ANALYTICAL SOLUTIONS OF THE LAPLACE TRANSFORM AND APPLICATIONS TO DIFFERENTIAL EQUATIONS

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ABSTRACT: Laplace decomposition strategy is applied to accomplish arrangement arrangements of nonlinear partial differential equation. The strategy depends chiefly upon some broad theorems on (express) specific arrangements of certain groups of partial differential equations with the Laplace transform and the extension coefficients of binomial arrangement. A significant favorable position of partial math is that it very well may be considered as an excessively set of number request analytics. Along these lines, fragmentary analytics can possibly accomplish what whole number request math can't. It has been assume that huge numbers of the tremendous future improvements will originate from the utilizations of fragmentary math to various fields. Laplace transform is a persuasive mathematical apparatus applied in different regions of designing and science. With the expanding unpredictability of designing issues, Laplace transforms help in taking care of complex issues with a direct methodology simply like the utilizations of move capacities to explain ordinary differential equations. It will permit us to transform partial differential equations into algebraic equations and afterward by understanding these algebraic equations. The obscure capacity by utilizing the Inverse Laplace Transform can be acquired.

I. INTRODUCTION

In genuine, general arrangements of certain equations, particularly kind of elliptic, are not found. Genuine fractional differential equation frameworks when number of autonomous factors are even can be transformed to a perplexing halfway differential equations.
The solving a complex equation can more easier with complex methods. For example,

$$u_{xx} + u_{yy} = 0$$

Laplace equation hasn't got general solution in *R*2, but it can be written

$$u_{z\bar{z}} = 0$$

$$\Delta = \frac{\partial^2}{\partial z \partial \bar{z}}$$

$$u = f(z) + g(\bar{z})$$

where f is expository, g is hostile to systematic self-assertive capacities. A fractional differential equation framework which has two genuine dependant and two genuine independant factors can be transformed to an intricate equation. For instance,

$$u_x - v_y = 0$$
$$u_y + v_x = 0.$$

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Cauchy Riemann system transforms to complex equation

$$w_{\bar{z}} = 0$$

where*w* = *u*+*iv*, *z* = *x*+*iy*. All arrangements of this unpredictable equation are logical capacities. Additionally any request complex differential equation can be transformed to genuine halfway differential equation framework which has two questions, two autonomous factors by seperating the genuine and imaginer parts. The arrangement of complex equation can be advanced helping arrangements of this genuine framework. In this examination, we explore arrangements of first request consistent coefficients complex equations with laplace transforms.

Laplace transform utilizing a few zones of science is an indispensable transform. We can comprehend ordinary differential equations, arrangement of ordinary differential equation, indispensable equations, fundamental differential equations, contrast equations, basic distinction equations and furthermore transform in electrical circuits. Subsequently we can understand partial differential equations by means of laplace transforms. Nonlinear differential equations can be tackled laplace decomposition strategy.

Let f(t) be definedfor t ≥ 0: Then the Laplace transform of f; which is denoted by L[f(t)]or by F(s), is defined by the following equation

$$\mathcal{L}[f(t)] = F(s) = \lim_{T\to\infty} \int_0^T f(t)e^{-st}dt = \int_0^\infty f(t)e^{-st}dt$$

The basic which characterized a Laplace transform is an ill-advised necessary. An inappropriate essential may join or wander, contingent upon the integrand. At the point when the ill-advised fundamental in united then we state that the capacity f(t) has a Laplace transform. So what sorts of capacities have Laplace transforms, that is, the thing that kind of capacities ensures a united inappropriate basic.

The Laplace transform is a useful asset in applied arithmetic and designing. For all intents and purposes each starting course in differential equations at the undergrad level presents this procedure for explaining direct differential equations. The Laplace transform is vital in specific regions of control hypothesis.

Given a function f(x) defined for 0<< 0 x) (sF (5) dx ex f x f L s F sx□□ □) (=)] ([=) (0

$$F(s) = L[f(x)] = \int_0^\infty f(x)e^{-sx}dx$$

at least for those for which the integral converges.

Let be a continuous function on the interval which is of exponential order, that is, for some and) (xf) [0,□□□□c0 > x

$$\sup \frac{|f(x)|}{e^{cx}} < \infty .$$

In this case the Laplace transform exists for all .c s >

Some of the useful Laplace transforms which are applied in this paper, are as follows:

For and) (=)] ([sFxf L) (=)] ([sGxgL

$$L[f(x) + g(x)] = F(s) + G(s),$$

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$$L[x^{\beta}] = \frac{\Gamma(\beta+1)}{s^{\beta+1}}, \qquad \beta > -1,$$

$$L[f^{(n)}(x)] = s^{n}F(s) - s^{n-1}f(0) - s^{n-2}f^{\prime(n-1)}(0),$$

$$L[x^{n}\ f(x)] = (-1)^{n}\ F^{(n)}(s),$$

$$L[\int_0^x f(t)dt] = \frac{F(s)}{s},$$

$$L[\int_0^x f(x-t)g(t)dt] = F(s)G(s).$$

II. APPLICATION OF THE LAPLACE TRANSFORM

It is basic in designing instruction to locate the point of view that the Laplace transform is only a hypothetical and mathematical idea (outside of this present reality) with no application in others regions. The transforms are considered as an apparatus to make mathematical estimations simpler. Notwithstanding, it is essential to see that "recurrence space" is conceivable acknowledge likewise in reality and applied in different territories like, for instance, financial matters.

"The most popular application of the Laplace transform is in electronic engineering, but it has also been applied to the economic and managerial problems, and most recently, to Materials Requirement Planning (MRP)"

It is pointed up that the procedure of the Laplace transform has discovered a developing number of utilizations in the fields of material science and innovation. For instance, there is probability of tackling issues in limiting by this strategy. With no loss of normal legitimacy, it is made realized that the rebate factor can generally be written in an exponential style which suggests that the current estimation of an "income" will acquire an extremely basic structure in the Laplace transform wording. This straightforwardness holds well for stochastic just as for deterministic financial cycles. It could be likewise applied to all mathematical rearrangements of the real world. Stochastic stock technique in which request is created by people isolated by free stochastic time spans while creation happens in clusters of changing sizes at various focuses in time. The subsequent cycles are investigated utilizing the Laplace transform strategy. At that point planned a summed up input-lattice to include necessities just as creation lead times by methods for z-transform in a discrete time portrayal. The hypothesis is reached out to consistent time utilizing the Laplace transform, which empowers it to include the chance of bunch creation at limited creation rates. Likewise they built up an essential strategy for how such security ace creation plans can be determinate in basic cases utilizing the Laplace transform technique.

III. APPLICATION OF LAPLACE TRANSFORM METHOD TO ODE

Let us apply Laplace transform on both sides of

$$y''(t) + y(t) = 1,$$

With initial condition y (0)=1 y'(0)= 1 we get

$$\begin{aligned} s^2 L\{y(t)\} - s - 1 + L\{y(t)\} &= \frac{1}{s} \\ L\{y(t)\} &= \frac{s+1}{s^2+1} + \frac{1}{s(s^2+1)} = \frac{1}{s} + \frac{1}{s^2+1} \\ y(t) &= 1 + \sin t. \end{aligned}$$

Hence we get the desired exact solution

$$x(t) = e^{-t} y(t) = e^{-t}(1 + \sin t).$$

IV. APPLICATION OF LAPLACE DECOMPOSITION METHOD TO ODE

Let us apply decomposition series technique in the Laplace transform method, wetake straightaway the equation

$$L\{y(t)\} = \frac{1}{s} + \frac{1}{s^2} + \frac{1}{s^3} - \frac{1}{s^2} L\{y(t)\}.$$

The Laplace decomposition series can be taken as

$$\begin{aligned} L\{y(t)\} &= \frac{1}{s} + \sum_{n=1}^{\infty} L\{y_n(t)\} \\ \frac{1}{s} + \sum_{n=1}^{\infty} L\{y_n(t)\} &= \frac{1}{s} + \frac{1}{s^2} + \frac{1}{s^3} - \frac{1}{s^2}\left[\frac{1}{s} + \sum_{n=1}^{\infty} L\{y_n(t)\}\right] \\ L\{y_1(t)\} + \sum_{n=2}^{\infty} L\{y_n(t)\} &= \frac{1}{s^2} - \frac{1}{s^2} \sum_{n=2}^{\infty} L\{y_{n-1}(t)\}. \end{aligned}$$

We can obtain an integration to compute L {yn(t)} as follows

$$\begin{aligned} L\{y_1(t)\} &= \frac{1}{s^2} \\ L\{y_2(t)\} &= -\frac{1}{s^2} L\{y_1(t)\} = -\frac{1}{s^4} \\ &\vdots \\ L\{y_n(t)\} &= -\frac{1}{s^2} L\{y_{n-1}(t)\} = \frac{(-1)^{n-1}}{s^{2n}} \\ &\vdots \end{aligned}$$

Hence by using Laplace decomposition series for L {y(t)}, we arrive at

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$$\begin{aligned} L\{y(t)\} &= \frac{1}{s} + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{s^{2n}}, \qquad s > 1 \\ &= \frac{1}{s} + \frac{1}{s^2+1} \\ y(t) &= L^{-1}\left\{\frac{1}{s}\right\} + L^{-1}\left\{\frac{1}{s^2+1}\right\} = 1 + \sin t \end{aligned}$$

and hence the desired solution is

$$x(t) = e^{-t} y(t) = e^{-t}(1 + \sin t) \ .$$

V. APPLICATION OF LAPLACE TRANSFORM METHOD TO DDE

Now multiplying both sides of by $e^{-st}$and integrate between !and, weobtain

$$\begin{aligned} \int_{\omega}^{\infty} y''(t)\ e^{-st} dt + \int_{\omega}^{\infty} y(t-\omega)\ e^{-st} dt &= \int_{\omega}^{\infty} 1 \cdot e^{-st} dt \\ \int_{0}^{\infty} y''(t) e^{-st} dt + e^{-\omega s} \int_{0}^{\infty} y(t) e^{-st} dt &= \frac{e^{-\omega s}}{s} \\ L\{y''(t)\} + e^{-\omega s} L\{y(t)\} &= \frac{e^{-\omega s}}{s} \\ s^2 L\{y(t)\} - s - 1 + e^{-\omega s} L\{y(t)\} &= \frac{e^{-\omega s}}{s} \end{aligned}$$

In step two, we have used y00(t) = 0, 0 _ t _ ! and in step four, we have usedy(0) = 1, y0(0) = 1.

$$\left(1 + \frac{e^{-\omega s}}{s^2}\right) L\{y(t)\} = \frac{1}{s} + \frac{1}{s^2} + \frac{e^{-\omega s}}{s^3}$$

$$\begin{aligned} L\{y(t)\} &= \frac{1}{s} + \frac{1}{s^2} \frac{1}{\left(1 + \frac{e^{-\omega s}}{s^2}\right)} \\ &= \frac{1}{s} + \frac{1}{s^2} + \sum_{n=1}^{\infty} (-1)^n \frac{e^{-n\omega s}}{s^{2n+2}}. \end{aligned}$$

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Now by applying inverse Laplace transform, we get

$$y(t) = 1 + t + \sum_{n=1}^{\infty} (-1)^n \frac{(t - n\omega)^{2n+1}}{(2n+1)!} e(t - n\omega).$$

Hence the exact solution in each interval is given by

$$y(t) = 1 + t + \sum_{n=1}^{N} (-1)^n \frac{(t - n\omega)^{2n+1}}{(2n+1)!}, \qquad N\omega \leq t \leq (N+1)\omega,$$
$$N = 1, 2, 3, \ldots \; .$$

when we allow $\varphi \rightarrow 0$, we get back the exact solution of the ODE, namely,y(t) = 1 + sin(t).

VI. CONCLUSIONS

The Laplace transform is an incredible asset in applied science and designing and have been applied for comprehending direct differential equations. In this paper, the use of Laplace transform is examined to get a definite arrangement of some straight fragmentary differential equations. The partial subordinates are portrayed in the Caputo sense which acquired by Riemann-Liouville fragmentary fundamental administrator. Tackling a few issues show that the Laplace transform is an incredible and productive methods for getting logical arrangement of straight fragmentary differential equations. The Laplace decomposition technique is without a doubt fine mix of both Laplace transform strategy and Adomian decomposition strategies. Laplace decomposition technique shows adaptability just as gives accommodation in the calculation of logical answers for both straight and nonlinear issues. Laplace decomposition strategy has created definite or surmised arrangement in every span with smooth calculation. Laplace transform strategies to unravel the time-fragmentary American alternative evaluating under system exchanging models. The estimation of the American alternative with system exchanging is defined as the answer for a free limit issue of time-fragmentary halfway differential equation framework. The Laplace transform is executed for the time factors and the subsequent framework PDEs are tackled logically. Thusly an arrangement of nonlinear algebraic equations for the free limits is gotten and fathomed utilizing secant strategies. At last mathematical Laplace reversal is applied to recoup the early exercise limits and the choice qualities. Examinations between the LTM and the benchmark FDM are made by means of mathematical models, which shows that the LTM is productive for valuing American alternatives with system exchanging. Nonetheless, the LTM is as yet trying for more mind boggling models like the system exchanging models with state-subordinate bounce disseminations.

VII. REFERENCES

[1] Podlubny, Geometric and physical interpretation of fractional integration and fractional differentiation, Fractional Calculus and Applied Analysis, (2002)

[2] S. Momani, N. T. Shawagfeh, Decomposition method for solving fractional Riccati differentialequations, Applied Mathematics and Computation, (2006)

[3] Manuel D. Ortigueira , Delfim F.M. Torres , Juan J. Trujillo, "Exponentials and Laplacetransforms on nonuniform time scales", Communications in Nonlinear Science NumericalSimulation, (2016)

[4] El-Sayed S. M. and Kaya D., "On the numerical solution of the system oftwo-dimensional Burgers equations by decomposition method", Appl. Mathand Comput.(2004.)

[5] Dr. SarojChoudharyApplications of Laplace Transform and Solution for Various Fractional Differential Equations(2015).

[6] A. V. Bobylev, The inverse Laplace transform of some analytic functions with an application to the eternal solutions of the Boltzmann equation, Applied Mathematics Letters, (2002),.

[7] S. Kazem, Exact solution of some linear fractional differential equations by laplace transform, International Journal of Nonlinear Science, (2013)

[8] Vaithyasubramanian study on applications of laplace transformation(2018)

[9] Wazwaz, A. M. (2010). The combined Laplace transform-Adomian decomposition method for handling nonlinear Volterraintegro-differential equations. *Appl. Math. Comput*

ISSN- 2394-5125 VOL 7, ISSUE 08, 2020

[10] Mohamed, M. A. and Torky, M. S., "Numerical solution of nonlinear system of parial differential equations by the Laplace decomposition method and the padeapproximation", American Journal of Computational Mathematics, (2013).
[11] Yusufoglu, E., "Numerical Solution of duffing equation by the Laplace decomposition algorithm", Applied Mathematics and Computation, (2006).
[12] Kazem, S.,"Exact Solution of Some Linear Fractional Differential Equations by Laplace Transform", International Journal of Nonlinear Science, (2013)
[13] Kexue, L., Jigen, P. "Laplace transform and fractional differential equations", Applied Mathematics Letters, (2011)
[14] Gupta, S., Kumar, D., Singh, J."Numerical study for systems of fractional differential equations via Laplace transform ", Journal ofthe Egyptian Mathematical Society, (2015)