

# **Rheology And Temperature Dependent Properties In Thermoviscoelasticity**

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## **ABSTRACT**

In the present paper Maxwell and Kelvin-Voigt solid are considered. Rheological properties of volume and as well as Rheological properties of density have been introduced in the basic equations of thermo-elasticity. Effects of temperature on mechanical properties of material are shown here. Equations are constructed in combinations of these properties.

**Keywords:** Visco-elasticity, Rheology, Kelvin-Voigt solid, Density property, Volume property.

## **1. INTRODUCTION**

Almost all solid bodies possess the property of regaining their original size or shapes if the deformations and the stresses which produce the deformations do not exceed certain limits. These limits are known as elastic limit and yield stress. Many solid bodies deform and recover their original size or configuration when deforming activity is removed. For the result of reduction of external forces from a body by which it comes back to its original configuration is called elasticity and the body is called elastic body. The theory of elasticity, often called elasticity in short, is the branch of solid mechanics which deals with the stresses and deformations in elastic solids produced by external forces.

When an elastic body is subject to temperature change, its elements tend to expand or contract. But due to the mutual constraint between the elements or due to external constraint in addition, such an expansion or contraction generally cannot proceed freely and thermal stresses are set up in the body. The motion of a body is governed by the mutual interaction between deformation and temperature field and hence coupling of deformation and temperature field occurs. The stress system in the solid gets modified with a temperature gradient term in the basic theory. It represents a generalization of both the theory of elasticity and theory of heat conduction in solids. In 1837, Duhamel [1] introduced the uncoupled thermoelasticity to the theory. The theory was developed by Neumann [2] in the year 1885. Applying classical theory of thermoelasticity in isotropic and anisotropic media, revolutionary works related to thermal stresses done by various research workers like Hilton [3], Boley [4], Nowacki [5], Sharma [6]. Biot [7] build up the coupled theory of thermoelasticity in the second half of the twentieth century. Valuable information's and important guidelines in classical theory of elasticity and linear theory of thermoelasticity can be obtained from the books of famous authors like Boley and Weiner [8], Nowacki [9], etc. These texts provide a wide range of review of earlier works on specific problems in this field and the related fields.

There has been some interest in the theory of hyperbolic thermoelasticity. So many authors have discussed thermoelasticity theory on the basis of modified Fourier's law of heat conduction. Lord and Shulman [10] constructed a new law of heat conduction by incorporating a flux rate term into the Fourier's law of heat conduction equation. Another hyperbolic thermoelasticity is based on temperature rate dependent thermoelasticity. Generalization of entropy production inequality that led to restrictions on a class of constitutive equations was proposed by Green and Laws [11]. In 1972, Green and Lindsay [12] obtained an explicit version of the constitutive equations by postulating temperature rate term into the constitutive equations. In 1986,

Chandrasekharaiah [13] referred a wave-like thermal signal as “second sound”. In this paper he presented a review of the literature on the theory of thermoelasticity with second sound.

Idea about viscoelasticity came into the mind of researchers in the second half of the nineteenth century. In 1865, Thomson [14] analyzed the elasticity and viscosity of metals. Viscous properties of solids were introduced in the theory of elasticity by Maxwell [15] in 1867. Later Kelvin [16] and Voigt [17] followed Maxwell and developed the theory of viscoelasticity. The governing equations and various discussions in this field of viscoelasticity and thermoviscoelasticity can be found in books of various authors like Bland [18], Eringen [19] etc. Some important works on viscoelastic materials have been completed by research workers. In 2002, Ezzat and Karamany [20], Ezzat et al. [21] give their efforts to analyzed the state space approach and they stand uniqueness and reciprocity theorems mainly on generalized thermoelasticity introducing the viscous effects.

Rheology is the study of the flow and deformation of matter, primarily in the liquid state, but also as soft solids. One of the major tasks of rheology is to empirically establish the relationships between deformations and stresses. In the field of thermoviscoelasticity two relaxation functions are well known one is characterizing by the rheological properties of volume and another is characterizing by the rheological properties of the density. Ezzat and Karamany [22], Aouadi and Karamany [23] investigated the new model of the equations of generalized thermoviscoelasticity for isotropic media taking into consideration the rheological properties of the volume. With the effect of the volume properties Ezzat et al. [24] solved the problem on viscoelastic materials in generalized thermoelasticity.

Leadermann [25] suggested the temperature-dependent mechanical properties of the materials. The idea on the temperature dependence of the mechanical properties of viscoelastic materials was proposed by Ferry [26]. Later on, Schwarzel and Stavermann [27], Muki and Sternberg [28] discussed the applications of such description. In 1976, Lomakin [29] analyzed how the mechanical properties like the modulus of elasticity, the co-efficient of thermal expansion, the thermal conductivity, the bulk modulus, density etc. depend on temperature.

## 2. THERMOELASTICITY

The linear dynamical theory of classical thermoelasticity consists of the coupled partial differential equation of motion and the Fourier heat conduction equation. The former equation, in terms of displacement field, is governed by a wave type hyperbolic equation, whereas, the later equation, for temperature field, is a parabolic type equation. For the heat flux vector  $\vec{q}$  and temperature gradient  $\vec{\nabla}T$ , the classical Fourier’s law of heat conduction is

$$\vec{q} = -k \vec{\nabla}T \quad (2.1)$$

and the classical parabolic-type heat transport equation with heat source term  $Q$  is

$$k\nabla^2 T = \rho C_E \frac{\partial T}{\partial t} - Q. \quad (2.2)$$

## 3. GENERALIZED THERMOELASTICITY

The non-classical theory of thermoelasticity was the significant area to the researchers. The heat transport equation (2.2) is a parabolic type partial differential equation. In the theory of thermoelastic diffusion the coupled thermoelastic model implies infinite speed of propagation of thermal waves. The conventional theory of thermoelasticity, based on the classical Fourier’s law of heat conduction predicts that if a material is subject to a thermal disturbance, the effects of the disturbance will be felt instantaneously at distances infinitely far from its source. This is physically unrealistic drawback as the experimental fact provides that the temperature field propagation takes places with finite velocity. Hyperbolic heat transport equation, in place of parabolic heat equation, admits wave-like thermal signals propagating with finite speed. Since both equation of motion and heat conduction equation in the theory of thermoelasticity, in this regards, are hyperbolic type wave equation so the theory is called hyperbolic thermoelasticity theory. To overcome that physically unrealistic drawback on the propagation of thermal signals suitable substitutions in the expressions for the heat flux vector have been formulated by several researchers. The approach is known as extended irreversible thermodynamics which

introduce time derivative of the heat flux vector in the classical Fourier’s law. Hyperbolic heat conduction theory is based on the modified Fourier’s law.

The modified Fourier’s law of heat conduction is

$$\vec{q} + \tau \frac{\partial \vec{q}}{\partial t} = -k \vec{\nabla} T \tag{3.1}$$

and corresponding heat transport equation with heat source term is

$$k \nabla^2 T = \left(1 + \tau \frac{\partial}{\partial t}\right) \left(\rho C_E \frac{\partial T}{\partial t} - Q\right). \tag{3.2}$$

Here  $\tau$  is a non-negative parameter interpreted as the time-lag between the temperature gradient and the resulting heat flux vector.  $\tau$  is also known as a characteristic of the theory and is a thermodynamical property of the material. The heat conduction equation (3.2) is a hyperbolic-type differential equation. In the absence of time-lag i.e. if  $\tau = 0$  the modified Fourier’s law of heat conduction equation (3.1) reduces to classical Fourier’s law represents by the equation (2.1). And with the same situation the heat transport equation (3.2) will convert into the equation (2.2).

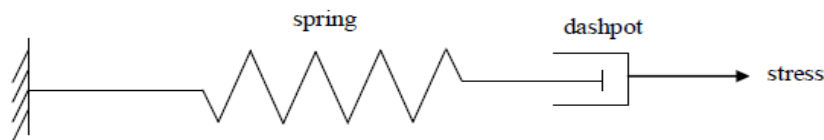
Generalized thermoelasticity theories have been developed with the objective of removing the paradox of infinite speed of heat propagation inherent in the conventional coupled dynamical theory of thermoelasticity in which the parabolic type heat conduction equation is based on Fourier’s law of heat conduction.

**4. VISCOELASTICITY WITH RHEOLOGICAL PROPERTIES**

Viscoelasticity is one of the principal types of inelastic behavior and it has enormous significance to pervasive categories of materials such as polymers, glass, solid propellants, electronic circuit boards, rocksalt, sandwich structures, concrete, biological and geological substances, etc. Under the influence of external loads all kind of solids not only deform but also flow upto a certain extent and disperse energy. Deformation and flow generate heat and vice-versa. Thus physical bodies possess the characteristics of both solid and fluids. Within the range of recoverable strains, the material deformation strain increases linearly with increasing applied stress. If the material deformation rate increases linearly with increase in applied stress then the material is viscous. If the material’s behaviors are characterized by combination of viscous and elastic components then the material is viscoelastic. Deformation of materials on application of external applied stress is either permanent or temporary. Both deformations can be a function of time or virtually independent of time. When both recoverable and permanent deformations occur together and are time dependent, we have viscoelastic deformation. The ratio of applied stress to the time-dependent strain decreases with increasing temperature. In general, an increase in temperature correlates to a logarithmic decrease in the time required to impart equal strain under a constant strain. That means it takes less work to stretch a viscoelastic material an equal distance at a higher temperature than it does at a lower temperature. The elastic theory, deals with the thermal effects to the viscoelastic materials traditionally known as theory of thermoviscoelasticity.

A spring represents the perfectly elastic behavior and a dashpot represents a pure viscous behavior. In combination of Hook’s law and Newton’s law for viscous, there are two types of viscoelastic solids.

- I) Maxwell Solid:- This type of solid is a series combination of a spring and a dashpot. In this case the stress constitutive equation involves the material time rate of the stress. Here the spring and dashpot support the same stress and strain in the element is the sum of the strain in the spring and the strain in the dashpot.



**Maxwell-solid**

Figure 1. Spring-dashpot series model

- II) Kelvin-Voigt Solid:- This kind of solid is the parallel combination of a spring and a dashpot. Here the stress depends on the strain and deformation-rate tensor. The strain in both spring and dashpot is the same at any time. The stress supported by the element is the sum of the stresses in the spring and dashpot.

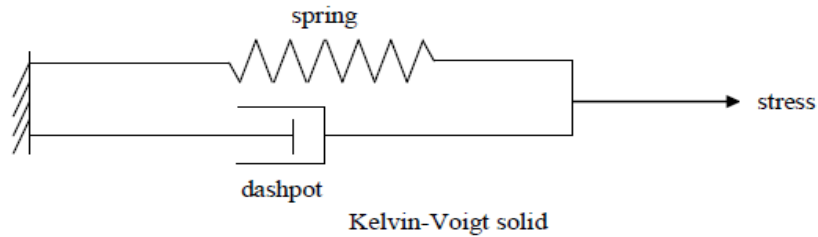


Figure 2. Spring-dashpot parallel model

III) Standard linear solid:- The standard linear solid model effectively combines the Maxwell model and a Hookean spring in parallel. A viscous material is modeled as a spring and a dashpot in series with each other, both of which are in parallel with a single spring. Under a constant stress, the modeled material will instantaneously deform to some strain, which is the elastic portion of the strain, and after that it will continue to deform and asymptotically approach a steady state strain. This last portion is the viscous part of the strain. Although the standard linear solid model is more accurate than the Maxwell and Kelvin-Voigt models in predicting material responses, mathematically it returns inaccurate result for strain under specific loading conditions and is rather difficult to calculate.

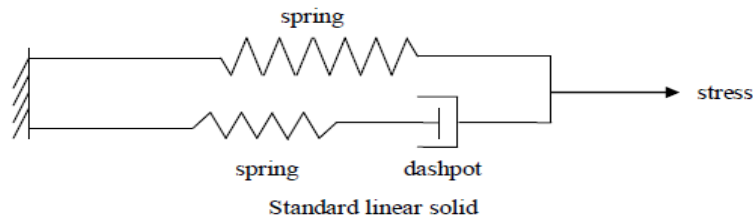


Figure 3. Spring-dashpot standard linear model

Materials used in applied science lab like metallurgy, material science, geological science, etc. does not behave like pure fluid or pure elastic. The characteristics of both fluid and elastic are found in these materials. Recently in the field of science, these viscoelastic materials are gaining importance. Uses of viscoelastic materials play an important role in nuclear physics, plasma physics, geophysics etc. The instruments broadband viscoelastic spectroscopy (BVS) and resonant ultrasound spectroscopy (RUS) are commonly used to test viscoelastic behavior of the materials. Analysis and study of the structure of natural viscoelastic composites such as synthetic composite, biomaterial and cellular solid is very important now a days.

The science of rheology is nearly a century in age. The rheology comes from Greek rheos meaning to flow. The Greek philosopher Heraclitus described rheology as panta rei- everything flows. Translated into rheological terms by Marcus Reiner this means everything will flow if you just wait long enough. Rheology is the study of the flow and deformation of matter, primarily in the liquid state, but also as soft solids or solids under conditions in which they respond with plastic flow rather than deforming elastically in response to an applied force. Rheology generally accounts for the behavior of non-Newtonian fluids, by characterizing the minimum number of functions that are needed to relate stresses with rate of change of strains. Many commonly-used materials and formulations exhibit complex rheological properties, whose viscoelasticity can vary depending upon the external conditions applied, such as stress, strain, timescale and temperature. Rheological properties impact all the stages of material use across multiple industries.

The rheological properties are always characterized by relaxation functions which depends on the non-negative time parameter  $t$ . On consideration of relaxation function  $R_1(t)$ , relaxation function characterizing the rheological volume properties  $R_2(t)$  and relaxation function characterizing the rheological properties of density  $R_3(t)$ , a compact form of expressions in view of Illiushin and Pobedria [30] may be consider as

$$\left. \begin{aligned} R_1(t) &= 2\mu \left( 1 - M_1 \int_0^t g_1(t) dt \right) \\ R_2(t) &= K \left( 1 - M_2 \int_0^t g_2(t) dt \right) \\ R_3(t) &= \rho \left( 1 - M_3 \int_0^t g_3(t) dt \right) \end{aligned} \right\} \quad (4.1)$$

Here the relaxation functions are decreasing functions of time such that (Karamany [31])

$$R_1(t) > 0, R_2(t) > 0, R_3(t) > 0 \text{ and } 3 R_2(t) - R_1(t) > 0, 0 \leq t < \infty.$$

And  $M_1, M_2, M_3$  are the non-negative empirical constants. The functions  $g_i(t), i = 1, 2, 3$  are generally chosen in the form of exponential functions (Koultunov [32]) in time parameter as

$$g_i(t) = e^{-\beta t} t^{\alpha-1}, i = 1, 2, 3. \quad (4.2)$$

For different values of  $\beta$  and  $\alpha$  satisfying the conditions  $\beta > 0$  and  $1 > \alpha > 0$ ,  $g_i(t)$  takes the different experimental data for the time range  $0 \leq t < \infty$ . The constant  $\alpha$  is usually taken in such a way that it satisfies the following inequality with empirical constants  $M_1, M_2, M_3$ .

$$0 \leq M_2 \leq M_3 \leq M_1 < \Gamma(\alpha)$$

The above equation (4.1) can be considered as zero rheological volume properties by setting the empirical constant  $M_2 = 0$ , i.e.  $R_2(t) = K$ . For  $M_3 = 0$ , i.e. for  $R_3(t) = \rho$  the equation will be one with zero rheological density properties. In case of consideration of  $M_2 = M_3 = 0$ , (i.e.  $R_2(t) = K$  and  $R_3(t) = \rho$ ) the above equation (4.1) will be one without rheological volume properties as well as without rheological density properties.

**5. TEMPERATURE DEPENDENT MECHANICAL PROPERTIES**

Experiments say that at high temperatures the mechanical properties of the materials may not be constant. At high temperatures, the mechanical properties of viscoelastic materials change with change in their temperature. In fact they are the functions of temperature. The consideration of this dependence widens the field area of the elasticity theory for researches.

Eventually mechanical properties are the functions of the temperature parameter and mathematical expressions for these functions give the clear idea about dependences. Some of the mechanical properties like Lamé's constant, bulk modulus, density and coefficient of linear thermal expansion for the thermoviscoelastic materials can be consider here in the form of

$$\left. \begin{aligned} \mu &= \mu_0 \psi_0(T) \\ K &= K_0 \psi_0(T) \\ \rho &= \rho_0 \psi_1(T) \\ \alpha_T &= \alpha_T^0 \psi_2(T) \end{aligned} \right\} \quad (5.1)$$

where  $\mu_0, K_0, \rho_0$  and  $\alpha_T^0$  are the constants and  $\psi_i(T); i = 0, 1, 2$  are the non-dimensional functions of temperature.  $\psi_i(T)$  can be taken in two different types of forms according as the empirical constants which are as follows

$$\left. \begin{aligned} \psi_i(T) &= 1 - \zeta_i(T - T_r), \quad i = 0, 1, 2 \\ &\text{or} \\ \psi_i(T) &= \exp(-\zeta_i^*(T - T_r)), \quad i = 0, 1, 2. \end{aligned} \right\} \quad (5.2)$$

According to Manson [33],  $\zeta_i$  and  $\zeta_i^*$  are known as empirical constants which are measured in  $1/^\circ\text{K}$  and  $\zeta_i > 0$  and  $\zeta_i^* > 0$  for  $i = 0, 1$  and  $\zeta_2 < 0, \zeta_2^* < 0$ .

For the consideration of infinitesimal temperature deviations from reference temperature it can be replace  $T$  by the reference temperature  $T_0$  in the equations (5.1) and (5.2). And in that case the equation (5.1) will reduced to one with the following equation.

$$\left. \begin{aligned} \psi_i(T_0) &= 1 - \zeta_i(T_0 - T_r), \quad i = 0,1,2 \\ &or \\ \psi_i(T_0) &= \exp(-\zeta_i^*(T_0 - T_r)), \quad i = 0,1,2. \end{aligned} \right\} \quad (5.3)$$

In above the equation (5.1) be consider for temperature dependent mechanical properties. Temperature independent case for mechanical properties can also be reduced from the equation (5.1) by considering  $\psi_i(T) = 1, i = 0, 1, 2$  and hence the elastic moduli in equation (5.1) reduced to  $\mu = \mu_0, K = K_0, \rho = \rho_0, \alpha_T = \alpha_T^0$ .

**6. SOME BASIC EQUATIONS**

In the theory of thermoviscoelasticity the stress, strains and temperature govern by some basic equations. The governing equations are considered for linear isotopic viscoelastic solids. Stress tensor and strain tensor and temperature are denoted by  $\sigma_{ij}, \epsilon_{ij}$  and  $T$  respectively.

**Classical theory**

Equation of motion without body force and heat source:

$$\sum_{j=1}^3 \sigma_{ij,j} = \rho \ddot{u}_i \quad (6.1)$$

The stress constitutive equation (Eringen [19]):

$$R^* \sigma_{kk} \delta_{ij} + 2 S^* \sigma_{ij} = P^* \epsilon_{kk} \delta_{ij} + 2 Q^* \epsilon_{ij}, \quad (6.2)$$

where  $P^*, Q^*, R^*, S^*$  are linear differential operators.

In particular for the involvement of Maxwell solid and Kelvin-Voigt solid the differential forms of constitutive equations are (Eringen [19]):

Maxwell form:

$$\dot{\sigma}_{ij} + \alpha_m \sigma_{kk} \delta_{ij} + 2 \beta_m \sigma_{ij} = \lambda_m \dot{\epsilon}_{kk} \delta_{ij} + 2 \mu_m \dot{\epsilon}_{ij}, \quad (6.3)$$

Kelvin-Voigt form:

$$\sigma_{ij} = \left( \lambda_e + \lambda_v \frac{\partial}{\partial t} \right) \epsilon_{kk} \delta_{ij} + 2 \left( \mu_e + \mu_v \frac{\partial}{\partial t} \right) \epsilon_{ij}, \quad (6.4)$$

where  $\lambda_e, \lambda_v, \mu_e, \mu_v, \alpha_m, \beta_m, \lambda_m, \mu_m$  are the material moduli.

The constitutive equation in functional form (Eringen [19]):

$$\sigma_{ij} = \lambda_e \epsilon_{kk} \delta_{ij} + 2 \mu_e \epsilon_{ij} + \int_{-\infty}^t \left\{ \lambda_v(t-s) \frac{\partial \epsilon_{kk}(s)}{\partial s} \delta_{ij} + 2 \mu_v(t-s) \frac{\partial \epsilon_{ij}(s)}{\partial s} \right\} ds \quad (6.5)$$

here  $\lambda_v, \mu_v$  are the memory function of time.

For classical coupled case the stress constitutive equations are (Eringen [19]):

In differential form:

$$\sigma_{ij} = -3K\alpha_T T \delta_{ij} + \left( \lambda_e + \lambda_v \frac{\partial}{\partial t} \right) \epsilon_{kk} \delta_{ij} + 2 \left( \mu_e + \mu_v \frac{\partial}{\partial t} \right) \epsilon_{ij}. \quad (6.6)$$

In functional form:

$$\begin{aligned} \sigma_{ij} = & -3K\alpha_T T \delta_{ij} + \lambda_e \epsilon_{kk} \delta_{ij} + 2 \mu_e \epsilon_{ij} \\ & + \int_{-\infty}^t \left\{ \lambda_v(t-s, T) \frac{\partial \epsilon_{kk}(s)}{\partial s} \delta_{ij} + 2 \mu_v(t-s, T) \frac{\partial \epsilon_{ij}(s)}{\partial s} \right\} ds \end{aligned} \quad (6.7)$$

here  $\beta, \lambda_e, \lambda_v, \mu_e, \mu_v$  are the functions of temperature.

**Generalized theory**

Equation of motion for generalized theory of thermoviscoelasticity will be the same as it was in (1.7.1) for classical theory.

i) For Lord-Shulman theory it contains one relaxation time  $\tau_1$  and basic equations are:

Equation of heat conduction:

$$k \nabla^2 T = \rho C_E (\dot{T} + \tau_1 \ddot{T}) + 3KT_0 \alpha_T (\dot{\epsilon} + \tau_1 \ddot{\epsilon}) \quad (6.8)$$

Stress constitutive equation:

a) Differential form

$$\sigma_{ij} = \left( \lambda_e + \lambda_v \frac{\partial}{\partial t} \right) \epsilon_{kk} \delta_{ij} + 2 \left( \mu_e + \mu_v \frac{\partial}{\partial t} \right) \epsilon_{ij} - 3K\alpha_T T \delta_{ij} \quad (6.9)$$

b) Functional form

$$\sigma_{ij} = \int_0^t R_1(t - \tau) \frac{\partial e_{ij}}{\partial \tau} d\tau + Ke\delta_{ij} - 3K\alpha_T(T - T_0)\delta_{ij} \quad (6.10)$$

ii) For Green-Lindsay theory it contains two relaxation times  $\tau_2, \tau_3$  and basic equations are:

Equation of heat conduction:

$$k\nabla^2 T = \rho C_E(\dot{T} + \tau_2 \ddot{T}) + 3KT_0\alpha_T \dot{e} \quad (6.11)$$

Stress constitutive equation:

a) Differential form:

$$\sigma_{ij} = \left(\lambda_e + \lambda_v \frac{\partial}{\partial t}\right) e \delta_{ij} + 2\left(\mu_e + \mu_v \frac{\partial}{\partial t}\right) \varepsilon_{ij} - 3K\alpha_T(T + \tau_3 \dot{T})\delta_{ij} \quad (6.12)$$

b) Functional form:

$$\sigma_{ij} = \int_0^t R_1(t - \tau) \frac{\partial e_{ij}}{\partial \tau} d\tau + Ke\delta_{ij} - 3K\alpha_T(T - T_0 + \tau_3 \dot{T})\delta_{ij} \quad (6.13)$$

iii) In generalized theory of thermoviscoelasticity the basic equations for composite of Lord-Shulman and Green-Lindsay theory are governed by combining above (i) and (ii)

Heat conduction equation:

$$k\nabla^2 T = \rho C_E(\dot{T} + \tau_2 \ddot{T}) + 3KT_0\alpha_T(\dot{e} + \tau_3 \ddot{e}) \quad (6.14)$$

Stress constitutive equation in functional form:

$$\sigma_{ij} = \int_0^t R_1(t - \tau) \frac{\partial e_{ij}}{\partial \tau} d\tau + Ke\delta_{ij} - 3K\alpha_T(T - T_0 + \tau_1 \dot{T})\delta_{ij} \quad (6.15)$$

Here  $\tau_1, \tau_2, \tau_3$  are relaxation times.  $\tau_1 = 0, \tau_2 = \tau_3 \neq 0$  correspond the Lord-Shulman theory and  $\tau_1 \neq 0, \tau_2 \neq 0, \tau_3 = 0$  correspond the Green-Lindsay theory.

iv) Basic equations for generalized thermoviscoelasticity characterize by rheological volume and density properties are

Heat conduction equation:

$$k\nabla^2 T = C_E \int_0^t R_3(t - \tau) \frac{\partial}{\partial \tau} \left( \frac{\partial T}{\partial \tau} + \tau_2 \frac{\partial^2 T}{\partial \tau^2} \right) d\tau + 3T_0\alpha_T \int_0^t R_2(t - \tau) \frac{\partial}{\partial \tau} \left( \frac{\partial e}{\partial \tau} + \tau_3 \frac{\partial^2 e}{\partial \tau^2} \right) d\tau \quad (6.16)$$

Stress constitutive equations in functional form:

$$S_{ij} = \int_0^t R_1(t - \tau) \frac{\partial e_{ij}}{\partial \tau} d\tau, \quad (6.17)$$

$$\sigma = \int_0^t R_2(t - \tau) \frac{\partial}{\partial \tau} \{e - 3\alpha_T(T - T_0 + \tau_1 \dot{T})\} d\tau \quad (6.18)$$

$$\sigma_{ij} = \int_0^t R_1(t - \tau) \frac{\partial e_{ij}}{\partial \tau} d\tau + \delta_{ij} \int_0^t R_2(t - \tau) \frac{\partial}{\partial \tau} \{e - 3\alpha_T(T - T_0 + \tau_1 \dot{T})\} d\tau \quad (6.19)$$

where

$$S_{ij} = \sigma_{ij} - \sigma\delta_{ij}; \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}); e_{ij} = \varepsilon_{ij} - \frac{e}{3}\delta_{ij}; e = \sum_{k=1}^3 \varepsilon_{kk}; \sigma = \frac{1}{3} \sum_{k=1}^3 \sigma_{kk}.$$

Depending upon the values of thermal relaxation times  $\tau_1, \tau_2, \tau_3$  the above equations can be reduced according as bellow.

- When  $\tau_1 = \tau_2 = \tau_3 = 0$  the above one corresponds the classical theory (CD-theory).
- When  $\tau_1 = 0, \tau_2 = \tau_3 \neq 0$  the above one corresponds the Lord-Shulman theory (LS-theory).
- When  $\tau_1 \neq 0, \tau_2 \neq 0, \tau_3 = 0$  the above one corresponds the Green-Lindsay theory (GL-theory).

## CONCLUSION

Generalized thermoelasticity theories have been developed with the objective of removing the paradox of infinite speed of heat propagation. Rheological volume property and density property have notable effect on the

constitutive equations. Mechanical properties are shown in functional form of temperature and consequence exposed in relative equations.

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