FRACTIONAL KINETIC EQUATION INVOLVING GENERALIZED J-GENERALIZED P-K MITTAG LEFFLER FUNCTION VIA SUMUDU TRANSFORM

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ABSTRACT
In this paper Generalized j-generalized p-k Mittag-Leffler function is used. We obtain the solution of fractional kinetic equation including of new function by using Laplace transform and Sumudu transform. The solution of fractional kinetic equation is expressed in the form of vompact form of Mittag-Leffler function etc are also considered.
Keywords and phrases Fractional kinetic equation, Laplace transform, Sumudu transform, Riemann-Liuoville fractional integral operator, Mittag-Leffler function.

I INTRODUCTION

Fractional calculus has been established and expliot in the different field of applied science and engineering . Due to its work varity of implementation in numerous topic such as wave like equation and diabetes model [see 13,14,29]. The fractional calculus got more importance, Due to its signification and relevance in various field [25-29] recently the fractional calculus and Mittag-Leffler function is generalily studied, it is one of the most specific tools to edit the annihilation of natural phenomena.

1- Preliminaries

In this section, we give some definitions, which will be useful in our further investigations.

1.1 Fractional kinetic equations Fractional kinetic equation play an important role in the field of applied science. It is always used in mathematical direction, physics dynamic system , control system and engineering. During the last decades,.The Haubold and Mathai [11] have given as follow fractional differential equation correlated with the rate changes of reaction $\mathcal{M} = \mathcal{M}(t)$ the destruction rate $d = d(\mathcal{M})$ and production rate $p = p(\mathcal{M})$

(1.1) $$d\mathcal{M}/dt = -d(\mathcal{M}_t) + p(\mathcal{M}_t)$$

Where $\mathcal{M}_t$ Is the function identified by

$$\mathcal{M}_t(t^*) = \mathcal{M}(t - t^*) \qquad t^* > 0.$$

Neglecting the irregularity in the quantity m(t),the particular case of (1.1) is formed as

(1.2) $$d\mathcal{M}/dt = -c_i \mathcal{M}_i(t)$$

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is part of the intial condition $\mathfrak{M}_i(t = 0)= \mathfrak{M}_0$ is the number of density of index i at time t= 0 . The equation of solution (1.1) is refered as

$$\mathcal{M}_i(t) = \mathcal{M}_0 e^{-c_i t}$$

we get intergationg the standard kinetic equation (1.2), without considering the index I

(1.3) $$\mathcal{M}(t)- \mathcal{M}_0 = c_0 D_t^{-1} \mathcal{M}(t)$$

Where ${}_0D_{-1}^{t}$ is the particular form of the Riemann—liouville operator ${}_0D_t^{-\gamma}$ is define by

$${}_0D_t^{-\gamma} f(t)=\frac{1}{\Gamma(\gamma)}\int_0^t((t-s)^{\gamma-1} f(s)ds, \quad (t>0, \quad R(\gamma) >0)$$

Haubold and Mathaisee(11) have given the fractional generalization form the standard kinetic equation(1.3)

(1.4) $$\mathcal{M}(t) - \mathcal{M}_0 = -c^{-\tau}\, {}_0D_t^{-\gamma} \mathcal{M}(t)$$

$$\mathcal{M}(t) = \mathcal{M}_0 \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(\gamma k+1)}(ct)^{\gamma k}$$

Later, the following fractional kinetic equation of the form are expressed by Saxena and Kalla see [25]

(1.5) $$\mathcal{M}(t) - \mathcal{M}_0 f(t) = -c_\gamma ({}_0D_t^{-\gamma} m)(t) \quad (R(\gamma) >0).$$

where $\mathcal{M}(t)$ refers to density number of a given species at every time t, $\mathcal{M}_0 = \mathcal{M}(0)$ is a density nember that species at time t=0, c is a constant and $f \in L(0,\infty)$.

1.2 Laplace transform The Laplace transform given by

$$L\{f(s)\} = \overline{f(t)} = \int_0^\infty e^{-ts} f(s)ds \qquad \mathbf{R}(t) > \alpha$$

The use of the Laplace transform to (1.5) gives

(1.6) $$L\{f(t)\} = \mathcal{M}_0 \frac{F(p)}{1+c^\gamma p^{-\gamma}}$$

$$= \mathcal{M}_0 \left(\sum_{n=0}^{\infty} -c^{\gamma n} p^{-nr}\right) F(p)$$

Where $n \in m_0, \ |\frac{c}{p}|<1$

1.3 Sumudu Transform:

The Sumudu transform over the set function

$A = \{ f(t) \mid \exists m, \eta_1, \eta_2 > 0, |f(t)| < m\, e^{|t|/r_j}, t \in (-1)^j \times [0,\infty]\}$ is given by

$$G(T) = S[f(t); T] = \int_0^\infty e^{-t} f(Tt)dt; \quad T \in (-\eta_1, \eta_2)$$

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1.4 Generalized J-generelized p-k Mittag-Leffler function

Suppose that k , p ∈ R^+ - {0}; $\theta, \vartheta, \rho \in$ C/ kZ^- with Re(Θ) > 0, Re(ν) > 0, Re(ϱ) > 0. j∈ N_0 and s ∈ (0,1)∪ N. $\lambda \neq -1, -2, -3 \ldots$ ..... Then Generalized j-generalized p – k Mittag-Leffler function denoted by ${}_p^jE_{k,\theta,\vartheta,\delta}^{\rho,s}$ (z) and defined as

$$ {}_p^jE_{k,,\theta\vartheta,\delta}^{\rho,s} = \prod_{i=1}^{r}\{\sum_{n=0}^{\infty} \frac{{}_{p_i}(\rho)_{(n+j)s,k}}{{}_p\Gamma_k(n\theta+\vartheta)} \frac{z^n}{(\lambda)_{n+j}}\} \quad (1.7) $$

Where ${}_p(\rho)_{ns,k}$ is Pochhammer (p,k) symbol and is defined by Gehlot et.al[ ] as

$$ {}_p(\rho)_{n.k} = \left(\frac{p\rho}{k}\right)\left(\frac{p\rho}{k}+p\right)\left(\frac{p\rho}{k}+2p\right)\ldots\ldots\ldots\left(\frac{p\rho}{k}+(n-1)p\right) $$

and ${}_p\Gamma_k(\rho)$ gamma function is defined as

$$ {}_p\Gamma_k(\rho) = \frac{1}{k}\lim_{n\to\infty} \frac{n!\,p^{n+1}(np)^{\frac{\rho}{k}-1}}{{}_p(\rho)_{n,k}} $$

$$ {}_p\Gamma_k(\rho) = \left(\frac{p}{k}\right)^{\frac{\rho}{k}}\Gamma_k(\rho) = \frac{p^{\frac{p}{k}}}{k}\Gamma\left(\frac{\rho}{k}\right) \quad (1.8) $$

Special case of Generalized j generalized p-k Mittag-Leffler function

(a) If we take $\lambda = 1$ in (1.7), the function reduce to the j-generalized p-k Mittag-Leffler function given by Gehlot

$$ {}_p^jE_{k,\theta,\vartheta,1}^{\rho,s}(z) = \sum_{n=0}^{\infty} \frac{{}_p(\rho)_{(n+j)s,k}}{{}_p\Gamma_k(n\theta+\vartheta)} \frac{z^n}{(n+j)} \qquad z \in C $$

(b) If we substitute j = 0 and $\lambda = 1$ in (1.7), the function reduce to the p-k Mittag – Leffler function studied by Gehlot [6] and is defined as

$$ {}_p^0E_{k,\theta,\vartheta,1}^{\rho,s}(z) = \sum_{n=0}^{\infty} \frac{{}_p(\rho)_{ns,k}}{{}_p\Gamma_k(n\theta+\vartheta)} \frac{z^n}{n!} \qquad z \in C $$

(c) If we put S = 1 and $\lambda = 1$ in (1.7). the function reduce to the j form Mittag –Leffler function given by

$$ {}_p^jE_{k,\theta,\vartheta,1}^{\rho,1}(z) = \sum_{n=0}^{\infty} \frac{{}_p(\rho)_{(n+j),k}}{{}_p\Gamma_k(n\theta+\vartheta)} \frac{z^n}{(n+j)!} \qquad z \in C $$

(d) If we take s = 1, $\lambda = 1$ and p=k in (1.7), the function reduce to the j form of k Mittag –Leffler function given by

$$ {}_k^jE_{k,\theta,\vartheta,1}^{\rho,1}(z) = \sum_{n=0}^{\infty} \frac{(\rho)_{(n+j),k}}{\Gamma_k(n\theta+\vartheta)} \frac{z^n}{(n+j)!} \qquad z \in C $$

(e) If we put s = 1, $\lambda = 1$ and j = 0 in (1.7), the function reduce to the generalized form of k Mittag-Leffler function given by

$${}_pE^{\rho,1}_{k,\theta,\vartheta,1}\,(z) = \sum_{n=0}^{\infty} \frac{{}_p(\rho)_{n,k}}{{}_p\Gamma_k(n\theta+\vartheta)} \frac{z^n}{n!} \qquad z \in C$$

(f) If we substitute j = 0, $\lambda = 1$ and p=k in (1.7) the function reduce to the Generalized k Mittag –Leffler function studied by Gehlot [5] and given by

$${}_kE^{\rho,s}_{k,,\theta,\vartheta,1}(z)= \sum_{n=0}^{\infty} \frac{{}_p(\rho)_{ns,k}}{{}_k\Gamma_k(n\theta+\vartheta)} \frac{z^n}{n!} = GE^{\rho,s}_{k,\theta,\vartheta}(z)$$

(g) If we take j = 0, s = 1 , $\lambda = 1$ and p=k in (1.7) the function reduce to the k Mittag –Leffler function studied by Dorrego [4] and is given by

$${}_kE^{\rho,1}_{k,,\theta,\vartheta,1}(z)= \sum_{n=0}^{\infty} \frac{(\rho)_{n,k}}{\Gamma_k(n\theta+\vartheta)} \frac{z^n}{n!} = E^{\rho}_{k,\theta,\vartheta}(z)$$

(h) If we substitute j = 0, k = 1, $\lambda = 1$ and p=k in (1.7) the function reduce to the Mittag –Leffler function studied by Shukla and Prajapati [21] and is given by

$${}_1E^{\rho,1}_{1,\theta,\vartheta,1}(z)= \sum_{n=0}^{\infty} \frac{(\rho)_{ns,k}}{\Gamma(n\theta+\vartheta)} \frac{z^n}{n!} = E^{\rho,s}_{\theta,\vartheta}(z)$$

(i) If we put p = k = s =1 = $\lambda = 1$ in (1.10) the function reduce to the L- Mittag –Leffler function given by Luque [10] and is defined as

$${}_1E^{\rho,1}_{1,\theta,\vartheta,1}(z)= \sum_{n=0}^{\infty} \frac{(\rho)_{n+j,k}}{\Gamma(n\theta+\vartheta)} \frac{z^n}{(n+j)!} = L^{\rho,j}_{\theta,\vartheta}(z)$$

(j) If we take j = 0, s = 1 ,k = 1, $\lambda = 1$ and p = k in (1.7) the function reduce to the Mittag –Leffler function studied by Prabhakar [17] and is given as

$${}_1E^{\rho,1}_{1,\theta,\vartheta,1}(z)= \sum_{n=0}^{\infty} \frac{(\rho)_n}{\Gamma(n\theta+\vartheta)} \frac{z^n}{n!} = E^{\rho}_{\theta,\vartheta}(z)$$

(k) If we put j = 0, s = 1 ,k = 1 p = k, $\lambda = 1$ and $\rho = 1$ in (1.7) the function reduce to the Mittag –Leffler function given by Mittag –Leffler [12] defined by

$${}_1E^{1,1}_{1,\theta,\vartheta,1}(z)= \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n\theta+\vartheta)} = E_{\theta,\vartheta}\ (z)$$

II MAIN RESULTS

n this part, we investigates the solutions of the fractional kinetic equation involving Generalized j-generalized p-k Mittag-Leffler function using the method of Laplace transform and Sumudu transform.

Theorem 2.1 Suppose that $\delta > 0$, $\gamma > 0$, p ,k > 0, j ∈ N_0 and s ∈ (0,1)∪ N, $\theta, \vartheta, \rho \in C$ with $R(\theta) > 0$, $R(\vartheta) > 0$, $R(\rho) > 0$, $\lambda \neq -1, -2$ ….. then the following equation

(2.1) $$\mathcal{N}(\tau) = \mathcal{N}_0\ {}^{j}_{p}E^{\rho,s}_{k,\theta,\vartheta,\lambda}\ (\tau) - \delta^{\gamma}\,{}_0D_t^{-\gamma}\,\mathcal{N}(\tau)$$

has a solution given by

(2.2) $$\mathcal{N}(\tau)=\mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)q,k}}{p^{\Gamma_k(n\theta+\vartheta)}} \frac{\Gamma(n+1)}{(\lambda)_{n+j}} \tau^n E_{\gamma,n+1}(-\delta^\gamma \tau^\gamma).$$

Where $E_{\gamma,n+1}$ (.) is the generalized Mittag-Leffe r function [17].

proof : the Laplace transform of Riemann Liouville fractional operator is presented as

(2.3) $\mathbf{L}\,({}_0D_t^{-v}\ f(\tau)\ ;t\,)= \tau^{-\gamma} G(t) \quad \mathrm{Re}(\tau) > 0.$

where $G(\tau) = \int_{n=0}^{\infty} e^{-\tau t} f(\tau)\, d\tau \quad \mathrm{Re}(t) > 0.$

Now, we will apply Laplace transform to both side side of equation (2.1) and applying (2.3) we have

$$\mathbf{L}\,(\mathcal{N}(\tau;t)) = \mathcal{N}_0\, \mathbf{L}\,(\ {}_p^jE_{k,\theta,\vartheta}^{\rho,s}\ (\tau;t)) - \delta^\gamma\, \mathbf{L}\,(({}_0D_t^{-\gamma}\ \mathcal{N}((\tau;t))$$

(2.4) $$\mathcal{N}(t) = \mathcal{N}_0(\int_0^\infty e^{-\tau t} \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)s,k}}{p^{\Gamma_k(n\theta+\vartheta)}} \frac{\tau^n}{(\lambda)_{n+j}}) - \delta^\gamma \tau^{-\gamma} \mathcal{N}(t)$$

By rearranging the sequence of integration and summation in the equation (2.4), we get

$$\mathcal{N}(t)(1+\delta^\gamma \tau^\gamma) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)s,k}}{p^{\Gamma_k(n\theta+\vartheta)}} \frac{1}{(\lambda)_{n+j}} \int_0^\infty e^{-\tau t} t^n\, dt$$

(2.5) $$= \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)s,k}}{p^{\Gamma_k(n\theta+\vartheta)}} \frac{1}{(\lambda)_{n+j}} \frac{\Gamma(n+1)}{t^{n+1}}$$

Equation (2.5) follows to

(2.6) $$= \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)s,k}}{p^{\Gamma_k(n\theta+\vartheta)}} \frac{\Gamma(n+1)}{(\lambda)_{n+j}} \{ t^{-(n+1)} \sum_{\mu=0}^{\infty} (-(\tfrac{t}{\delta}))^{-\vartheta}]^\mu$$

Now, we will apply inverse Laplace transform on both side the equation (2.6) and applying

$\mathbf{L}^{-1}(t^{-\gamma};t) = \frac{t^{\gamma-1}}{\Gamma(\gamma)} \quad (\mathrm{Re}(\gamma) > 0).$

We obtain,

$$\mathbf{L}^{-1}(\mathcal{N}(t)) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)s,k}}{p^{\Gamma_k(n\theta+\vartheta)}} \frac{\Gamma(n+1)}{(\delta)_{n+j}} \mathbf{L}^{-1}(\sum_{\mu=0}^{\infty} (-1)^\mu \delta^{\mu\gamma} t^{-(n+1+\mu\gamma)}$$

$$\mathcal{N}(\tau) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)s,k}}{p^{\Gamma_k(n\theta+\vartheta)}} \frac{\Gamma(n+1)}{(\lambda)_{n+j}} \{\sum_{\mu=0}^{\infty} \delta^{\mu\gamma} \tau^{n+\mu\gamma}\}$$

$$= \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)s,k}}{p^{\Gamma_k(n\theta+\vartheta)}} \tau^n \frac{\Gamma(n+1)}{(\lambda)_{n+j}} \{ \sum_{\mu=0}^{\infty} \frac{(-1)^\mu}{\Gamma(n+\mu\gamma+1)} \delta^{\mu\gamma} \tau^{\mu\gamma}\}$$

$$\mathcal{N}(\tau) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)s,k}}{p^{\Gamma_k(n\theta+\vartheta)}} \frac{\Gamma(n+1)}{(\lambda)_{n+j}} \tau^n E_{\gamma,n+1}(-\delta^\gamma \tau^\gamma)$$

Theorem 2.2 Suppose that $\delta > 0$, $\gamma > 0$, p ,k > 0, j ∈ N_0 and s ∈ (0,1)∪ N , θ, ϑ, ρ, ∈ C with R(θ) > 0, R(ϑ) > 0, R(ρ) > 0. then the following equation

(2.7) $$\mathcal{N}(\tau) = \mathcal{N}_0\ {}_p^jE_{k,\theta,\vartheta,\lambda}^{\rho,s}(\tau) - \delta^\gamma\, {}_0D_t^{-\gamma}\, \mathcal{N}(\tau)$$

has a solution given by

(2.8) $$\mathcal{N}(\tau) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)q,k}}{p^{\Gamma_k(n\theta+\vartheta)}} \frac{\Gamma(n+1)}{(\lambda)_{n+j}} t^{n-1} E_{\gamma,n}(-\delta^\gamma t^\gamma).$$

Where $E_{\gamma,n}$ (.) is the generalized Mittag-Leffer function[17] .

proof : the sumudu transform of Riemann Liouville fractional operator is presented as

(2.9) $S({}_0D_t^{-v}\ f(t)\ ;\tau\,)= (\tau)^\gamma G(\tau) \quad \mathrm{Re}(\tau) > 0.$

where $G(\tau) = \int_{n=0}^{\infty} e^{-t} f(t\tau)\, dt$

Now, we will apply the Sumudu transform to both side side of equation (2.7) and applying (2.9) we obtain

$S(\mathcal{N}(\tau);t) = \mathcal{N}_0\, S({}_p^jE_{k,\theta,\vartheta}^{\rho,s}(\tau)) - \delta^\gamma S({}_0D_t^{-\gamma}\mathcal{N}(\tau))$

(2.10) $\mathcal{N}(t) = \mathcal{N}_0 \int_{n=0}^{\infty} e^{-t} \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)q,k}}{p^{\Gamma_k}(n\theta+\vartheta)} \frac{(\tau t)^n}{(\lambda)_{n+j}} dt - \delta^\gamma \tau^\gamma \mathcal{N}(t)$

By rearranging the sequence of integration and summation in the equation (2.10), we get

$$\mathcal{N}(t)(1+\delta^\gamma\tau^\gamma) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)s,k}}{p^{\Gamma_k}(n\theta+\vartheta)} \frac{\tau^n}{(\lambda)_{n+j}} \int_0^{\infty} e^{-t} t^n\, dt$$

(2.11) $$= \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)s,k}}{p^{\Gamma_k}(n\theta+\vartheta)} \frac{\Gamma(n+1)\tau^n}{(\lambda)_{n+j}}$$

Equation (2.11) follows to

(2.12) $$= \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)s,k}}{p^{\Gamma_k}(n\theta+\vartheta)} \frac{\Gamma(n+1)}{(\lambda)_{n+j}} (1+\delta^\gamma\tau^\gamma)^{-1}$$

Now, we will apply inverse Sumudu transform on both side the equation (2.12) and applying $S^{-1}(\tau^\gamma;t) = \frac{t^{\gamma-1}}{\Gamma(\gamma)}$ $(Re(\gamma) > 0)$.

We obtain,

$$S^{-1}(\mathcal{N}(t)) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)s,k}}{p^{\Gamma_k}(n\theta+\vartheta)} \frac{\Gamma(n+1)}{(\lambda)_{n+j}} S^{-1}\left(\sum_{\mu=0}^{\infty}(-1)^\mu \delta^{\mu\gamma}\tau^{-(n+\mu\gamma)}\right)$$

$$\mathcal{N}(\tau) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)s,k}}{p^{\Gamma_k}(n\theta+\vartheta)} \frac{\Gamma(n+1)}{(\lambda)_{n+j}} \sum_{\mu=0}^{\infty} \frac{(-1)^\mu}{\Gamma(n+\gamma\mu)} \delta^{\mu\gamma} t^{n+\mu\gamma-1}$$

$$= \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)s,k}}{p^{\Gamma_k}(n\theta+\vartheta)} \frac{\Gamma(n+1)}{(\lambda)_{n+j}} t^{n-1} \sum_{\mu=0}^{\infty} \frac{(-1)^\mu}{\Gamma(n+\mu\gamma)} \delta^{\mu\gamma} t^{\mu\gamma}$$

$$= \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)s,k}}{p^{\Gamma_k}(n\theta+\vartheta)} \frac{\Gamma(n+1)}{(\lambda)_{n+j}} t^{n-1} \sum_{\mu=0}^{\infty} \frac{(-1)^\mu}{\Gamma(n+\mu\gamma)} (\delta^\gamma t^\gamma)^\mu$$

$$= \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)s,k}}{p^{\Gamma_k}(n\theta+\vartheta)} \frac{\Gamma(n+1)}{(\lambda)_{n+j}} t^{n-1} \sum_{\mu=0}^{\infty} \frac{(-\delta^\gamma t^\gamma)^\mu}{\Gamma(n+\mu\gamma)}$$

$$= \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)s,k}}{p^{\Gamma_k}(n\theta+\vartheta)} \frac{\Gamma(n+1)}{(\lambda)_{n+j}} t^{n-1} E_{\gamma,n}(-\delta^\gamma t^\gamma)$$

III SPECIAL CASE

Corollary 3.1 If we set $\lambda = 1$,in theorem 2.1 then we obtain the following equation

$$\mathcal{N}(\tau) = \mathcal{N}_0\ {}_p^jE_{k,\theta,\vartheta,\lambda}^{\rho,s}(\tau) - \delta^\gamma {}_0D_t^{-\gamma}\mathcal{N}(\tau)$$

has a solution given by

$$\mathcal{N}(\tau) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{p^{(\rho)(n+j)q,k}}{p^{\Gamma_k}(n\theta+\vartheta)} \frac{\Gamma(n+1)}{\Gamma(n+j+1)} \tau^n E_{\gamma,n+1}(-\delta^\gamma\tau^\gamma).$$

Where $E_{\gamma,n+1}$ (.) is the generalized Mittag-Leffe r function [17].

Corollary 3.2 If we put j = 0 and $\lambda = 1$ in theorem 2.1 then we obtain the following equation

$$\mathcal{N}(\tau) = \mathcal{N}_0\ {}_p^0E_{k,\theta,\vartheta,1}^{\rho,s}(\tau) - \delta^\gamma {}_0D_t^{-\gamma}\mathcal{N}(\tau)$$

has a solution given by

$$\mathcal{N}(\tau) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{{}_p(\rho)_{ns,k}}{{}_p\Gamma_k(n\theta+\vartheta)} \tau^n E_{\gamma,n+1}(-\delta^\gamma \tau^\gamma).$$

Where $E_{\gamma,n}$ (.) is the generalized Mittag-Leffer function[17] .

Corollary3.3 If we take s = λ = 1, in theorem 2.1 then we obtain the following equation

$$\mathcal{N}(\tau) = \mathcal{N}_0 \, {}_p^jE_{k,\theta,\vartheta,1}^{\rho,1}(\tau) - \delta^\gamma {}_0D_t^{-\gamma} \mathcal{N}(\tau)$$

has a solution given by

$$\mathcal{N}(\tau) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{{}_p(\rho)_{(n+j),k}}{{}_p\Gamma_k(n\theta+\vartheta)} \frac{\Gamma(n+1)}{\Gamma(n+j+1)} \tau^n E_{\gamma,n+1}(-\delta^\gamma \tau^\gamma).$$

Where $E_{\gamma,n}$ (.) is the generalized Mittag-Leffer function [17].

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Corollary 3.4 If we substitute p = k and s = λ = 1, in theorem 2.1 then we obtain the following equation

$$\mathcal{N}(\tau) = \mathcal{N}_0 \, {}_p^jE_{k,\theta,\vartheta,1}^{\rho,1}(\tau) - \delta^\gamma {}_0D_t^{-\gamma} \mathcal{N}(\tau)$$

has a solution given

$$\mathcal{N}(\tau) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{(\rho)_{(n+j),k}}{\Gamma_k(n\theta+\vartheta)} \frac{\Gamma(n+1)}{\Gamma(n+j+1)} t^n E_{\gamma,n+1}(-\delta^\gamma \tau^\gamma).$$

Where $E_{\gamma,n}$ (.) is the generalized Mittag-Leffer function[17].

Corollary 3.5 If we take j = 0 and s = λ = 1, in theorem 2.1 then we obtain the following equation

$$\mathcal{N}(\tau) = \mathcal{N}_0 \, {}_p^0E_{k,\theta,\vartheta,1}^{\rho,1}(\tau) - \delta^\gamma {}_0D_t^{-\gamma} \mathcal{N}(\tau)$$

has a solution given by

$$\mathcal{N}(\tau) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{{}_p(\rho)_{n,k}}{{}_p\Gamma_k(n\theta+\vartheta)} \tau^n E_{\gamma,n+1}(-\delta^\gamma \tau^\gamma).$$

Where $E_{\gamma,n}$ (.) is the generalized Mittag-Leffer function [17].

Corollary 3.6 If we substitute p = k **,** j= 0 and λ = 1, in theorem 2.1 then we obtain the following equation see Agrawal et al.[1]

$$\mathcal{N}(\tau) = \mathcal{N}_0 \, GE_{k,\theta,\vartheta}^{\rho,s}(\tau) - \delta^\gamma {}_0D_t^{-\gamma} \mathcal{N}(\tau)$$

has a solution given by

$$\mathcal{N}(\tau) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{{}_k(\rho)_{ns,k}}{{}_k\Gamma_k(n\theta+\vartheta)} \tau^n E_{\gamma,n+1}(-\delta^\gamma \tau^\gamma).$$

Where $E_{\gamma,n}$ (.) is the generalized Mittag-Leffer function[17].

Corollary 3.7 If we take p = k j = 0 and s = λ = 1, in theorem 2.1 then we obtain the following equation see Agrawal et al.[1]

$$\mathcal{N}(\tau) = \mathcal{N}_0\, E^{\rho}_{k,\theta,\vartheta}(\tau) - \delta^{\gamma}\,{}_0D_t^{-\gamma}\,\mathcal{N}(\tau)$$

has a solution given by

$$\mathcal{N}(\tau) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{(\rho)_{n,k}}{\Gamma_k(n\theta+\vartheta)}\, \tau^n E_{\gamma,n+1}(-\delta^{\gamma}\tau^{\gamma}).$$

Where $E_{\gamma,n}$ (.) is the generalized Mittag-Leffer function[17].

Corollary 3.8 If we set j = 0 and p = k = s = λ = 1, in theorem 2.1 then we obtain the following equation see Agrawal et al.[1]

$$\mathcal{N}(\tau) = \mathcal{N}_0\, E^{\rho,s}_{\theta,\vartheta}(\tau) - \delta^{\gamma}\,{}_0D_t^{-\gamma}\,\mathcal{N}(\tau)$$

has a solution given by

$$\mathcal{N}(\tau) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{(\rho)_{ns}}{\Gamma(n\theta+\vartheta)}\, \tau^n E_{\gamma,n+1}(-\delta^{\gamma}\tau^{\gamma}).$$

Where $E_{\gamma,n}$ (.) is the generalized Mittag-Leffer function[17].

Corollary 3.9 If we substitute p = k = s = λ = 1, in theorem 2.1 then we obtain the following equation

$$\mathcal{N}(\tau) = \mathcal{N}_0\, L^{\rho,j}_{\theta,\vartheta}(z)(\tau) - \delta^{\gamma}\,{}_0D_t^{-\gamma}\,\mathcal{N}(\tau)$$

Has a solution given by

$$\mathcal{N}(\tau) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{(\rho)_{n+j,k}}{\Gamma(n\theta+\vartheta)} \frac{\Gamma(n+1)}{\Gamma(n+j+1)}\, \tau^n E_{\gamma,n+1}(-\delta^{\gamma}\tau^{\gamma}).$$

Where $E_{\gamma,n}$ (.) is the generalized Mittag-Leffer function[17] .

Corollary 3.10 If we take j = 0 and p = k = s = λ = 1, in theorem 2.1 then we obtain the following equation

$$\mathcal{N}(\tau) = \mathcal{N}_0\, E^{\rho}_{\theta,\vartheta}(\tau) - \delta^{\gamma}\,{}_0D_t^{-\gamma}\,\mathcal{N}(\tau)$$

has a solution given by

$$\mathcal{N}(\tau) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{(\rho)_{n}}{\Gamma(n\theta+\vartheta)}\, \tau^n E_{\gamma,n+1}(-\delta^{\gamma}\tau^{\gamma}).$$

Where $E_{\gamma,n}$ (.) is the generalized Mittag-Leffer function[17] .

Corollary 3.11 If we substitute j= 0, ρ = p = k = s = λ = 1, in theorem 2.1 then we obtain the following equation

$$\mathcal{N}(\tau) = \mathcal{N}_0 \, E_{\theta,\vartheta}\,(z)\,(\tau) - \delta^{\gamma}\,{}_0D_t^{-\gamma}\,\mathcal{N}(\tau)$$

has a solution given by

$$\mathcal{N}(\tau) = \mathcal{N}_0 \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n\theta+\vartheta)}\, \tau^n E_{\gamma,n+1}\,(-\delta^{\gamma}\tau^{\gamma})\,.$$

Where $E_{\gamma,n}$ (.) is the generalized Mittag-Leffer function [17] .

Similarly, we can deduce new and known result from theorem (2.2).

IV CONCLUSION

In this paper, by using Laplace transform and Sumudu transform technique we find a new solution of fractional kinetic equation which include Mittag-Leffler function. In this above solution of Generalized j- generalized p-k Mittag-Leffler function also done and it will help in generating new result and further studying with new ideas.

REFERENCES

[1]. P. Agrawal. M. Chand ,D. Baleanu, D.O.Regan and Shilpi jain :"On the solution of certain fractional kinetic equations involving k –Mittag-Leffler function Advance in Differential Difference Equation 2018".

[2]. I. Batalov, A. Batalova : " Crittical dynamics in system controlled by fractional kinetic equation , Physics A-392 602-61(2013)".

[3]. V.N. Borodikhin :" Fractional differential kinetic equation and growth of the nuclei of a new phase at phase transitions, Phys Lett. A 376 1952-1954(2012)".

[4]. M. Chand , P.Agrawal and Z. Hammouch:" Certain sequences involving product of k-Bessel function. Int. J. Appl. Comput. Math.4, 101(2018)".

[5]. A. Chouhan, S. D. Purohit and S. D. Saraswat:" An alternative method for solving generalized differential equation of fractional order Kragujev. J. Math 37(2), 299-306 (2013)".

[6]. G. A. Dorrego and R. A. Cerutti:" The K-Mittag-Leffler function . Int. J. Contemp. Math. Sciences, Vol. 7(2012) No . 15, 705-716".

[7]. K. S. Gehlot:" The Generalized K- Mittag-Leffler function . Int. J. Contemp. Math. Sciences, Vol. 7(2012) No 45,2213-2219".

[8]. K. S. Gehlot:" The p-k Mittag-Leffler function, Palestine Journal of Mathematics, Vol . 7(2) (2018) , 628-632".

[9]. K. S. Gehlot:" The j-Generalized p-k Mittag-Leffler function,"

[10]. R. E. Gutierrez , J. M. Rosario , J. T. Machado:" Ractional order calculus basic concepts and engineering applications. Math. Probl Eng 2010".

[11]. H .J. Haubold and A. Mathai:" The Fractional kinetic equation and thermonuclear functions. Astrophys. Space Sci. 327. 53-63(2010)".

[12]. D. Kumar :" Solution of fractional kinetic equation by a class of integral transform of pathway type. J. Math. Phys 54 Article ID 043509(2013)".

[13]. D. Kumar , J. Singh , D. Baleanu , S. Rathore ;" Analysis of a fractional model of Ambartsumian equation . Eur. Phys. J. Plus 133 , 259 (2018)".

[14]. D. Kumar , J. Singh , S.D.Purohit , R.Swroop:" A hybrid analytical algorithum for nonlinear fractional wave-like equation , Math .Model.Nat. Phenorm 14,304(2019) (ISSN:0973-5348)".

[15].L.L.Luque:" On a Generalized Mittag-Leffler function, International Journal of Mathematical Analysis, Vol. 13 , 2019, no. 5, 223-234".

[16]. K. S. Millar and B. Ross :". An Introduction to the Fractional Calculus and Fractional Differential Equations. Wiley. New York (1991)".

[17]. G. Mittag-Leffler.:" Sur la nouvellefonction $E_\alpha(z)$ C. R. Acad . Sci. Pari 137(1903) 554-558".

[18]. K. S. Nisar, S. D. Purohit, S. R. Mondal :" Generalized fractional kinetic equations involving generalized Struve function of the first kind .J. King Saud Univ, Sci. 28 167-171(2016)".

[19].K. Oldham and J. Spanier.:"Fractional Calculus Theory and Applications of Differentiation and Integration of Arbitrary Order . Academic Press, New York (1974)".

[20]. I. Petras.:" Stability of fractional-order system with ratinal order a survey. Fract. Calc. Appl. Anal. 12. 269-298 (2009)".

[21]. I. Podlubny :". Fractional Differential Equations. Academic Press, New York (1999).

[17]. T. R. Prabhakar. A singularintegral equation with a generalized Mittag-Leffler function in the kernel. Yokohama Math. J. 19(1971), 7-15".

[22]. S. D. Purohit.:" Solution of fractional partial differentiation equations of quantum mechanics. Appl.Math. Mech 5(5). 639-651 (2013)".

[23]. S.D. Purohit and S. D. Kalla.:" On fractional partial differential equation related to quantum mechanics. J. Phy. A. Math. Sci. 66, 397-408(2011)".

[24]. A. Saichev, M. Zaslavsky:" Fractional kinetic equation : solution and applications. Chaos 7, 753-764(1997)".

[25]. R. K. Saxena and S. L. Kalla.:" On the solution of certain fractional kinetic equations. Appl. Math. Comput. 199. 504-511".

[26]. R.K.Saxena and A.M. Mathai. H.J.Haubold :"On fractional kinetic equations . Astrophys Space Sci. 282, 281-287 (2002)".

[27]. R.K.Saxena and A.M. Mathai. H.J.Haubold :"On generalized fractional kinetic equations, Physica A 344 , 657-664 (2004".

[28]. R.K.Saxena and A.M. Mathai. H.J.Haubold :"Solution of generalized fractional reaction – diffusion equations .Astrophys. Space Sci. 305, 305-313(2006)".

[29]. J. Singh , D. Kumar, D. Baleanu :" On the analysis of fractional diabetes model with exponential law . Adv. Differ. Equ. 2018, 231 (2018)".

[30]. A. K. Shukla and J. C. Prajapati .:" On the generalization of Mittag-Leffler function and its properties. Journal of Mathematical Analysis and Application, 336(2007) 797-811".

[31]. A. Wiman.:" Uber den fundamental Staz in der Theories der Funktionen $E_\alpha(z)$, Acta Math. 29(1905) 191-201".

[32]. G. M. Zaslavsky :" Fractional kinetic equation for Hamiltonian chaos.Physica D76 110-122(1994)".