} factory.

s_j – The unit sales price in the j^{th} market.

t_{ij} – The unit cost of transportation from the i^{th} factory to the j^{th} factory.

I_{ij} – The per unit profit based on items sent from i^{th} factory to the j^{th} factory.

$I_{ij} = s_j - [p_j + t_{ij}\ ]$;for every $i = 1,2 \ldots, m$ & $j = 1,2, \ldots, n$

x_{ij} – The number of units to the transported from i^{th} factory to j^{th} market.

$s_{i1}, s_{i2}, \ldots, s_{ik}$ be the different level of supply of the i^{th} factor. ($i = 1,2, \ldots, m$)

ss_i– The simulated supply of the i^{th} factory.

$d_{j1}, d_{j2}, \ldots, d_{jk}$ be the different levels of demand at the market j.

sd_j– The simulated demand of the j^{th} market.

Consider the proposed model

	s_1	s_2	.....	s_n	*Stochastic*
Market →	M_1	M_2	.....	M_n	*Supply*
↓ *Factory*					
p_1 F_1	t_{11}	t_{12}		t_{1n}	$s_{11}, s_{12}, \ldots, s_{1k}$
p_2 F_2	t_{21}	t_{22}		t_{2n}	$s_{21}, s_{22}, \ldots, s_{2k}$
.					.....
.					......
.					......
p_m F_m	t_{m1}	t_{m2}		t_{mn}	$s_{m1}, s_{m2}, \ldots, s_{mk}$
Stochastic Demand	$d_{11}, d_{12}, \ldots, d_{1k}$	$d_{21}, d_{22}, \ldots, d_{2k}$	... .	$d_{n1}, d_{n2}, \ldots, d_{nk}$	

i^{th} Factory:

Consider the stochastic supply of the i^{th} Factory.

$$s_{i1}, s_{i2}, \ldots, s_{ik}.$$

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Construct a discrete distribution with frequencies. Evaluate the corresponding probability distribution and apply the concept of Monte Carlo Simulation technique to get the simulated supply ss_i for the i^{th} factory.

Similarly evaluate all the simulated supplies $ss_i (i = 1,2 \ldots, m)$.

j^{th} Market:

Consider the stochastic demand of the j^{th} market.

$d_{j1}, d_{j2}, \ldots, d_{jk}$

Construct a discrete distribution with frequencies. Evaluate the corresponding probability and apply the concept of Monte Carlo Simulation technique to find the simulated demand sd_j.

Similarly, evaluate all the simulated demands $sd_j \ (j = 1,2 \ldots, n)$.

Evaluate the per unit profit I_{ij} for each factory based on every Market.

$I_{ij} = s_j - p_i - t_{ij};\ for\ all\ (i = 1,2 \ldots, m)\ and\ for\ all\ (j = 1,2 \ldots, n)$

Then this structure can be solved for maximizing the profit with the help of Linear Programming approach.

The corresponding Linear Programming model can be given as:

Maximize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} I_{ij}\, x_{ij}$

$\sum_{j=1}^{n} x_{ij} = ss_i;\ for\ all\ i = 1, 2, \ldots, m$

$\sum_{i=1}^{m} x_{ij} = sdj;\ for\ all\ j = 1, 2, \ldots, n$

$$x_{ij} \geq 0;\ for\ all\ i = 1, 2, \ldots, m\ and\ for\ all\ j = 1, 2, \ldots, n$$

The same can be computed using TORA package and the optimum solution can be evaluated.

The reduced model is as follows:

Market → / *Factory* ↓	M_1	M_2	.....	M_n	*Simulated Supply*
F_1	x_{11} I_{11}	x_{12} I_{12}	......	x_{1n} I_{1n}	ss_1
F_2	x_{21} I_{21}	x_{22} I_{22}	......	x_{2n} I_{2n}	ss_2
.					.
.					.
F_m	x_{m1} I_{m1}	x_{m2} I_{m2}	......	x_{mn} I_{mn}	ss_m
Simulated Demand	sd_1	sd_2	.....	sd_n	

Example:1

Find the optimum solution.

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5	4	3	s1
2	7	1	s2
4	3	1	s3
d1	d2	d3	

s1	10	9	12	7	14	12	10	15	7	8	14	10	11	9	7
s2	20	17	18	19	16	20	14	16	15	13	14	21	20	18	17
s3	15	14	13	12	10	11	12	13	16	14	13	14	12	15	11
d1	5	4	3	4	5	2	6	6	2	1	1	5	6	1	2
d2	10	9	8	7	10	6	8	9	11	6	5	7	8	9	10

3

The main difference is that here the demand and supply are stochastic. As such, one cannot carry on as such. Using Monte Carlo Simulation, the estimated demand and supply can be evaluated. Using the simulated demand and supply get the optimum solution.

Consider the stochastic values of s_1 and use Monte Carlo method.

s1	Probability	Cumulative probability	Tag numbers
7	0.20	0.20	00-19
8	0.07	0.27	20-26
9	0.13	0.40	27-39
10	0.20	0.60	40-59
11	0.07	0.67	60-66
12	0.13	0.80	67-79
14	0.13	0.93	80-92
15	0.07	1.00	93-99

Trials	Random Numbers	Simulated (s1)
1	36	9
2	32	9
3	84	14
4	08	7
5	96	15
6	52	10

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7	24	8
8	88	14
9	56	10
10	72	12

s_1=108/10=10.8=11

Similarly, proceeding like this for s_2, s_3, d_1, d_2 and d_3.

s_2=176/10=17.6=18; s_3=133/10=13.3=14; d_1=38/10=3.8= 4

d_2=85/10=8.5 = 9; d_3=64/10=6.4=7

The simulated demand and supply are rounded to the nearest integers.

Using this simulated demand, the problem reduced to

5	4	3	11
2	7	1	18
4	3	1	14
4	9	7	

Converting the modified TPP in to an LPP and solving the same gives the optimum solution as x_{14}=11; x_{21}=4; x_{23}=2; x_{24}=12; x_{32}=9; x_{33}=5 and the total transportation cost, hence the required optimum cost is Rs. 42.

Example: 2

Find the optimum solution.

10	7	3	1	s1
5	2	4	7	s2
1	3	2	4	s3
2	4	6	6	s4
d1	d2	d3	d4	

s1	10	9	12	7	14	12	10	15	7	8	14	10	11	9	7
s2	20	17	18	19	16	20	14	16	15	13	14	21	20	18	17
s3	15	14	13	12	10	11	12	13	16	14	13	14	12	15	11
s4	50	49	48	27	27	45	44	43	50	42	44	43	40	27	42
d1	5	4	3	4	5	2	6	6	2	1	1	5	6	1	2
d2	10	9	8	7	10	6	8	9	11	6	5	7	8	9	10
d3	8	6	7	5	4	8	7	3	5	6	7	8	4	6	7
d4	20	21	14	16	17	18	19	20	21	20	17	17	16	14	15

The simulated demand and supply are rounded to the nearest integers.

s_1=108/10=10.8 = 11; s_2=176/10=17.6 = 18; s_3=133/10=13.3 = 14

s_4=432/10=43.2= 44; d_1=38/10=3.8=4; d_2=85/10=8.5=9

d_3=64/10=6.4=7; d_4=180/10=18=18

The given problem is reduced to,

10	7	3	1	11
5	2	4	7	18
1	3	2	4	14
2	4	6	6	44
4	9	7	18	

Converting the modified TPP in to an LPP and solving the same gives the optimum solution as x_{14}=11; x_{21}=9; x_{25}=9; x_{33}=7; x_{34}=7; x_{41}=4; x_{45}=40 and the total transportation optimum cost is 79.

3. Conclusion

This novel type of Transportation Model is extremely necessary in the real-life decision-making situation in any Organization.This makes the new insight into the current scenario.This method is really simple and lucid to apply.

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