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COORDINATE MEASUREMENT ERROR IN SATELLITE NAVIGATION SYSTEM DURING IONOSPHERIC DISTURBANCES

Vladimir P. Pashintsev[1] Sergey V. Melnikov[2] Alexander F. Chipiga[3] MarkV.Peskov[4] Alexey V. Lyakhov[5] Gennadiy I. Linets[6]

[1]Doctor of Engineering Sciences, Professor, Professor at theDepartment ofInformationSafetyinAutomatedSystems
pashintsevp@mail.ru
[2]Doctor of Engineering Sciences, Associate Professor Head of theDepartment of Information Communications
kbytw@mail.ru
[3]Candidate ofEngineering Sciences, Professor Head of theDepartment of Information Safety in Automated Systems
chipiga.alexander@gmail.com
[4]Associate Professor at the DepartmentofInformationSafetyinAutomatedSystems
mvpeskov@hotmail.com
[5]Senior Lecturer at theDepartment of Information Safety in Automated Systems
lyachov_st@mail.ru
[6]Postgraduate at theDepartment ofInformation Communications
territoreer@yandex.ru

North-Caucasus Federal University Pushkina Str., Stavropol, 355009 Russia
E-mail:mvpeskov@hotmail.com, +7928325 37 68

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Abstract: A procedure has beendeveloped for estimation of a pseudorange measurement errorin the navigation equipment of users ofthe satellite navigation systems (SNS), operated at single and double frequencies, in conditions ofionospheric disturbances, accompanied bysimultaneous formation ofdifferent(large, medium, and small) scale irregularities. As a result, the analytical dependences ofthe pseudorange measurement erroron frequency parameters of transmitted SNS signals (carrier frequencies andspectrum width), signal/noiseratio at the input of a measuring receiverandsmall-scale fluctuations in the total electron content of the ionosphere onits average value have been established. It has been shown thatin conditions of artificial ionospheric disturbances, accompanied by formation ofintensesmall-scale irregularities, the use ofdouble-frequency SNSsin combination with spread spectrum (10 MHz) signals will enable not a decrease, butan increase in the pseudorange measurement error as compared tosingle-frequency SNSs. It is related to anarrowingofthe coherence bandwidthof transionospheric channel,when artificial disturbances occur, to the values of 1.2…0.5 MHzand theoccurrence ofseverefrequency-selective fadingof the received signals.
Keywords:satellite navigation system, unmanned aerial vehicle, total electron content, small-scale irregularities, frequency-selective fading, pseudorange measurement error.

Introduction

Nowadays, unmannedaerialvehicles (UAVs) have found an extensive use in solving various problems in the arctic latitudes[1]. UAV positioning error is determined bythe airborne receiverof the navigation equipment of users (NEU) within the GLONASS satellite navigation system (SNS) orGPS(GlobalPositioningSystem), which ensures an order of magnitude better accuracy of calculating coordinatesthan theinertial navigation system[1, 2].WhenSNS NEUis located onUAV, the resulting accuracy of positioning is higher compared to ground-based placement due to removal ofthe component of error, stemming fromthe multipath propagation of the signal received from the Earth. The drawback of applyingSNSis in reduction of the accuracy of defining coordinatesin the arctic latitudes.

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According to [3],in these conditions the error in determiningUAVposition may increase to 10 and more metres, since large-scale irregularities ($l > 100$km) are formed in the ionosphere.

The SNSs are currently designed so that a negative effect is taken into account on the pseudorange measurement error (δR), which defines an error in measuring coordinates ($\delta x = (1.5 \ldots 3)\delta R$),regular disturbances in the ionosphere and its large- and medium-scale irregularities [5, 6, 15,16]. To eliminate theionospheric component ofthe pseudorange measurement error,double-frequency(f_{02} =1.6 GHz and f_{01}=1.2 GHz) mode of SNS operation is used. However, this mode has brought about a threefold increase in the noise component δR . Hence, to decrease it, a very high energy-related signal/noise ratio $E_r/N_0 = 35\ldots48$dB at the measuring receiver's input is employed in SNS, and when SNS operates at double frequencies, phase-shift keyed pseudonoise signals (PSK PNS) with spread spectrum bandwidth$\Delta F_0 = \Delta F_{0S}$=10MHz, instead of PSK PNS with narrow bandwidth$\Delta F_0 = \Delta F_{0N}$=1MHz,are transmitted [16]. But when designing double-frequency spread spectrumSNS NEU, the negative effect of small-scale ionospheric irregularities on their noise-induced error is neglected.

It has been known [4], thatwhen large-scale irregularities move in the ionosphere,smaller irregularities are continuously formed. Therefore,during natural ionospheric disturbances in the arctic (and equatorial) latitudes, medium-scale ($l \approx 1\ldots100$km) and small-scale ($l \approx 100\ldots1000$m) irregularities may occur in addition to the large-scale ($l > 100$km) ones [5-7]. Such different scale irregularities may also arise duringartificial (human-induced) disturbances in the ionosphere due to itsradioheating, injection of easily ionizablechemical substances (for example, barium), operation of missile enginesin the ionosphere, explosions, etc. [5, 6]. Here, small-scale ionospheric irregularities produce the most deleterious effect upon the accuracy of determining UAV position using SNS NEU, rather than large- or medium-scale ones,by the following reasons.

As is known [4, 8-11], in the polar and equatorial latitudes a considerable reduction is often seenin the quality parameters of SNS and satellite communication systems (SCSs) sinceintense fluctuationsin the amplitude of received signals (orflickering, fading, scintillations) occur. These fluctuations are due to the formation of small-scale (of about 100…1000m in size) ionospheric irregularities andscattering (diffraction) of radio waves within these irregularities, what specifies their multipath propagation in the transionospheric channel. In conditions of artificial ionospheric disturbances owing to the release ofbarium,the intensity of small-scale irregularities mayincrease to the point that the coherence band of thetransionospheric channel of transmittingsignals at the carrier frequency f_0 =1.6GHzcan narrow down to the values of$\Delta F_C < 1$MHz [12-14]. Therefore,with conventional values of spectrum width of the signalsused in SNS (ΔF_0 =1MHzor 10MHz), the condition$\Delta F_0 \geq \Delta F_C$of arising frequency-selective fading (FSFs) inthe transionospheric channel may be met. SinceFSFsare characterized byfluctuations (distortions) in the envelope shapeof received signals, there will be energy losses inthe correlation receiver of signals, which result in the increased probability of error in the satellite communication systems [13], along withthe pseudorange measurement error(δR) in SNS, which defines theerror in measuring coordinates ($\delta x = (1.5 \ldots 3)\delta R$).

Hence, the questions of estimating theerror in measuring pseudorange in the SNS NEU,operated in the double-frequencywideband mode,in conditions of ionospheric disturbances, accompanied by simultaneous formation of different (large, medium, and small) scale irregularities, are pertinent.

Today, to monitor the ionosphere and study its influence on the process of radio wave propagation,there have been widely used special-purposedouble-frequency GISTM-receivers (GISTM – GNSS Ionospheric Scintillation and TEC Monitor), allowing to determine, aside from solving a navigation problem, the total electron content (TEC)of the ionosphere (N_T) by the signals fromthe satellite navigation systemsGLONASS/GPS [5, 7, 17-19]. The most advanced GISTM-receiver NovAtel GPStation-6 can measure TEC of the ionosphere along the radiowave propagation paths fromnavigation space vehicles (NSV)to NEU and define its statistical characteristics: mean-square deviation $\sigma_{\Delta N_T}$ of small-scale TEC fluctuations with respect to the average value $\overline{N_T}$ (which takes account of large- and medium-scale variations) [17, 20].The results of monitoring $\sigma_{\Delta N_T}$ enable determination ofthe coherence bandwidth ofthe transionospheric channel $\Delta F_C \sim \frac{f_0}{\sigma_{\Delta N_T}}$ [21].

The intention in this article is to develop the procedure for estimation of pseudorange measurement errorin the navigation equipment of SNS users,operated in single and double-frequency modes, in conditions of ionospheric disturbances, accompanied by simultaneous formation of different (large, medium, and small) scale irregularities. As a result of developing the procedure, there have to be found the analytical dependences $\delta R = \psi(f_0, F_0, E_r/N_0, \sigma_{\Delta N_T}, \overline{N_T})$ of pseudorange measurement error in SNS NEU on frequency parameters of transmitted SNS signals (carrier frequencies f_0 and spectrum bandwidth F_0), signal/noise ratio (

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E_r/N_0) at the measuring receiver's input and mean-square deviation of small-scale fluctuations in the ionospheric TEC ($\sigma_{\Delta N_T}$) on the average value ($\overline{N_T}$).

To attain the objective stated, the following problems need to be solved:

1)the effect of various scale ionospheric irregularities onthe pseudorange determination errorin the single-frequency SNS NEU shall be analysed.

2)the effect of small-scale ionospheric irregularities on the pseudorange determination error in the single-frequency SNS NEU shall be estimated.

3) the effect of small-scale ionospheric irregularities on the pseudorange determination error in the double-frequency SNS NEU shall be estimated.

4) pseudorange measurement errors in single and double-frequency SNS NEU in conditions of ionospheric disturbances shall be predicted.

1. Analysis of the effect of various scale ionospheric irregularities on the error in determining pseudorange of single-frequency SNS NEU

It is known [15-17], that the error in determining the coordinates in single-frequency($f_0 = f_{02}$ =1.6 GHz) SNS NEU$\delta x = G\delta R$dependsongeometric factor$G$(usually,G=1.5…3) and error in measuring pseudorange δR_i to each of the four(i=1…4) navigation space vehicles (NSV), incorporated into a constellation. The error in measuring pseudorange δR to any NSV depends on the errors of measuring the delay in the signal propagation time $\delta\tau = \delta R/c$ (with the speed of light$c = 3 \cdot 10^8 \frac{m}{sec}$) and is defined as the sum of several components

$$\delta R = \delta R_{ion} + \delta R_t + \delta R_{mp} + \delta R_n + \delta R_{\text{ts}} + \delta R_{\text{oth}} = c\delta\tau = c\left(\delta\tau_{ion} + \delta\tau_t + \delta\tau_{\text{mp}} + \delta\tau_n + \delta\tau_{\text{ts}} + \delta\tau_{\text{oth}}\right),(1)$$

which are caused by different reasons: effect of the ionosphere ($\delta R_{ion} = c\delta\tau_{ion}$=2…45m), troposphere ($\delta R_t$=$c\delta\tau_t$=0.25…2m), multipath propagation (δR_{mp}=$c\delta\tau_{\text{mp}}$ ≈3...9m), receiver noises ($\delta R_n = c\delta\tau_n$ ≈2…6m), displacement of time scales ($\delta R_{\text{ts}} = c\delta\tau_{\text{ts}}$=1…2m), and a number of other factors ($\delta R_{\text{oth}} = c\delta\tau_{\text{oth}}$).

Analysing the budget of errors in determining pseudoranges (1) shows [15-17] that the components associated with the effect of the ionosphere δR_{ion}, multipath propagation δR_{mp} and receiver noisesδR_n are the major contributors.When SNS NEU is placed on UAV, the effect of multipath propagation of radiowaves from the Earth is almost eliminated ($\delta R_{\text{mp}} \approx 0$). Therefore, to a first approximation, the error in determining pseudorange (1) to NSV, when SNS NEU is positioned on UAV, may be considered as defined by its ionospheric$\delta R_{ion} \approx$ 2 ... 45mand noiseδR_n ≈2…6mcomponents:$\delta R \approx \delta R_{ion} + \delta R_n$ ≈4…51m.

The presentedvalues ofionospheric and noise components of the error in determining SNS NEU pseudorange have been obtained exclusive of the effect of ionospheric irregularities. To analyse their effect, a model of the ionosphere needs to be developed taking into consideration various scale irregularities and the model of radiowave propagation (RWP) through such ionosphere into SNS.

It is customary [15-17, 22] to take into consideration the variation in electron concentration $N(h)$ by height$h$in various layers (D, E, F) exclusive of horizontal $\rho = (x, y)$ irregularities, when the ionospheric model is described in navigation and communication tasks. It is known [4-7] that in conditions of the arctic and equatorial latitudes, migrating ionospheric disturbances are often seen, accompanied by simultaneous formation of irregularitiesin electron concentration (EC)of various scales [6]: large (ρ=600…1200km), medium (ρ =1…600km), small (ρ=0.1…1km). Hence, the model ofvariations in the ionosphericECacross the space$N(h,\rho)$ should take into account the deviations of various scale irregularities $\Delta N(h,\rho)$ from the average value of $\overline{N}(h)$, which describes only heightwise EC variations:

$$N(h,\rho) = \overline{N}(h) + \Delta N_L(h,\rho) + \Delta N_M(h,\rho) + \Delta N_S(h,\rho), \quad (2)$$

where$\Delta N_L(h,\rho)$and$\Delta N_M(h,\rho)$- large-scale andmedium-scaleirregularities, $\Delta N_S(h,\rho) \equiv \Delta N(h,\rho)$- small-scale EC irregularities.

Fig.1 presents the model ofthe ionosphere, which describes: 1) thetypical dependence of EC on height $N(h) \equiv \overline{N}(h)$ in layers (D, E, F) of the mid-latitude ionosphere,whichis invariant(i.e. it is homogeneous)along horizontal coordinates $\rho = (x, y)$); 2)variations in EC across space (h, ρ) inclusive of large-scale deviations$\Delta N_L(h,\rho)$from background $\overline{N}(h)$:$\overline{N}(h) + \Delta N_L(h,\rho)$; 3) variations in EC across the space inclusive of the sum of large- and medium-scale deviations$\overline{N}(h) + \Delta N_L(h,\rho) + \Delta N_M(h,\rho) = \overline{N}(h,\rho)$;4) variations in the

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ionospheric ECacross the space inclusive of small-scale$\Delta N_S(h,\rho) \equiv \Delta N(h,\rho)$fluctuationswith respect to the average value $\overline{N}(h,\rho)$.

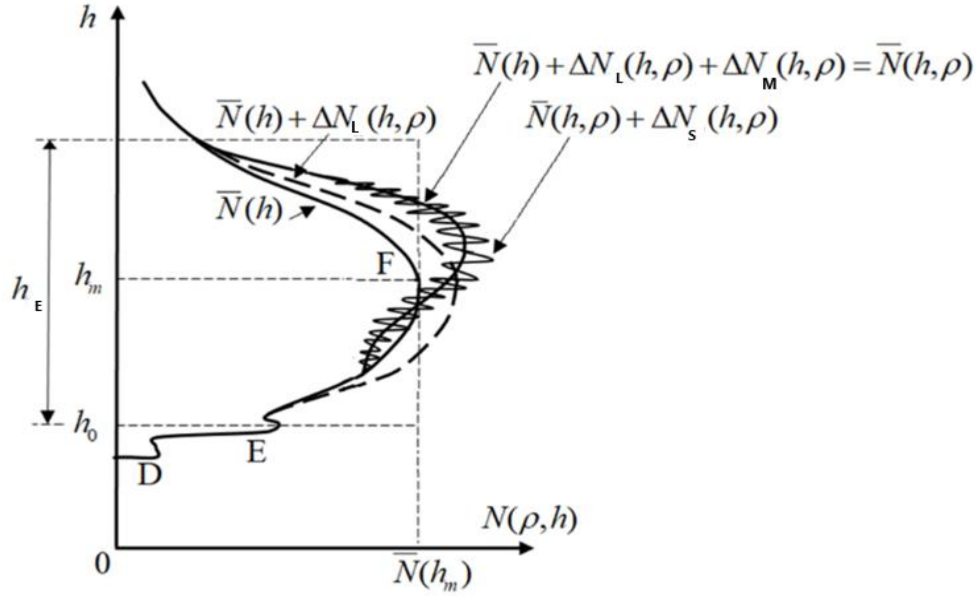

Fig.1. Model of the ionosphere with consideration for spatial variation in the electron concentrationin large-, medium-, and small-scale irregularities

The ionospheric component of the error in determining pseudorange to NSV in conditions of the mid-latitude normal ionosphereis known to be described [15,16] by the expression, where only the heightwise variation $N(h) \equiv \overline{N}(h)$ in the electron concentration within the ionospheric layers is taken into consideration:

$$\delta R_{ion} = c \cdot \delta\tau_{ion} = c\left(\frac{40.3 N_T \sec\alpha}{c f_0^2}\right) = 40.3 f_0^{-2} \sec\alpha \int_0^{h_{NSV}} N(h)dh \equiv$$

$$\equiv 40.3 f_0^{-2} \sec\alpha \int_0^{h_{NSV}} \overline{N}(h)dh = 40.3 f_0^{-2} \overline{N}(h_m) h_E \sec\alpha = \frac{40.3 \overline{N}_T \sec\alpha}{f_0^2}, \quad (3)$$

where40.3-coefficient of $\frac{m^3}{sec^2}$dimension (in some sources [11, 17]this coefficient is assumed to be 40.4);

$$N_T = \int_0^{h_{NSV}} N(h)dh = N(h_m)h_E \equiv \int_0^{h_{NSV}} \overline{N}(h)dh = \overline{N}(h_m)h_E = \overline{N}_T \quad (4)$$

- the average value of the total electron content (TEC) of the ionosphere along the path from NSV(at height$h_{\text{NSV}} \approx$ 20000km) to the NEU receiver(exclusive of EC irregularities); $N(h) \equiv \overline{N}(h)$ – variation by height h in the electron concentration in the homogeneous ionosphere;h_E- equivalent thickness of the ionosphere (400…500km) with the EC, equal to its maximum value $N(h_m) \equiv \overline{N}(h_m)$ at the maximum ionization height $h = h_m = 300...400$ km; α – zenith angle of radiowave propagation (RWP), f_0 - carrier frequency[Hz]. In accordance with expressions(2) and(4),the spatial variations in the total electron content in the nonhomogeneous ionosphere are defined as

$$N_T(\rho) = \int_0^{h_{NSV}} N(h,\rho)dh = \int_0^{h_{NSV}} \overline{N}(h)dh + \int_0^{h_{NSV}} \Delta N_L(h,\rho)dh + \int_0^{h_{NSV}} \Delta N_M(h,\rho)dh + \int_0^{hNSV} \Delta N_S(h,\rho)dh =$$

$$= \overline{N}_T + \Delta N_{TL}(\rho) + \Delta N_{TM}(\rho) + \Delta N_{TS}(\rho) = \overline{N}_T(\rho) + \Delta N_{TS}(\rho), \quad (5)$$

where$\Delta N_{TS}(\rho) \sim \Delta N_S(h,\rho)$–small-scale TEC fluctuationsalong the horizontal coordinates $\rho = (x, y)$ with respect to theaverage(background) value$\overline{N}_T(\rho) = \overline{N}_T + \Delta N_{TL}(\rho) + \Delta N_{TM}(\rho)$, that takes into account large- and medium-scale EC irregularities.

To estimate the error in determining pseudorange$\delta R \approx \delta R_{ion} + \delta R_n$and SNS NEU coordinates$\delta x = G\delta R \approx G(\delta R_{ion} + \delta R_n)$taking into account the effect of the TECof the ionosphere with large-, medium-, and small-scale irregularities (5), the model of the RWP process from one ofNSV (for example, NSV No.1) through the ionosphere to the single-frequency SNS NEU and the specificfeatures of processing the received signals (Fig.2) shall be analysed.

Letthe signal, transmitted from NSV No.1,with complex envelope $\dot{S}_t(t)$, energy E_t and carrier frequency $f_0 = \omega_0/2\pi$, be described by the expression[24]

$$s_t(t) = \sqrt{2}\,\mathrm{Re}\left(\dot{S}_t(t)\exp\left(j\omega_0 t\right)\right) = \sqrt{2}\,\mathrm{Re}\left(\sqrt{E_t}\,\dot{f}(t)\exp\left(j\omega_0 t\right)\right), \quad (6)$$

where $\dot{f}(t) = \dot{S}_t(t)\big/\sqrt{E_t}$ - standardcomplex envelope.

Apparently, with no ionospheric irregularities, when,in conformity with (5) $\Delta N_{TL}(\rho) = \Delta N_{TM}(\rho) = \Delta N_{TS}(\rho) =$ 0and $\overline{N}_T(\rho) = \overline{N}_T$, the time of signal propagation from NSV to NEU receiver over distance $R_0 = c\tau_0$ increases by the value of the ionospheric error$\delta\tau_{ion}$ and is determined according to (3) as

$$\tau = \tau_0 + \delta\tau_{ion} = \tau_0 + \frac{40.3\overline{N_T}\sec\alpha}{cf_0^2}. \quad (7)$$

Recall that in line with [4, 5] small-scale EC irregularities measure ρ =0.1…1km. However, in terms oftransionospheric RWP conditionsit is more convenient to consider assmall-scalesuch EC irregularities, the sizes of which ($\rho \equiv l$) do not exceed the dimension of the first Fresnel zone$l \le l_F$. For SNS it measures about $l_F \approx \sqrt{h_m\lambda_0} = \sqrt{h_m c / f_0} \approx 200\ldots400$m. It is obvious that the occurrence of irregularities which size is greater than the dimension of the Fresnel zone $l > l_F$ (i.e. large- and medium-scale ones)causes the additional (as compared to$\delta\tau_{ion} \sim \overline{N_T}$) time delay in propagation of all the wave front segmentsin the ionosphere$\Delta\tau_{ion} \sim (\Delta N_{TL}(\rho) + \Delta N_{TM}(\rho)$.

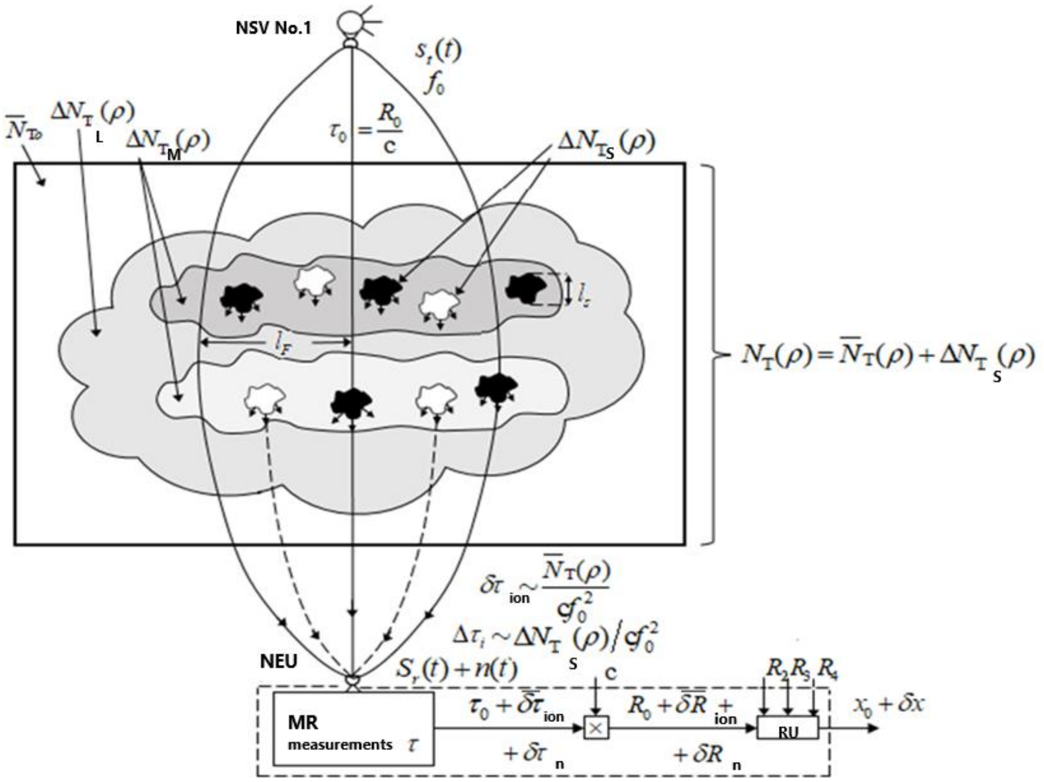

Fig 2. Model of the radiowave propagation process through the ionosphere to SNS taking into account the effect of large-, medium-, and small-scale irregularities

That is why the average value of the ionospheric TEC taking into account large- and medium-scale irregularities $\overline{N}_T(\rho) = \overline{N}_T + \Delta N_{TL}(\rho) + \Delta N_{TM}(\rho)$that appears in (5), defines the average time of signal propagation from NSV to SNS NEU

$$\overline{\tau} = \tau_0 + \frac{40.3\overline{N_T(\rho)}\sec\alpha}{cf_0^2} = \tau_0 + \frac{40.3\left(\overline{N_T} + \Delta N_{TL}(\rho) + \Delta N_{TM}(\rho)\right)\sec\alpha}{cf_0^2} = \tau_0 + \overline{\delta\tau_{ion}}, \quad (8)$$

where

$$\overline{\delta\tau_{ion}} = \frac{40.3\left(\overline{N_T} + \Delta N_{TL}(\rho) + \Delta N_{TM}(\rho)\right)\sec\alpha}{cf_0^2} = \delta\tau_{ion} + \Delta\tau_{\text{ion L}} + \Delta\tau_{\text{ion M}}$$

-is the average value of the ionospheric error in delay time taking into account large-($\Delta\tau_{\text{ion L}}$)and medium-scale ($\Delta\tau_{\text{ion M}}$) ionospheric irregularities.

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The presence of small-scale spatial ($\rho = \rho_i$) fluctuations in the ionospheric TEC $\Delta N_{TS}(\rho) \equiv \Delta N_T(\rho_i)$defines the emergence of relative phase shifts in the various segments (ρ_i) of the front of the wave (beams)propagated into the point of receiving signals[11]

$$\Delta\varphi_i = \omega_0 \Delta\tau_i = 2\pi f_0 \Delta\tau_i = \frac{80.6\pi \Delta N_{TS}(\rho) \sec\alpha}{c f_0} \equiv \frac{80.6\pi \Delta N_T(\rho_i) \sec\alpha}{c f_0} \quad (9)$$

andthe relative delay time of these beams

$$\Delta\tau_i = \frac{\Delta\varphi_i}{2\pi f_0} \equiv \Delta\tau(\rho_i) = \frac{40.3 \Delta N_{TS}(\rho) \sec\alpha}{c f_0^2} \equiv \frac{40.3 \Delta N_T(\rho_i) \sec\alpha}{c f_0^2}. \quad (10)$$

As per [11, 24, 25], the ratios between $\Delta\tau_i$ and the values of carrier frequency f_0 and spectrum width ΔF_0 of the transmitted signal specify the conditions for the occurrence of interference fading in the received signals of different types. When the condition $\Delta\tau_i << 1/f_0$ is met, generalRician(non-selective) fading arises, and when $1/f_0 << \Delta\tau_i << 1/\Delta F_0$, Raileigh fading emerges. The last expression ($\Delta\tau_i << 1/F_0$) reflects the conditionwhenno FSF is found in the received signals,which can be more illustrative if written in the form$\Delta F_0 << \frac{1}{\Delta\tau_i} \approx \Delta F_C$, where$\Delta F_C \approx \frac{1}{\Delta\tau_i}$- coherence bandwidth (undisturbed transmission) of the radiochannel. If the spectrum width of the transmitted signal exceeds the channel coherence bandwidth$\Delta F_0 \geq \Delta F_C \approx \frac{1}{\Delta\tau_i}$, the received signals will be exposed to FSF, accompanied by not only fluctuations in the amplitude and phase of the received signal, but random distortions of its envelope shape.

It is clear that as the small-scale TEC fluctuations increase$\Delta N_{TS}(\rho) \equiv \Delta N_T(\rho_i)$, the relative time delays of incoming beams (10) $\Delta\tau_i \sim \frac{\Delta N_T(\rho_i)}{f_0^2} \equiv \frac{\Delta N_{TS}(\rho)}{f_0^2}$ in transionospheric radiochannel (Fig.2) in the ionosphere will be longer, what will result in satisfying the conditions for emergence of Rician fading ($\Delta\tau_i << 1/f_0$) first, thenRaileigh fading ($1/f_0 << \Delta\tau_i << 1/\Delta F_0$), andthereafter, FSF ($\Delta F_0 \geq \Delta F_C \approx \frac{1}{\Delta\tau_i}$) in the received signals.

According to expressions (8-10),when the condition of FSF occurrence in transionospheric radiochannel is met

$$\Delta F_0 \geq \Delta F_C \approx \frac{1}{\Delta\tau_i} = \frac{c f_0^2}{40.3 \Delta N_T(\rho_i)} \quad (11)$$

the signal at the input of the SNS NEU receiver is described by the following expression [11]

$$s_r(t) = \sqrt{2}\,\mathrm{Re}\left\{\dot{S}_r(t) \exp\left(j\omega_0(t - \bar{\tau})\right)\right\} = \\ = \sqrt{2}\,\mathrm{Re}\left\{\sqrt{E_t} \int_{-\infty}^{\infty} \dot{b}(\Delta\tau) \dot{f}(t - \bar{\tau} - \Delta\tau) d\Delta\tau \exp\left(j\omega_0(t - \bar{\tau})\right)\right\}, \quad (12)$$

where $\bar{\tau}$ - the average delay time of the received signal is determined according to (8);

$$\dot{S}_r(t) = \sqrt{E_t} \int_{-\infty}^{\infty} \dot{b}(\Delta\tau) \dot{f}(t - \bar{\tau} - \Delta\tau) d\Delta\tau \quad (12a)$$

- complex envelope of the received signal, which isdefined by the complexlow-frequency impulse function $\dot{b}(\Delta\tau)$ of the channel. The last function is relatedthrough theFourier transform

$$\dot{b}(\Delta\tau) = \int_{-\infty}^{\infty} \dot{K}(\omega) \exp(j\Omega\Delta\tau) d\omega / 2\pi \quad (13)$$

to the complex transfer function $\dot{K}(\omega)$ of transionospheric channel at frequency $\omega = \omega_0 + \Omega = 2\pi(f_0 + F)$ within the signal spectrum width $\pm F_0 = 2F_0 = \Delta F_0$. It depends on the fluctuations in the delay time of incoming beams (10) $\Delta\tau(\rho_i) \sim \Delta N_T(\rho_i) / f_0^2$ with respect to their average value $\bar{\tau}$ as[11]

$$\dot{K}(\omega) = \sqrt{K_{\mathrm{att}}} \sum_{i=1}^{M} exp\left(-j(\omega_0 + \Omega)\Delta\tau(\rho_i)\right) \\ = \sqrt{K_{\mathrm{att}}} \sum_{i=1}^{M} exp\left(-\frac{j 80.6\pi (f_0 + F) \Delta N_T(\rho_i) \sec\alpha}{c f_0^2}\right), \quad (14)$$

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where$\sqrt{K_{att}}$is the attenuation coefficient by amplitude (identical to all incoming beams)in the transionospheric channel (related to the effect of free space and thermal wave absorption in the ionosphere).

The average power of the received signal (12) with FSFis determined using itscomplex conjugate function $\overset{*}{S}_r(t)$ as[11]

$$\bar{P}_r(t) = \overline{\dot{S}_r(t) \cdot \overset{*}{S}_r(t)} = E_t \int_{-\infty}^{\infty} \left| \dot{f}(t-\bar{\tau}-\Delta\tau) \right|^2 \sigma(\Delta\tau)\, d\Delta\tau \ , \qquad (15)$$

where $\sigma(\tau)$ – channel scattering function by the delaytime of beams, defined as

$$\sigma(\Delta\tau) = \overline{\dot{b}(\Delta\tau)\overset{*}{b}(\Delta\tau)} = \overline{\left| \dot{b}(\Delta\tau) \right|^2} . \qquad (16)$$

The analysis of expressions (16) and (13, 14) signifies that there is a dependence $\sigma(\Delta\tau) = \psi\left(\Delta\tau(\rho_i) \approx \frac{1}{\Delta F_C}\right)$of scattering function of transionospheric channel onits coherence bandwidthΔF_C. However, this dependence is not analytical.

According to Fig.2,the single-frequency SNS NEU comprises (Fig. 1) a receiver measuring (MR) the average delay time $\bar{\tau}$ of signal $s_r(t)$, received from NSV No.1 at the carrier frequency $f_0 \approx 1.6$ GHz, lightspeed (c) multiplier device,andreprocessing unit (RU).

The additive mixture of received signal (12) and fluctuation noise $n(t)$ of white Gaussian noise (WGN) type, associated withinterior MR noises,is supplied to the measuring receiver's input:

$$y(t) = s_r(t) + n(t) = \sqrt{2}\,\mathrm{Re}\left\{ \dot{S}_r(t) \exp\left(j\omega_0 (t-\bar{\tau}) \right) \right\} + n(t) . \qquad (17)$$

At the output of thesingle-frequency measuring receiver, the estimation of the average delay time of incoming signal(8) and the noise error in measuring the delay in the received signal are formed:

$$\bar{\tau} + \delta\tau_n = \tau_0 + \overline{\delta\tau_{ion}} + \delta\tau_n = \tau_0 + \delta\tau_{ion} + \Delta\tau_{\text{ion L}} + \Delta\tau_{\text{ion M}} + \delta\tau_n . \qquad (18)$$

Hence it follows that the noise error in measuring the delay time in the received signal$\delta\tau_n$should depend on WGN characteristics ($n(t)$) and complex envelope of the received signal $\dot{S}_r(t)$, which, in accordance with (12-16),depends on the ratio between frequency parameters of transmitted signals(f_0, ΔF_0) and the parameters of fluctuations of small-scale ionospheric irregularities $\Delta N_{TS}(\rho) \equiv \Delta N_T(\rho_i)$.

At the output of the (lightspeed c) multiplier, the result of measuring pseudorange to NSV No.1 in the single-frequency SNS NEU is formed, which, corresponding to(8, 18) and (1),can be written as follows

$$(\bar{\tau} + \delta\tau_n) \cdot c = \left(\tau_0 + \overline{\delta\tau_{ion}} + \delta\tau_n\right) \cdot c = (\tau_0 + \delta\tau_{ion} + \Delta\tau_{\text{ion L}} + \Delta\tau_{\text{ion M}}) \cdot c + \delta\tau_n \cdot c = R_0 + \delta R_{\text{ion}} + \Delta R_{\text{ion L}} + \Delta R_{\text{ion M}} + \delta R_n = R_0 + \overline{\delta R_{ion}} + \delta R_n = R_0 + \delta R = R, \qquad (19)$$

where

$$\overline{\delta R_{ion}} = \delta R_{ion} + \Delta R_{ionL} + \Delta R_{ionM} = \frac{40.3\left(\overline{N_T} + \Delta N_{TL}(\rho) + \Delta N_{TM}(\rho)\right) \sec\alpha}{f_0^2} = \frac{40.3\overline{N_T}(\rho) \sec\alpha}{f_0^2} \qquad (20)$$

-is the average value of ionospheric error in measuring pseudorange taking into account large- and medium-scale variations in the ionospheric TEC.

The reprocessing unit (RU) employs the results of measuring pseudorange toNSV No.1 (19) $R_1 = R_0 + \overline{\delta R_{ion}} + \delta R_n = R_0 + \delta R_{\text{ion}} + \Delta R_{\text{ion L}} + \Delta R_{\text{ion M}} + \delta R_n$and three otherNSVs ($R_2$, R_3, R_4), obtained in a similar way, to determine NEU coordinates with the error

$$\delta x = G\delta R = G\left(\overline{\delta R_{ion}} + \delta R_n\right) = G(\delta R_{ion} + \Delta R_{\text{ion L}} + \Delta R_{\text{ion M}} + \delta R_n) = Gc\left(\overline{\delta\tau_{ion}} + \delta\tau_n\right) =$$
$$= Gc(\delta\tau_{ion} + \Delta\tau_{\text{ion L}} + \Delta\tau_{\text{ion M}} + \delta\tau_n) = \delta x_{ion} + \Delta x_{\text{ion L}} + \Delta x_{\text{ion M}} + \delta x_n = \overline{\delta x_{ion}} + \delta x_n , \qquad (21)$$

where$\overline{\delta x_{ion}} = G\overline{\delta R_{ion}} = Gc\overline{\delta\tau_{ion}}$isthe average value of ionospheric error in determining SNS NEU coordinates, with consideration for the effect of large- and medium-scale irregularities; $\delta x_n = G\delta R_n = Gc\delta\tau_n$ is the noise component of determining NEU coordinates; G–geometric factor, which depends on the NSV No.1…NSV No.4 position with respect to NEU and typically [15-17] amounts to$G \approx 1.5…3$.

The effect of small-scale fluctuations in the ionospheric TEC $\Delta N_{TS}(\rho) = \Delta N_T(\rho_i)$on measurement of the average delay time of incoming signal $\bar{\tau}$ is that they are responsible for forming the received signal(12), the complex envelope $\dot{S}_r(t)$ of which according to(14)is prone to frequency-selective fading (distortions), what is exactly the

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cause of variation in the noise error in determining the delay time$\delta\tau_n$of the received signal and pseudorange$\delta R_n = c\delta\tau_n$to NSVin the single-frequency SNS NEU.

Thus, the development of the ionospheric model(Fig.1) and transionospheric RWP(Fig.2) with consideration for the effect of different scale EC irregularities has allowed to justifythat large- and medium-scale variations in the ionospheric TEC$\left(\Delta N_{TL}(\rho), \Delta N_{TM}(\rho)\right)$are responsible for an increase in the average value of ionospheric component $\overline{\delta R_{ion}}$ of pseudorange measurement error$\delta R = \overline{\delta R_{ion}} + \delta R_n$in SNS NEU. Here, the obtained expression (20) creates theanalytical dependence$\overline{\delta R_{ion}} = \psi\left(f_0 = f_{02}, \overline{N_T(\rho)}\right)$of ionospheric component of the pseudorange measurement errorin the single-frequency SNS NEU on carrier frequency $f_0 = f_{02}$ of the signal and average (background) value of the ionospheric TEC$\overline{N}_T(\rho) = \overline{N}_T + \Delta N_{TL}(\rho) + \Delta N_{TM}(\rho)$, that takes into accountlarge- and medium-scale EC irregularities.

The obtained results of the qualitative analysis have demonstrated that small-scale TEC fluctuations ($\Delta N_{TS}(\rho) \equiv \Delta N_T(\rho_i)$) define the relative delay time of incoming beams (9) $\Delta\tau_i \sim \Delta N_T(\rho_i) / f_0^2$ and contribute to theoccurrence of FSF in the received signals(11-14), followed by distortions of the envelope shape.This fading may cause an increase in the noise component ofpseudorange measurement error$\delta R_n$of the single-frequency SNS NEU.

2. Procedure for estimation ofthe effect of small-scale ionospheric irregularities on the error in determining pseudorange of single-frequency SNS NEU

To establish the analytical dependence $\delta R_n = \psi\left(f_0, F_0, \frac{E_r}{N_0}, \sigma_{\Delta N_T}\right)$of noise pseudorange measurement errorin the single-frequency SNS NEU on frequency parameters of transmitted SNS signals (carrier frequency $f_0 = f_{02}$ and spectrum bandwidth F_{0y}, F_{0n}),signal/noise ratio (E_r / N_0) and characteristics of small-scale fluctuations of the ionospheric TEC$\Delta N_{TS}(\rho) \equiv \Delta N_T(\rho_i)$ (their mean-square deviation $\sigma_{\Delta N_T}$ with respect to the average value $\overline{N_T}$), thecircuit of processing the received signals in the receiver that measures the average delay time of signalsin the single-frequency SNS NEU (Fig.2)requires concretization.

The noise error in measuring the average delay time ($\overline{\tau}$) of the received signal ($\delta\tau_n = \frac{\delta R_n}{c}$) is characterised by itsmean-square deviation (MSD)$\sigma_\tau = (\overline{\delta\tau_n^2})^{0.5}$. The minimum value $\sigma_\tau = (\overline{\delta\tau_n^2})^{0.5}$is provided at theoutput of the circuitof the optimumnon-coherent (NC) measuring receiver $\overline{\tau}$, which can be realized (Fig.3) usingmatched filter (MF), envelope detector (ED) and maximum selection circuit (MSC) [26].

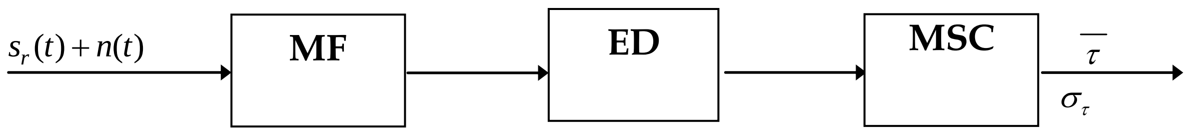

Fig. 3. Circuit of the optimumnon-coherent receiver measuring the average delay time ($\overline{\tau}$) of received signal

To obtain the expression for estimation of the MSD of measuring the delay time of the received signal with FSF (12)at the output of the SNS NEU receiver $\sigma_{\tau(f)}$, the scattering function of transionospheric channel $\sigma(\Delta\tau)$ needs to be defined. The analysis of expressions (16) and (13-15) implies thatthere is a relationship$\sigma(\Delta\tau) = \psi\left(\Delta\tau(\rho_i) \approx \frac{1}{\Delta F_C}\right)$between the scattering function of transionospheric channel andits coherence bandwidth, but no more than qualitatively speaking. The more exact dependence$\sigma(\Delta\tau) = \psi(\Delta F_C)$can be derived through constructing not multipath, butstructure-physical models of the transionospheric channel [11]. Such models allow to obtain the required dependence$\sigma(\Delta\tau) = \psi(\Delta F_C)$asthe product of the attenuation radio-channel coefficient (K_{att}) andnormalized scattering function$\sigma_{norm}(\Delta\tau)$withGaussian probability law:

$$\sigma(\Delta\tau) = K_{\text{att}}\sigma_{norm}(\Delta\tau) = K_{\text{att}}\left(\sqrt{2\pi}\sigma_{\tau 2}\right)^{-1} exp\left(-\frac{\Delta\tau^2}{2\sigma_{\tau 2}^2}\right). \quad (22)$$

Here, parameter $\sigma_{\tau 2}$ is related to the coherence bandwidth$\Delta F_C$of the transionospheric channel through the following expression[11]

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$$\sigma_{\tau 2} = \sigma_\varphi (1 + d_1^2/2)^{\frac{1}{2}}/\omega_0 = \sigma_{\tau 0}(1 + d_1^2/2)^{\frac{1}{2}} = \sigma_{\tau 1}/\sqrt{2} = 1/\sqrt{2}\pi \Delta F_C, \quad (23)$$

where

$$\Delta F_C = \frac{\sqrt{2} f_0}{\sigma_\varphi (1+d_1^2/2)^{\frac{1}{2}}} = \frac{\sqrt{2} f_0}{\sigma_\varphi D_1} = \frac{\sqrt{2}}{\sigma_{\tau 0} D_1}. \quad (24)$$

According to (24), the value $\Delta F_C \sim \frac{1}{\sigma_{\tau 0}} = \frac{\omega_0}{\sigma_\varphi}$is determined bythe MSD of fluctuations in the relative delay time of various segments of the wavefront at the output of the non-homogeneous ionosphere $\sigma_{\tau 0}$ (analogue of the relative delay time of incoming beams (10)$\Delta\tau_i = \frac{40.3\Delta N_T(\rho_i) \sec\alpha}{c f_0^2}$) and the MSD offluctuations in the phase front of this wave $\sigma_\varphi = \omega_0 \sigma_{\tau 0}$ (analogue of the relative phase shiftsin incoming beams (9)$\Delta\varphi_i = \omega_0 \Delta\tau_i = \frac{80.6\pi \Delta N_T(\rho_i) \sec\alpha}{c f_0}$), and the value of diffraction parameter $d_1^2 \ge 1$.

The MSD of fluctuations in the phase front of the outgoing wave $\sigma_\varphi = \omega_0 \sigma_{\tau 0}$, included into (24),depends on the MSD of small-scale fluctuations in the ionospheric TEC$\left(\overline{\Delta N_T^2(\rho_i)}\right)^{0.5} \equiv \left(\overline{\Delta N_{TS}^2(\rho)}\right)^{0.5} = \sigma_{\Delta N_T}$according to expression [8, 11, 12]

$$\sigma_\varphi \approx \frac{80.6\pi \left(\overline{\Delta N_T^2(\rho_i)}\right)^{0.5} \sqrt{\sec\alpha}}{c f_0} = \frac{80.6\pi \sigma_{\Delta N_T} \sqrt{\sec\alpha}}{c f_0}. \quad (25)$$

Parameter D_1 from (24) is defined as [11, 12]

$$D_1 = (1 + d_1^2/2)^{1/2} \ge 1, \quad (26)$$

where

$$d_1^2 = \frac{(3h_u^2 - 3h_u h_E + h_E^2) c^2 \sec^2\alpha}{192\pi^2 f_0^2 (l_m l_0)^2} = (3 h_u h_l + h_E^2) \frac{c^2 \sec^2\alpha}{192\pi^2 f_0^2 (l_m l_0)^2}. \quad (27)$$

Here,$h_u \approx 600$km–height of the upper boundary of the ionosphere of$h_E \approx 500 \ldots 400$km equivalent thickness; $h_l = h_u - h_E \approx 100$km - height of the lower boundary of the ionosphere; $l_m \approx 1$ mand $l_0 \approx 200...400$ m – minimum and maximum sizes of small-scale irregularities in the ionospheric EC. In accord with (26),parameter $D_1 \ge 1$grows asthe equivalent ionospheric thickness h_E, height of its lower boundaryh_l and zenith angle α increase, and carrier frequency $f_0$and sizes l_m, l_0 of small-scale irregularities decrease: $D_1 = \psi(h_l, h_E, \alpha, f_0^{-1}, l_m^{-1}, l_0^{-1})$.

According to (24-26), the coherence bandwidth of the transionospheric radiochannel is described by expression [11]

$$\Delta F_C \approx \frac{\sqrt{2} c f_0^2}{80.6\pi D_1 \sigma_{\Delta N_T} \sqrt{\sec\alpha}}, \quad (28)$$

which corresponds by the form to the qualitative expression (11) $\Delta F_C \approx \frac{1}{\Delta\tau_i} = \frac{c f_0^2}{40.3\Delta N_T(\rho_i)}$.

Thus, expression (12) for the received signal and its average power (15) fully describes the RWP model in transionospheric channel with FSF, and expressions(30-35) relate $\sigma(\Delta\tau) = \psi\left(\Delta F_C \sim \frac{f_0}{\sigma_{\Delta N_T}}\right)$of the scattering function of transionospheric channel toits coherence band and MSD of small-scale TEC fluctuations in the ionosphere.

According to (28),condition (11) for occurrence of FSF in the signalsreceived in transionospheric channel can be written in the revised form as follows

$$\Delta F_0 \ge F_C \approx \frac{\sqrt{2} c f_0^2}{80.8\pi D_1 \sigma_{\Delta N_T} \sqrt{\sec\alpha}}, \text{ or } \sigma_{\Delta N_T} \ge \frac{\sqrt{2} c f_0^2}{80.8\pi D_1 \Delta F_0 \sqrt{\sec\alpha}}. \quad (29)$$

It is known [27, 28], that whenthe additive mixture (17) $y(t) = s_r(t) + n(t)$ of WGNand signal (12) with FSF and known scattering function (22-27) acts upon the input of the optimumNC measuring receiver (Fig.3),the minimum MSD of measuring the average delay time of the received signal $\bar{\tau}$ at the receiver's output is provided, which can be written as follows:

$$\sigma_{\tau(f)} = \left(2\bar{E}_r \left(\frac{\bar{E}_r \eta_f}{N_0(N_0 + \bar{E}_r \eta_f)}\right) (\Delta\Omega_E \mu_f)^2\right)^{-0.5} = \left(\left(2\bar{E}_r \left(\frac{\bar{E}_r \eta_f}{N_0(N_0 + \bar{E}_r \eta_f)}\right)\right)^{0.5} \left(\sqrt{\pi} \Delta F_0 \mu_f\right)\right)^{-1}. \quad (30)$$

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Here, $\overline{E}_r = E_r$ - the average energy of the received signal, equal to its energy in the channel with no fading($E_r =$ EtKatt; N_0 - power spectral density ofWGNat the input ofthe optimum receiver; $\overline{E_r}/N_0 = \overline{h^2} = E_r/N_0 = h^2$ - the average signal/noise ratio at the input of the measuring receiver, equal to its value when no fading occurs; $\Delta\Omega_E = \sqrt{\pi}\Delta F_0$–the effective signal spectrum width; $\eta_f \leq 1$–coefficient of energy losses (i.e. of the decrease in the average energy of received signal $\overline{E}_r$) during correlation processing of the FSF-prone signal; $\mu_f \leq 1$–coefficient of narrowing of the effective spectrum width $\Delta\Omega_E$of signal due to FSF.

The mentionedcoefficients$\eta_f$and$\mu_f$depend on the degree of FSF (i.e. $\frac{\Delta F_0}{\Delta F_C}$ratio) in received signals, which, in its turn, depends on frequency parameters of transmitted signals (f_0, ΔF_0) and MSD of small-scale fluctuations in the ionospheric TEC ($\sigma_{\Delta N_T}$):

$$\eta_f = \left(1 + \frac{4\Delta F_0^2}{\pi\Delta F_C^2}\right)^{-0.5} = \left(1 + \frac{4\Delta F_0^2}{\pi}\left(\frac{80.8\pi D_1 \sigma_{\Delta N_T}\sqrt{sec\ \alpha}}{\sqrt{2}cf_0^2}\right)^2\right)^{-0.5} \leq 1; \tag{31}$$

$$\mu_f = \left(1 + \frac{4\Delta F_0^2}{\pi\Delta F_C^2}\right)^{-\frac{3}{4}} = \left(1 + \frac{4\Delta F_0^2}{\pi}\left(\frac{80.8\pi D_1 \sigma_{\Delta N_T}\sqrt{sec\ \alpha}}{\sqrt{2}cf_0^2}\right)^2\right)^{-\frac{3}{4}} \leq 1. \tag{32}$$

Expression (30) to determine the MSD of measuring the average delay time $\overline{\tau}$ of the received signal with FSF in SNS NEU at high values of signal/noise ratio ($\overline{E_r}/N_0$), when condition$\overline{E}_r\eta_f/N_0 >> 1$ is met, is reduced to the form

$$\sigma_{\tau(f)} \approx \left(\left(\frac{2\overline{E}_r}{N_0}\right)^{0.5} \Delta\Omega_E \mu_f\right)^{-1} \approx \left(\left(\frac{2\overline{E}_r}{N_0}\right)^{0.5} \sqrt{\pi}\Delta F_0 \mu_f\right)^{-1}. \tag{33}$$

When MSD $\sigma_{\Delta N_T}$ of small-scale fluctuations in the ionospheric TEC declines to the values, at which, according to (28), the condition of initiating Raileigh fadingin received signals is met:

$$\Delta F_0 << \Delta F_C \approx \frac{\sqrt{2}cf_0^2}{80.8\pi D_1 \sigma_{\Delta N_T}\sqrt{sec\ \alpha}}, \text{ or } \sigma_{\Delta N_T} << \frac{\sqrt{2}cf_0^2}{80.8\pi D_1 \Delta F_0 \sqrt{sec\ \alpha}}, \tag{34}$$

coefficient$\mu_f \approx 1$and expression (33) is reduced to the known [24, 26] form

$$\sigma_\tau \approx \left(\left(\frac{2\overline{E}_r}{N_0}\right)^{0.5} \Delta\Omega_E\right)^{-1} = \left(\left(\frac{2\overline{E}_r}{N_0}\right)^{0.5} \sqrt{\pi}\Delta F_0\right)^{-1} = \left(\left(\frac{2E_r}{N_0}\right)^{0.5} \sqrt{\pi}\Delta F_0\right)^{-1}, \tag{35}$$

typical of theMSD of measuring the average time $\overline{\tau}$ of delay in the received signal with Raileigh fading. The last equation shows that value σ_τ , when Raileigh fading occurs and ratio $\overline{E_r}/N_0 >> 1$, remains almost unchanged as compared with no-fading cases (since $\overline{E_r}/N_0 = E_r/N_0$).

In accordance with expression (35),formula (33) for determining MSD of measuring the average time $\overline{\tau}$ of delay in the received signal with FSF in SNS NEU can be written in the form of the following product

$$\sigma_{\tau(f)} \approx \left(\left(\frac{2\overline{E}_r}{N_0}\right)^{0.5} \sqrt{\pi}\Delta F_0\right)^{-1} \mu_f{}^{-1} = \sigma_\tau \mu_f^{-1}. \tag{36}$$

As perexpressions (19,33, 36), MSD of noise error in determining pseudorange in single-frequency (i.e. at carrier frequency f_0 $\approx$1.6 GHz) SNS NEU (Fig.2) with optimum measuring circuit (Fig.3), when FSF in the received signals occurs in the general case,is defined as

$$\sigma_{R(f)} = \left(\overline{\delta\tau_n^2}\right)^{0.5} c = \sigma_{\tau(f)} c \approx \sigma_\tau \mu_f^{-1} c = \left(\left(\frac{2\overline{E}_r}{N_0}\right)^{0.5} \sqrt{\pi}\Delta F_0\right)^{-1} c\mu_f{}^{-1} = \sigma_R \mu_f^{-1}, \tag{37}$$

where

$$\sigma_R = \sigma_\tau c = \left(\left(\frac{2\overline{E}_r}{N_0}\right)^{0.5} \sqrt{\pi}\Delta F_0\right)^{-1} c = \left(\left(\frac{2E_r}{N_0}\right)^{0.5} \sqrt{\pi}\Delta F_0\right)^{-1} c \tag{38}$$

- MSD of noise error in determining pseudorange in single-frequency SNS NEU, when Raileigh fading occurs (or in no-fading cases), and coefficient$\mu_f^{-1}$is described, according to (43),by expression

$$\mu_f^{-1} = \left(1 + \frac{4\Delta F_0^2}{\pi\Delta F_C^2}\right)^{\frac{3}{4}} = \left(1 + \frac{4\Delta F_0^2}{\pi}\left(\frac{80.8\pi D_1 \sigma_{\Delta N_T}\sqrt{sec\ \alpha}}{\sqrt{2}cf_0^2}\right)^2\right)^{\frac{3}{4}} \geq 1 \tag{39}$$

andcharacterises an increase in the MSD of noise error in determining pseudorange (σ_R) in the single-frequency SNS NEU as the degree of FSF (i.e.$\frac{\Delta F_0}{\Delta F_C}$ratio) in the received signals grows.

As per (37), when MSD $\sigma_{\Delta N_T}$ of small-scale fluctuations in the ionospheric TEC increases to the values, at which the condition of initiating FSF (37) $\frac{\Delta F_0}{\Delta F_C} \geq 1$ is met, the MSD of measuring pseudorange in SNS NEU will grow as compared to the case when Raileigh fading occurs (35) or with no Raileigh fading: $\sigma_{R(f)} \approx \sigma_R \mu_f^{-1} > \sigma_R$.

Thus, the analysis of the optimum circuit for measuring delay time ($\bar{\tau}$) of received signal (Fig.3) and the assessment of its noise error$\sigma_\tau = (\overline{\delta\tau_n^2})^{0.5}$enabled the development of the procedure for estimation of the effect of small-scale ionospheric irregularities on the error in determining pseudorange of the single-frequency SNS NEU. It comprises the following stages:

1) establishment of relationship(22-24) $\sigma(\Delta\tau) = \psi\left(\Delta F_C \sim \frac{f_0}{\sigma_{\Delta N_T}}\right)$between the scattering function of transionospheric channel and its coherence band$\Delta F_C$and analytical dependence of this band (25-28) $\Delta F_C \sim \frac{f_0}{\sigma_{\Delta N_T}}$on carrier frequency and MSD ofsmall-scale TEC fluctuations in the ionosphere$\sigma_{\Delta N_T} = \left(\overline{\Delta N_T^2(\rho_i)}\right)^{0.5}$;

2) establishment of analytical dependence (37) of the MSD of measuring the noise error when determining pseudorange$\sigma_{R(f)} = \sigma_R \mu_f^{-1}$on coefficient(39) $\mu_f^{-1} = \psi\left(\frac{\Delta F_0}{\Delta F_C}\right)$of increasing the MSD of the noise error in determining pseudorange (38) $\sigma_R = \psi(\overline{E}_r / N_0, \Delta F_0)$ in the channels withRaileigh fading (or without fading), as the degree of FSF ($\frac{\Delta F_0}{\Delta F_C}$) of received signals increases.

The developed procedure has allowed to establish theanalytical dependence $\sigma_{R(f)} = \psi\left(f_0, F_0, \frac{\overline{E}_r}{N_0}, \sigma_{\Delta N_T}\right)$of the noise error in measuring pseudorange in the single-frequency SNS NEUon frequency parameters of transmitted SNS signals (carrier frequency f_0 and spectrum bandwidth F_0), signal/noise ratio $\overline{E_r}/N_0$ at the receiver's input and mean-square deviation of small-scale fluctuations in the ionospheric TEC$\sigma_{\Delta N_T} = \left(\overline{\Delta N_T^2(\rho_i)}\right)^{0.5}$in the form of expressions (37-39).

It will be recalled that the error in measuring pseudorange in the single-frequency SNS NEU is defined by sum (19) $\delta R = \overline{\delta R_{ion}} + \delta R_n$of its ionospheric and noise components, which, taking into account the MSD $\sigma_R = (\overline{\delta R_n^2})^{0.5}$of the latter and expressions (20) and (37),can be written as follows

$$\delta R = \overline{\delta R_{ion}} + \delta R_n \equiv \overline{\delta R_{ion}} + \sigma_{R(f)} = \frac{40.3\overline{N_T}(\rho) \sec\alpha}{f_0^2} + \sigma_R \mu_f^{-1}, \quad (40)$$

wheremultipliers (38)$\sigma_R = \psi\left(\Delta F_0, E_r/N_0\right)$and (39)$\mu_f^{-1} = \psi(\Delta F_0, f_0, \sigma_{\Delta N_T})$depend on frequency parameters of transmitted SNS signals (f_0, F_0), signal/noise ratio (E_r/N_0) at the measuring receiver's input and MSD of small-scale fluctuations in the ionospheric TEC ($\sigma_{\Delta N_T}$).

Thus, as a result of the conducted qualitative analysis of the effect of different-scale ionospheric irregularities and the development of the procedure for estimating the effect of small-scale ionospheric irregularities on the error in determining pseudorangeof the single-frequency SNS NEU, there has been found its dependence $\delta R = \psi\left(f_0, F_0, E_r/N_0, \sigma_{\Delta N_T}, \overline{N_T}(\rho)\right)$ on frequency parameters of transmitted SNS signals (carrier frequency f_0 and spectrum bandwidth F_0), signal/noise ratio (E_r/N_0) at the measuring receiver's input and mean-square deviationof small-scale fluctuations of the total electron content of the ionosphere $\sigma_{\Delta N_T}$ with respect to the average value$\overline{N}_T(\rho) = \overline{N}_T + \Delta N_{TL}(\rho) + \Delta N_{TM}(\rho)$, that takes into account large- and medium-scale irregularities, as a set of expressions (40, 19-20, 37-39).

3. Procedure for estimation of the effect of small-scale ionospheric irregularities on the error in determining pseudorange of double-frequency SNS NEU

As it has been stated in the introduction, to reduce the ionospheric component ($\delta R_{ion} = \delta\tau_{ion} c$) of determining the pseudorange of SNS NEU on UAV, the operation mode at two carrier frequencies ($f_{01} \approx 1.2$ GHzand $f_{02} \approx 1.6$

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GHz) can be used. It will be demonstrated that the measuring receiver, functioning in the double-frequency operation mode (at the upper $f_{02} = f_{0u} \approx 1.6$GHz and lower $f_{01} = f_{0L} \approx 1.25$GHz frequencies), used in NEU (Fig.2) makes it possible to entirely eliminate the ionospheric error(20) $\overline{\delta R_{ion}} = \overline{\delta \tau_{ion}} c = \delta R_{ion} + \Delta R_{ionL} + \Delta R_{ionM}$ in measuring pseudorange of the received signal delay time, including its large-scale and medium-scale components,i.e.to ensure that$\overline{\delta R_{ion}} = \overline{\delta \tau_{ion}} c = \delta R_{ion} + \Delta R_{ionL} + \Delta R_{ionM} =0$. However,it is conceivable that thedouble-frequency NEU operation mode can cause a significant increase in the noisepseudorange measurement errorin circumstances when FSFemerges, the analytical expression for which remains unknown.

The algorithm [16] for measuring the delay time of the signal received in the double-frequency SNS NEU is known based on thespecified ratio of frequencies $m = f_{01} / f_{02}$ and the results of the delay time (τ_2, τ_1) measurement at each of the two frequencies:

$$\tau(2f) = \left(\tau_2 / \left(1-m^2\right)\right) - \left(m^2 \tau_2 / \left(1-m^2\right)\right). \tag{41}$$

In line with this algorithm for measuring the delay time of the signal received in the double-frequency SNS NEU, a double-frequency circuit of measuring the delay time of the received signal, when the ratio between the lower and the upper frequencies is $m = \frac{f_{01}}{f_{02}} = \frac{1.25 \cdot 10^9}{1.6 \cdot 10^9} = \frac{7}{9}$, is presented inFig.4.

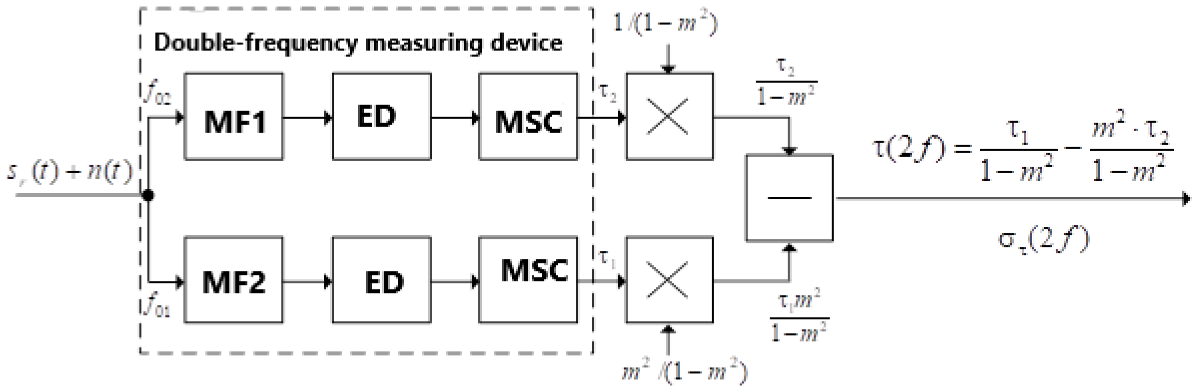

Fig. 4. Double-frequency (f_{02}, f_{01}) circuit of measuring the received signal delay time in SNS NEU

With RWPat carrier frequencies f_{02} and f_{01} through the ionosphere with different-scale irregularities, the delay time of signals measured at the output of each channel of processing of the double-frequency circuit (Fig.5), is determined similarly to the delay time (18, 8) in the single-frequency measuring circuit (Fig.3):

$$\tau_2 \equiv \tau(f_{02}) = \overline{\tau_2} + \delta\tau_{n2} = \tau_0 + \overline{\delta\tau_{ion2}} + \delta\tau_{n2} = \tau_0 + \delta\tau_{n2} + \frac{40.3\overline{N_T(\rho)}\sec\alpha}{c f_{02}^2} = \tau_0 + \delta\tau_{n2} + \frac{A}{f_{02}^2}; \tag{42a}$$

$$\tau_1 \equiv \tau(f_{01}) = \overline{\tau_1} + \delta\tau_{n1} = \tau_0 + \overline{\delta\tau_{ion1}} + \delta\tau_{n1} = \tau_0 + \delta\tau_{n1} + \frac{40.3\overline{N_T(\rho)}\sec\alpha}{c f_{01}^2} = \tau_0 + \delta\tau_{n1} + \frac{A}{f_{01}^2}, \tag{42b}$$

where$A = \frac{40.3\overline{N_T(\rho)}\sec\alpha}{c} = \frac{40.3\left(\overline{N}_T + \Delta N_{TL}(\rho) + \Delta N_{TM}(\rho)\right)\sec\alpha}{c}$.

According to (41, 42), the measured delay time of the received signal at the output of the subtracting unit of the double-frequency circuit (Fig. 5) shall be defined as

$$\tau(2f) = \frac{\tau_2}{1-m^2} - \frac{m^2 \tau_1}{1-m^2} = \frac{1}{1-\left(\frac{f_{01}^2}{f_{02}^2}\right)}\left(\left(\tau_0 + \frac{A}{f_{02}^2} + \delta\tau_{n2}\right) - \frac{f_{01}^2}{f_{02}^2}\left(\tau_0 + \frac{A}{f_{01}^2} + \delta\tau_{n1}\right)\right) =$$

$$= \tau_0 + \frac{\delta\tau_{n2}}{1-m^2} - \frac{m^2 \delta\tau_{n1}}{1-m^2} = \tau_0 + \delta\tau_n(2f).$$

The comparative analysis of this expressionwith (42) shows that when the double-frequency circuit(Fig.4) is used in SNS NEU(Fig.2), the measured signal delay time will differ from the true value $\tau_0 = R_0 / c$ by the value of error $\delta\tau(2f) = \delta\tau_n(2f)$, related only to the noise component, since there will be no time delayscaused bythe ionosphere with large- and medium-scale irregularities$\overline{N}_T(\rho) = \overline{N}_T + \Delta N_{TL}(\rho) + \Delta N_{TM}(\rho)$:

$$\overline{\delta\tau_{ion}}(2f) = \delta\tau_{ion}(2f) + \Delta\tau_{ion\,L}(2f) + \Delta\tau_{ion\,M}(2f) = 0. \tag{43a}$$

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Therefore, according to (20) the average value of the ionospheric error in measuring pseudorange taking into account the effect of large- and medium-scale variations in the ionospheric TEC in the double-frequencySNS NEU is zero:

$$\overline{\delta R_{ion}}(2f) = \overline{\delta \tau_{ion}}(2f)c = \delta R_{ion}(2f) + \Delta R_{\text{ion L}}(2f) + \Delta R_{\text{ion M}}(2f) = 0. \quad (43b)$$

As per expressions (40, 41), noise error in the double-frequency circuit for measuring the delay time when $m = f_{01}/f_{02} = 7/9$ is defined as

$$\delta\tau_n(2f) = \frac{\delta\tau_{n2}}{1-m^2} - \frac{m^2\delta\tau_{n1}}{1-m^2} \equiv \frac{\delta\tau_n(f_{02})}{1-m^2} - \frac{m^2\delta\tau_n(f_{01})}{1-m^2} = 2.53\delta\tau_n(f_{02}) - 1.53\delta\tau_n(f_{01}). \quad (44)$$

Obviously, when condition (11, 29) of FSF occurrence is met, the noise error $\delta\tau_{n2} \equiv \delta\tau_n(f_{02})$ in measuring the delay time of signal (12)received at frequency $f_0 = f_{02} \approx 1.6$GHzat the output of the 2nd channel of optimum NC processing in thedouble-frequency measuring device (Fig. 4) will be described similarly to the noise error in the single-channel circuit(Fig.3) of measuring delay time(48):

$$\sigma_{\tau(f)}(f_{02}) = \left(\overline{\delta\tau_n^2(f_{02})}\right)^{0.5} \approx \left(\left(\frac{2\overline{E}_r}{N_0}\right)^{0.5}\sqrt{\pi}\Delta F_0\right)^{-1}\mu_f^{-1}(f_{02}) = \sigma_\tau\mu_f^{-1}(f_{02}), \quad (45)$$

where $\mu_f^{-1}(f_{02}) \geq 1$–coefficient ofthe increase in the MSD of noise error in determining the delay time of received signals(35) σ_τ in SNS NEU at the upper frequency $f_0 = f_{02}$ due to FSF, defined according to (39) as

$$\mu_f^{-1}(f_{02}) = \left(1 + \frac{4\Delta F_0^2}{\pi\Delta F_C^2(f_{02})}\right)^{\frac{3}{4}} = \left(1 + \frac{4\Delta F_0^2}{\pi}\left(\frac{80.8\pi D_1(f_{02})\sigma_{\Delta N_T}\sqrt{\sec\alpha}}{\sqrt{2}cf_{02}^2}\right)^2\right)^{\frac{3}{4}} \geq 1, \quad (46)$$

whereparameter $D_1(f_{02}) = (1 + d_1^2(f_{02})/2)^{1/2} \geq 1$ is described by expressions (26, 27) when $f_0 = f_{02}$. Similarly to (45),the MSD of the noise error $\delta\tau_{n1} \equiv \delta\tau_n(f_{01})$in measuring the delay time of the received signal with FSF is defined at the lower frequency $f_0 = f_{01} \approx 1.2$ GHzat the output of the 1st channel of the double-frequency measuring device (Fig. 4):

$$\sigma_{\tau(f)}(f_{01}) = \left(\overline{\delta\tau_n^2(f_{01})}\right)^{0.5} \approx \left(\left(\frac{2\overline{E}_r}{N_0}\right)^{0.5}\sqrt{\pi}\Delta F_0\right)^{-1}\mu_f^{-1}(f_{01}) = \sigma_\tau\mu_f^{-1}(f_{01}), \quad (47)$$

where

$$\mu_f^{-1}(f_{01}) = \left(1 + \frac{4\Delta F_0^2}{\pi\Delta F_C^2(f_{01})}\right)^{\frac{3}{4}} = \left(1 + \frac{4\Delta F_0^2}{\pi}\left(\frac{80.8\pi D_1(f_{01})\sigma_{\Delta N_T}\sqrt{\sec\alpha}}{\sqrt{2}cf_{01}^2}\right)^2\right)^{\frac{3}{4}} \geq 1. \quad (48)$$

It is usually believed [16] that noise errors in each channel of measuring the delay time of signals in the double-frequency circuit (Fig. 4) will be independent. Hence, in accordance with (44) the MSDof noise error in measuring the delay time of signals in the double-frequency circuit (Fig. 5) is determined as

$$\sigma_{\tau(f)}(2f) = \left(\left(2.53\sigma_{\tau(f)}(f_{02})\right)^2 + \left(1.53\sigma_{\tau(f)}(f_{01})\right)^2\right)^{0.5}, \quad (49)$$

where $\sigma_{\tau(f)}(f_{02})$and$\sigma_{\tau(f)}(f_{01})$are defined according to (45, 46) and (47, 48).

By (45, 47), expression (49) for the MSD of measuring the average time $\overline{\tau}$ of delay in the received signal with FSF in the double-frequency NEU can be written in the most compact form as

$$\sigma_{\tau(f)}(2f) = \left(\left(2.53\sigma_\tau\mu_f^{-1}(f_{02})\right)^2 + \left(1.53\sigma_\tau\mu_f^{-1}(f_{01})\right)^2\right)^{0.5} =$$

$$= \sigma_\tau\left(\left(2.53\mu_f^{-1}(f_{02})\right)^2 + \left(1.53\mu_f^{-1}(f_{01})\right)^2\right)^{0.5}. \quad (50)$$

where σ_τ is determined according to (35), $\mu_f^{-1}(f_{02})$and$\mu_f^{-1}(f_{01})$–according to (46, 48).

In concordance withexpressions (37, 38, 50), the MSDof noise pseudorange measurement errorin the double-frequency SNS NEU (Fig. 2) is defined as

$$\sigma_{R(f)}(2f) = \sigma_{\tau(f)}(2f)c = \sigma_\tau c\left(\left(2.53\mu_f^{-1}(f_{02})\right)^2 + \left(1.53\mu_f^{-1}(f_{01})\right)^2\right)^{0.5} = \sigma_R\mu_f^{-1}(2f), \quad (51)$$

wherethe MSDof noisepseudorange measurement errorin the single-frequency NEU with no FSF is determined by (38)$\sigma_R = \sigma_\tau c = \left(\left(\frac{2\overline{E}_r}{N_0}\right)^{0.5}\sqrt{\pi}\Delta F_0\right)^{-1}c$, and coefficient$\mu_f^{-1}(2f)$of the increase in theMSD of noisepseudorange measurement errorin the double-frequency SNS NEU in FSF conditions is defined as

$$\mu_f^{-1}(2f) = \left(\left(2.53\mu_f^{-1}(f_{02})\right)^2 + \left(1.53\mu_f^{-1}(f_{01})\right)^2\right)^{0.5}. \quad (52)$$

Its value, according to (60), is$\mu_f^{-1}(2f) \approx 3$with no FSF(when$\mu_f^{-1} = 1$) and$\mu_f^{-1}(2f) > 3$- when FSF occurs.

Thus, the procedure has been developed for estimation of the effect of small-scale ionospheric irregularities onpseudorange measurement errorin the double-frequency SNS NEU; it comprises two stages:

1) justification for eliminating the ionospheric pseudorange measurement error in the received signal, including its large-scale and medium-scale components (43b):$\overline{\delta R_{ion}}(2f) = \delta R_{ion}(2f) + \Delta R_{\text{ion L}}(2f) + \Delta R_{\text{ion M}}(2f)$ =0, fromthe double-frequency SNS NEU (Fig.4);

2) establishment of the analytical dependence of theMSD of measurement of the noise error in determining pseudorangein the double-frequency NEU (51)$\sigma_{R(f)}(2f) = \sigma_R \mu_f^{-1}(2f)$in the form of the product of the noisepseudorange measurement errorMSD (38) $\sigma_R = \psi(\overline{E}_r / N_0, \Delta F_0)$ in the channels with Raileigh fading (or without fading) and coefficient (52) $\mu_f^{-1}(2f) = \psi\left(\mu_f^{-1}(f_{02}); \mu_f^{-1}(f_{01})\right) = \psi(f_{02}, f_{02}, \Delta F_0, \sigma_{\Delta N_T})$), characterisingincrement σ_R when the degree of FSF ($\frac{\Delta F_0}{\Delta F_C(f_{02})}$, $\frac{\Delta F_0}{\Delta F_c(f_{01})}$) in received signals grows due to the increased MSD of small-scale TEC fluctuations $\sigma_{\Delta N_T}$ in the ionosphere during its disturbances.

The analytical dependence$\sigma_{R(f)}(2f) = \psi\left(f_{02}, f_{02}, F_0, \frac{\overline{E}_r}{N_0}, \sigma_{\Delta N_T}\right)$of noise error in measuring pseudorange in the double-frequency SNS NEUon frequency parameters of transmitted SNS signals (carrier frequencies f_{02} and f_{01}, spectrum bandwidth F_0), signal/noise ratio $\overline{E}_r / N_0$ at the receiver's input and mean-square deviation $\sigma_{\Delta N_T}$ of small-scale fluctuations in the ionospheric TEC, as a set of expressions (51, 52, 38, 46, 48), is the result of development ofthe above procedure.

4.Predicting of pseudorange measurement error in the double-frequency SNS NEU operation modein conditions of ionospheric disturbances

Examination of the above results indicates that pseudorange measurement errors in SNS NEU in single-frequency and double-frequency operation modescansubstantiallygrowduring ionospheric disturbances, accompanied by an increase in TEC deviations (5) inlarge$\Delta N_{TL}(\rho)$, medium$\Delta N_{TM}(\rho)$and small$\Delta N_{TS}(\rho)$scale irregularities with respect to the average value $\overline{N}_T$. That is why to predictSNS NEU pseudorange measurement error in conditions ofionospheric disturbances, first it is necessary to provide rationale for the intervals of possible TEC increments ($\Delta N_{TL}(\rho)$, $\Delta N_{TM}(\rho)$and$\Delta N_{TS}(\rho)$).

It is known [23], thatthe value of TEC of the normal ionosphere (NI) averaged for all the seasons in the middle latitudes amounts to$\overline{N}_T \approx 10^{17} \frac{\text{el}}{m^2}$=10TECUat nightand $\overline{N}_T \approx 5 \cdot 10^{17} \frac{\text{el}}{m^2}$=50TECUby day. In conditions of natural ionospheric disturbances (NID) in the arctic and equatorial latitudes, the average TEC value is invariant as compared to themiddle latitudes, i.e. $\overline{N}_T \approx$ 10…50TECU [23], but large- and medium-scale TEC variations are seen, that achieve the following values [6]: ΔN_{TL}=1…5TECU; ΔN_{TM}=0.2…2TECU. Thus,TEC variations, complementary to $\overline{N}_T \approx$ 10…50TECU, amount to$\Delta N_{TL}+\Delta N_{TM}$=1.2…7TECUand the background TEC value in case of NID is in the following range

$$\overline{N_T(\rho)} = \overline{N}_T + \Delta N_{TL}(\rho) + \Delta N_{TM}(\rho) = (10 \ldots 50) + (1.2 \ldots 7) = 1.2 \ldots 57 \text{ TECU}. \quad (53)$$

Let us analyse the variations in small-scale ionospheric TEC fluctuations, characterised by the MSD value $\sigma_{\Delta N_T} = \left(\overline{\Delta N_{TS}^2(\rho)}\right)^{0.5}$, in conditions of normal middle-latitude ionosphere and its natural and artificial disturbances.

According to experimental data [12],during artificial ionospheric disturbances (AID) initiated by the injection of easily ionizable chemical substances (of barium type), the MSD of fluctuations in the phase wave front at the ionosphere output (25) $\sigma_\varphi \approx \frac{80.8\pi\sigma_{\Delta N_T}\sqrt{\sec\alpha}}{cf_0}$at $\alpha = 0^\circ$ and $f_0 \approx$ 1.4 GHzmay reach the values σ_φ =100…500 rad. At the carrier frequency $f_0 \approx$ 1.6 GHz, it corresponds to the increase in the MSD of small-scale fluctuations to values $\sigma_{\Delta N_T} \approx 14...70$ TECU.

The known [23] experimentson studying small-scale irregularities in the normal ionosphere (NI)in the middle latitudesinvolved no measurements of the MSD of small-scale TEC fluctuations ($\sigma_{\Delta N_T}$) andwere limited to the values of intensity of these irregularities, which amount to$\beta_i \approx 3 \cdot 10^{-3}$by dayand$\beta_i \approx 10^{-2}$at night. In conditions of natural ionospheric disturbances (NID) in the polarand equatorial latitudes, the intensity of small-scale irregularities increases to values$\beta_i \approx 0.1$and$\beta_i \approx 0.2$, respectively [23].However, the intensity β_i of small-scale

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irregularities is related to the ratio between the MSD of small-scale TEC fluctuations $\sigma_{\Delta N_T} = \left(\overline{\Delta N_{TS}^2(\rho)}\right)^{0.5}$ and the average value $\overline{N}_T$ through the known [29] relationship:

$$\beta_i = \left(\frac{\sigma_{\Delta N_T}}{\overline{N}_T}\right)\left(\frac{h_E \sec \alpha}{\sqrt{\pi} l_S}\right)^{0.5}, \quad (54)$$

where l_S –typical (average) size of small-scale irregularities in the ionospheric EC.

Ifthe average TEC value is defined with consideration for not only homogeneous background $\overline{N}_T$, but large- and small-scale variations as well, in line with (5) $\overline{N_T(\rho)} = \overline{N}_T + \Delta N_{TL}(\rho) + \Delta N_{TM}(\rho)$, expression (54) is rearranged in the following form

$$\beta_i = \left(\frac{\sigma_{\Delta N_T}}{\overline{N_T(\rho)}}\right)\left(\frac{h_E \sec \alpha}{\sqrt{\pi} l_S}\right)^{0.5}.$$

Hence it follows that the expressionfor determining the MSD of small-scale TEC fluctuations in the ionosphere is as follows

$$\sigma_{\Delta N_T} = \beta_i \left(\overline{N}_T + \Delta N_{TL}(\rho) + \Delta N_{TM}(\rho)\right)\left(\frac{h_E \sec \alpha}{\sqrt{\pi} l_S}\right)^{0.5}. \quad (55)$$

If l_S =400m is assumed to be the typical size ofsmall-scale irregularities, then according to (53, 55) with consideration for values$\beta_i \approx 3 \cdot 10^{-3}$by day and$\beta_i \approx 10^{-2}$at night at $\alpha = 0^\circ$, the MSD of small-scale TEC fluctuations in the middle latitude ionosphereamounts to$\sigma_{\Delta N_T} \approx (1.3 \dots 7) \cdot 10^{-2}$TECUby day and$\sigma_{\Delta N_T} \approx 0.04 \dots 0.2$TECUat night. In the polar and equatorial latitudes (inclusive of values$\beta_i \approx 0.1$and$\beta_i \approx 0.2$) the MSD of small-scale TEC fluctuations is$\sigma_{\Delta N_T} \approx 0.4 \dots 2$ TECUand$\sigma_{\Delta N_T} \approx 0.8 \dots 4$ TECU, respectively.

The following conclusions are apparent from the conducted analysis of the TEC variation during ionospheric disturbances:

1)in the normal middle latitude ionosphere, the variations in the average TEC value amount to $\overline{N_T} \approx 10 \dots 50$ TECU and the variations in the MSD of small-scale fluctuations –$\sigma_{\Delta N_T} \approx 10^{-2} \dots 0.2$TECU;

2) natural ionospheric disturbances in the arctic and equatorial latitudesare characterisedby an increase in the background TEC valueto$\overline{N_T(\rho)} = \overline{N}_T + \Delta N_{TL}(\rho) + \Delta N_{TM}(\rho) \approx 11.2 \dots 57$TECUowing to the occurrence of large- and medium-scale irregularities and an increase in the MSD of small-scale TEC fluctuations to values$\sigma_{\Delta N_T} \approx 0.4 \dots 4$ TECU;

3) artificial ionospheric disturbances may cause an increase in the MSD of small-scale TEC fluctuationsto values $\sigma_{\Delta N_T} \approx 14 \dots 70$ TECUat the constant background TEC value$\overline{N_T(\rho)} \approx 11.2 \dots 57$TECU.

In accordance with the specified ranges of increments in TEC variations and the results obtainedin par.1-4,inFig.5 there have been given the graphs of the dependence of pseudorange measurement errorin single-frequency(19) $\delta R = \overline{\delta R_{ion}} + \sigma_{R(f)}$and double-frequency (49)$\delta R = \sigma_{R(f)}(2f)$SNS NEUon variation in TEC parameters (average value $\overline{N_T(\rho)}$ and MSD of small-scale fluctuations $\sigma_{\Delta N_T}$) in conditions of natural (NID) and artificial (AID) ionospheric disturbances.

Curve 1 reflects dependencies (8, 19, 20, 37-39) of pseudorange measurement error$\delta R = \overline{\delta R_{ion}} + \sigma_{R(f)}$in single-frequency ($f_0 = f_{02} \approx$1.6GHz) SNS NEUon the increase in the MSD of small-scale TEC fluctuations in NID conditions (in the range of $\sigma_{\Delta N_T}$ =0.1…4TECU) and in AID conditions (in the range of $\sigma_{\Delta N_T}$ =0.1…70TECU). It is constructed for the cases of transmitting the signals with narrow spectrum bandΔF_{0N}=1MHzat the maximum background TEC value with consideration for large- and medium-scale variations $\overline{N_T(\rho)} \approx 57$ TECUat $\alpha = 0^\circ$, $\overline{h^2} = \overline{E}_r / N_0$ =35 dB. In accord with (20) and (37-39),the ionospheric component of pseudorange measurement erroramounts to$\overline{\delta R_{ion}}$ ≈8.6m, and the noise component grows fromσ_{RN} ≈2mat $\sigma_{\Delta N_T}$ =0.1 TECUto $\sigma_{R(f)N}$ ≈3.2mat $\sigma_{\Delta N_T}$ =70 TECU,since the condition of FSF occurrence is met ($\Delta F_0 \geq \Delta F_C$)in the received signals. The resulting value of pseudorange measurement error in the single-frequency SNS NEU (curve 1) amounts to$\delta R \approx 8.6 + 2 = 10.6$mwith NID ($\sigma_{\Delta N_T}$ =4 TECU) and increases to$\delta R \approx 8.6 + 3.2 = 11.8m$when AID take place ($\sigma_{\Delta N_T}$ =70 TECU).

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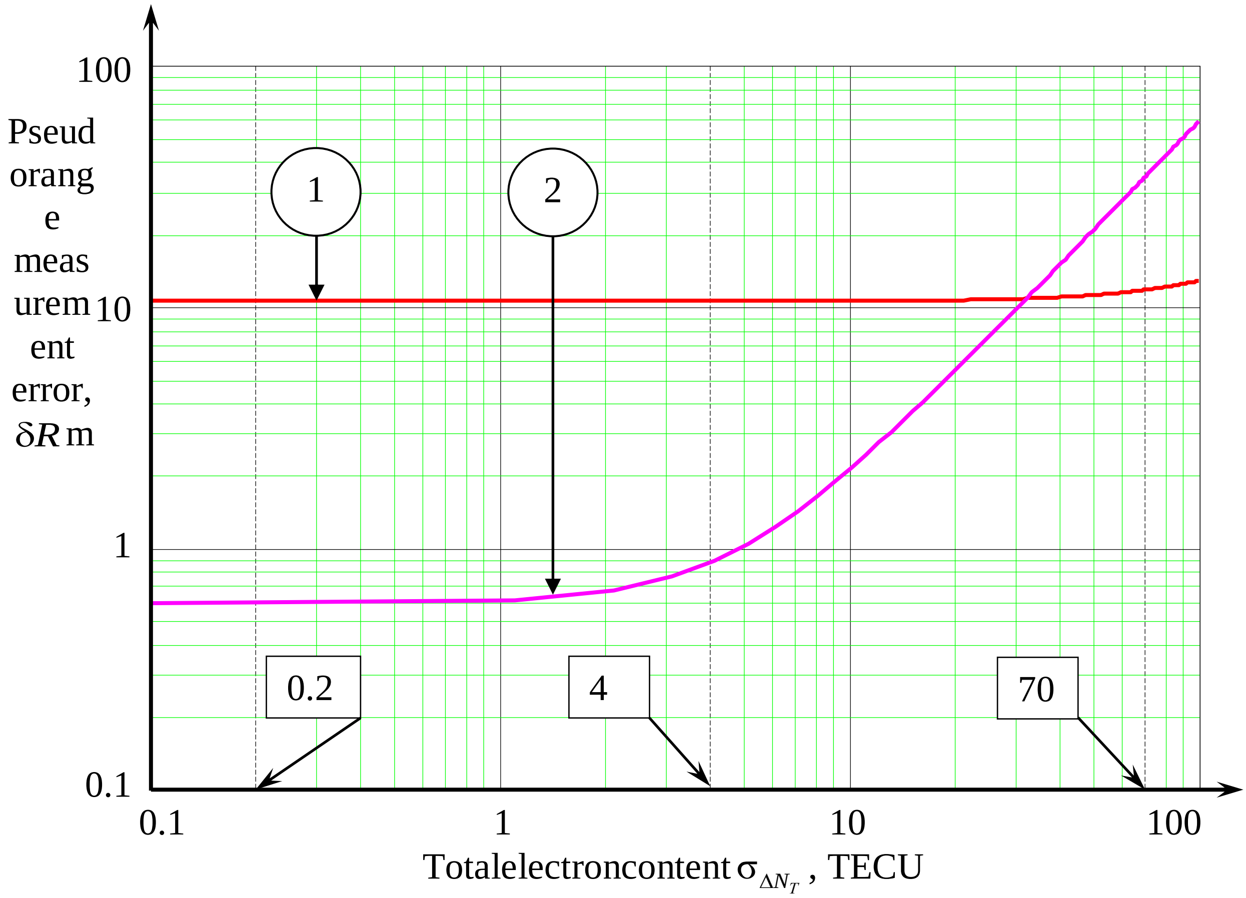

Fig. 5. Dependencies of pseudorange measurement error in single- and double-frequency navigation equipment of SNS users on variation in small-scale TEC fluctuations in the ionosphere

Curve 2 reflects dependencies (43, 38, 51) of pseudorange measurement error$\delta R = \sigma_{R(f)}(2f) = \sigma_R \mu_f^{-1}(2f)$in double-frequency ($f_0 = f_{02} \approx$1.6GHz, $f_{01} \approx$1.2GHz) SNS NEU on the increase in the MSD of small-scale TEC fluctuations in NID conditions (in the range of $\sigma_{\Delta N_T}$ =0.1…4TECU) and in AID conditions (in the range of $\sigma_{\Delta N_T}$ =0.1…70 TECU). It is constructed assuming noionospheric component of error (43) δR_{ion} $(2f)$=0 for the cases of transmittingthe signals withspread spectrum band ΔF_{0S}=10MHzat $\alpha = 0^\circ$, $\overline{h^2} = \overline{E}_r / N_0$ =35dB. According to (38, 46, 48, 51) and Fig.5,the pseudorange measurement error in the double-frequency NEUamounts toδR $(2f)_S$ ≈0.6min conditionsof normal ionosphere ($\sigma_{\Delta N_T} \leq$0.2 TECU), in NID conditions ($\sigma_{\Delta N_T} \leq$4 TECU) pseudorange measurement errorincreases toδR $(2f)_S$ ≈0.9mdue to the occurrence of FSF in the received signals ($\frac{\Delta F_{0S}}{\Delta F_C} > 1$). In conditions of AID, an increase in small-scale TEC fluctuationsto the maximum value $\sigma_{\Delta N_T}$ =70 TECUmakes thepseudorange measurement errorgrow toδR $(2f)_S = \sigma_{R(f)}(2f)_S$ ≈35m, due toa narrowing oftransionospheric channel coherence band(28) toΔF_C ≈1.2…0.5MHzand the occurrence of severe FSF ($\frac{\Delta F_{0S}}{\Delta F_C} >> 1$) in the received signals.

Thus, the use ofdouble-frequency SNS NEUin combination with spread spectrum band signals (ΔF_{0S}=10MHz) in conditions of the normal ionosphere and NID ensures higher values of pseudorange measurement error(δR $(2f)_S$=0.6…0.9m) as compared to single-frequency NEU (where $\delta R \approx$10.6…11.8m). However, in AID conditions, this error can grow to the values δR $(2f)_S$ ≈35m, considerably exceeding error $\delta R \approx$11.8min these conditions of single-frequency NEU.

It should be noted thatin the case of oblique propagation of radiowaves through the ionosphere at angle $\alpha \approx 70^\circ$, pseudorange measurement errors (δR) in the navigation equipment of SNS users, given in Fig.5, can increase 3…11times at the same values of statistical TEC parametersin the ionosphere.Thus, pseudorange

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measurement errorgrows in the single-frequency NEU from $\delta R \approx$ 26m with NID to 45m – with AID, and in the double-frequency NEU – from 0.6m to 384m. When geometric factor$G$=2,these values of pseudoranges specify an increase in the error in determining the coordinates in the single-frequency SNS NEU from$\delta x = G\delta R$ ≈52mwith NID to 90m – with AID, and in the double-frequency NEU – from 1.2mto 768m.

Results

1. As a result of the conducted qualitative analysis and the development of the model of the ionosphere(Fig.1) and transionospheric RWP (Fig.2),it has been established thatlarge- and medium-scale variations in the ionospheric TEC$(\Delta N_{TL}(\rho), \Delta N_{TM}(\rho))$causean increase in the average value of ionospheric component$\overline{\delta R_{ion}}$ of pseudorange measurement error$\delta R = \overline{\delta R_{ion}} + \delta R_n$in the single-frequency SNS NEU, and small-scale TEC fluctuations ($\Delta N_{TS}(\rho) \equiv \Delta N_T(\rho_i)$) define the relative time delays of incoming beams (9) $\Delta\tau_i \sim \Delta N_T(\rho_i)/f_0^2$. Theycontribute to the occurrence of FSF (11-14) in the received signals. The latter are accompanied by distortions of the envelope shapeof received signals and bring about an increase in the noise component of pseudorange measurement errorδR_nin the single-frequency SNS NEU.

2. The analytical dependence (20) $\overline{\delta R_{ion}} = \psi(f_0, \overline{N}_T, \Delta N_{TL}, \Delta N_{TM})$has been established of the ionospheric componentof pseudorange measurement errorin the single-frequency SNS NEU on carrier frequency $f_0 = f_{02}$ ofthe signal and the average (background) value of the ionospheric TEC$\overline{N}_T(\rho) = \overline{N}_T + \Delta N_{TL}(\rho) + \Delta N_{TM}(\rho)$ that takes into account large- and medium-scale EC irregularities.

3. The procedure has been developed for estimation of the effect of small-scale ionospheric irregularities on the pseudorange measurement errorof single-frequency SNS NEU, which makes it possible to establish the analytical dependence $\sigma_{R(f)} = \psi\left(f_0, F_0, \frac{\overline{E}_r}{N_0}, \sigma_{\Delta N_T}\right)$of the noise component of pseudorange measurement erroron frequency parameters of transmitted SNS signals (carrier frequency f_0 and spectrum bandwidthF_0), signal/noise ratio$\overline{E}_r/N_0$at the receiver's input and mean-square deviation of small-scale fluctuations in the ionospheric TEC$\sigma_{\Delta N_T} = \left(\overline{\Delta N_T^2(\rho_i)}\right)^{0.5}$ as a set of expressions (37-39).

4. The procedure has been developed for estimation of the effect of small-scale ionospheric irregularities on the error in determining pseudorange of the double-frequency SNS NEU, which allows to obtain the analytical dependence $\sigma_{R(f)}(2f) = \psi\left(f_{02}, f_{01}, F_0, \frac{\overline{E}_r}{N_0}, \sigma_{\Delta N_T}\right)$of the noisepseudorange measurement error onfrequency parameters of transmitted SNS signals (carrier frequencies f_{02} and f_{01}, spectrum bandwidthF_0), signal/noise ratio $\overline{E}_r/N_0$ at the receiver's input and mean-square deviation of small-scale fluctuations in the ionospheric TEC $\sigma_{\Delta N_T}$ as a set of expressions (51, 52, 38, 46, 48).

5.The comparative analysis of the results of predicting thepseudorange measurement errorin single-frequency and double-frequency operation modes of SNS NEU in conditions of the normal and disturbed ionosphere(Fig.5) shows that the use of double-frequency SNS NEU in combination with spread spectrum band signals (ΔF_{0S}=10MHz) in conditions of the normal ionosphere and NID ensures higher values of pseudorange measurement error($\delta R(2f)_S$ =0.6…0.9m) as compared to single-frequency NEU (where$\delta R \approx$ 10.6…11.8m). However, in AID conditions, this error can grow to values $\delta R(2f)_S \approx$35m, considerably exceeding error $\delta R \approx$ 11.8min these conditions of single-frequency NEU due to a narrowing of transionospheric channel coherence bandwidth during artificial ionospheric disturbancesto the values$\Delta F_C \approx$1.2…0.5MHzand the occurrence of severe FSF ($\Delta F_{0S} >> \Delta F_C$) in received signals. In the case of oblique propagation of radiowaves through the ionosphere at angle $\alpha \approx 70^\circ$, the pseudorange measurement errorin conditions of AIDcan grow in the single-frequency NEU to $\delta R \approx$45m, in the double-frequency NEU – to 384m. When geometric factor$G$=2, these values of pseudoranges specifyan increase in the errorin determining the coordinates of single-frequency SNS NEU to$\delta x = G\delta R$ ≈90mwith AID, and of double-frequency SNS NEU - to 768m.

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