Stimulation of Power of Near Mean Graph

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Abstract
Let a graph with p vertices and q edges be G = (V , E) and let f: V (G) →{0 , 1, 2,. An injection is a, q − 1, q + 1}. Graph G is said to have a close mean marking if there is an induced injective map f for each edge: E(G) {1, 2,. , q} that is defined by

$$f^*(uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even,} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd.} \end{cases}$$

We expand this definition to Smarandachely close m-mean labeling (as in [9]) if the induced Smarandachely m-labeling f *≥ is specified for each edge e = uv and an integer m ≥ 2,

$$f^*(e) = \left\lceil \frac{f(u)+f(v)}{m} \right\rceil.$$

The Smarandachely near m-mean graph is considered a graph that acknowledges a Smarandachely near mean m-labeling. A near mean graph (NMG) is considered the graph that accepts near mean marking. We have shown in this paper that the graphs Pn, Cn, K2,n are close mean graphs and the graphs Kn(n > 4) and K1,n(n > 4) are not close mean graphs.
Keywords: Labeling, near mean labeling, near mean graph, Smarandachely near mean labeling, Smarandachely near m-mean graph.

Introduction

Rosa developed alpha-graph valuation in 1966. Golomb implemented delicate marking concurrently. The harmonious marking of a graph was adopted by Graham and Sloance in 1980. In the Gallian analysis, many graph labelings have been applied. In Ponraj, the Mean Labeling definition was implemented and .Close Graceful Labeling was researched adopted by S.El-Zanati, M.Kenig and C.VandenEynden by Frucht and Close al-labeling. This motivates one to describe the Close Mean Labeling definition as follows:

Let G be a graph of (p, q)-. The assignment f: V(G) ⁇ {0 , 1, 2, ..., q-1, q+1} is a vertex marking of G. The induced edge labeling f * for a vertex labeling f is described by

$$f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil,$$

For every G edge uv where [x] denotes the least integer greater than or equal to x as the least integer. If the induced edge labeling is f*(E) = {1, 2, .., q}, the vertex labeling f is considered Close Mean Labeling of G. If a graph G has Near Mean Naming, then a Near Mean Graph (NMG) is labeled G.

Notice that the K4 and K1,4 graphs are not the mean graphs. Therefore, there are maps that are not Mean Graphs, but Similar Mean Graphs. In this portion, on Close Mean Graphs, we research several simple theorems and also find the total number of possible mean labels of a given graph. We are still researching certain graphs with Very Mean Marking. It also examines the close meanness of the family of trees and joining diagrams.

By a map, we mean a plain, undirected, finite line. V (G) and E(G) respectively represent the vertex set and edge set of the graph G denoted. Let f: V(G) belong to {0, 1, 2,. An injection is a., q − 1, q + 1}. Graph G is said to have a close mean marking if there is an induced injective map f for each edge: E(G) {1, 2,. ., q} which is described by

$$f^*(uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even,} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd.} \end{cases}$$

We extend this notion to Smarandachely near m-mean labeling (as in [9]) if for each edge e = uv and an integer m ≥ 2, the induced Smarandachely m-labeling f ∗ is defined

$$f^*(e) = \left\lceil \frac{f(u)+f(v)}{m} \right\rceil .$$

A graph that admits a Smarandachely near mean m-labeling is called Smarandachely near mean graph. A path Pn is a graph of length n − 1 · Kn and Cn are complete graph and cycle with n vertices respectively. Terms and notations not used here are as in [2].

Preliminaries

In [3], mean marking was adopted. Let G be the graph of (p, q). In[4], we showed that Book Bn, Ladder Ln, Grid Pn×Pn, Prism Pm×C3 and Ln ⊙ K1 graphs are approximately average graphs. We have shown in[5] that Join Graphs, K2 + mK1, K1 n + 2K2, Sm + K1Pn + 2K1 and Double Fan are close average graphs. In[6], we demonstrated that Bi-star, Sub-division Bi-star Pm ⊖ 2K1, Pm ⊖3K1, Pm⊖K1,4 and Pm ⊖ K1,3 are close average graphs for the family of trees. Relevant graph types of triangular snake, quadrilateral snake, C+n, Sm,3, Sm,4, and parachute graphs are seen in [7] as near mean graphs. In[8], we showed that the C3 and C4 armed and double armed crown graphs are close mean graphs. We showed in this paper that the graphs Pn, Cn, K2,n are close mean graphs and that the graphs Kn(n > 4) and K1,n(n > 4) are not close mean graphs.

Near Mean Graphs

Theorem 3.1 The path Pn is a near mean graph.

Proof Let Pn be a path of n vertices with V (Pn) = {u1, u2, . . ., un} and E(Pn) = *{(uiui+1)/*i = 1, 2, . . ., n − 1}. Define f : V (Pn) → {0, 1, 2, . . ., n − 1, n + 1} by

f(ui) = i − 1, 1 ≤ i ≤ n

f(un) = n + 1.

Clearly, f is injective. It can be verified that the induced edge labeling given by f ∗ (uiui+1) = i(1 ≤ i ≤ n) are distinct. Hence, Pn is a near mean graph.

Example 3.2 A near mean labeling of P4 is shown in Figure 1.

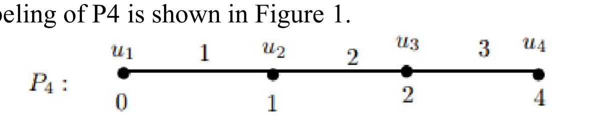

Figure 1: P4

Theorem 3.3 Kn,(n > 4) is not a near mean graph.

Proof Let f: V(G) = {0 , 1, 2,.. ., q−1, q + 1}. We would use either 0 and 1 as vertex labels, or 0 and 2 as vertex labels, to get the edge mark 1.

In any case, 0 must have a certain vertex number. We would have either q − 1 and q + 1 as vertex labels in the same manner as edge mark q, or q − 2 and q + 1 as vertex labels. Let u be a vertex whose mark of vertex is 0.

Case i If I Grab the edge label q. Assign q − 1 and q + 1 vertex codes to the vertices w and x and x respectively.

Subcase a. Let v be a vertex whose vertex label is 2, then get the same label on the edges of vw and ux.

Subcase b. Let v be a vertex with 1. as its vertex mark.

If q is strange, then the edges of vw and ux get the same name. Similarly, when q is also, the uw and vw edges obtain the same label as well as the ux and vx edges obtain the same label.

CaseiiTo get the edge label q, add the vertex labels q − 2 and q + 1 respectively to the vertices w and x.

Subcase a. Let the vertex whose vertex number is 1. be v.

For n > 4, there should be a vertex whose vertex label is either 3 or 4 to get edge label 2. Let the z (say) be it. The edges ux and wz have the same label and the edges uz and vz have the same edge label if the vertex label of z is 3. The edges vx and wz have the same name while the vertex mark of z is 4.

Subcaseb. Let v be a vertex whose mark 2 is the vertex.

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For n > 4, there should be a vertex, say z, whose vertex label is either 3 or 4, to get edge label 2. The ux and wz edges get the same mark when the vertex mark of z is 3. Suppose that z's vertex number is 4. If q is the same mark, then the edges of ux and wz are the same.

If q is odd, then vw and ux have the same marking on the sides. Kn(n ≥5) is thus not a close-mean table.

Remark 3.4 K2, K3 and K4 are near mean graphs.

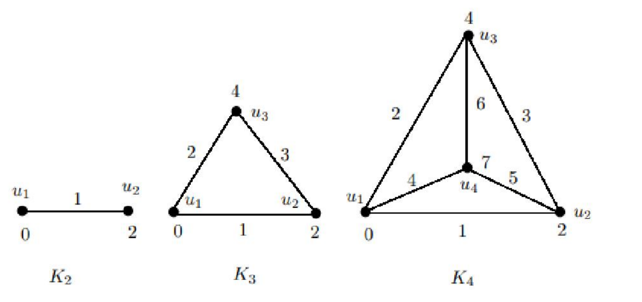

Figure 2: K2, K3, K4

Theorem 3.5 A cycle Cn is a near mean graph for any integer n ≥ 1.

Proof Let V (Cn) = (u1, u2, u3, . . ., un, u1} and E(Cn) = {[(uiui+1) : 1 ≤ i ≤ n − 1] ∪ (u1un)}.

Case i Let n be even, say n = 2m.

Define f : V (Cn) → {0, 1, 2, . . ., 2m, 2m + 2} by

f(ui) = i − 1, 1 ≤ i ≤ m.

f(um+j) = m + j, 1 ≤ i< m.

f(un) = 2m + 1.

Clearly f is injective. The set of edge labels of Cn is {1, 2, . . ., q}.

Case ii. Let n be odd, say n = 2m + 1.

Define f : V (Cn) → {0, 1, 2, . . ., 2m − 1, 2m + 1} by

f(ui) = i − 1, 1 ≤ i ≤ m

f(um+j) = m + j, 1 ≤ j ≤ m.

f(u2m+1) = 2m + 2.

Clearly f is injective. The set of edge labels of Cn is {1, 2, . . ., q}.

Example 3.6 A near mean labeling of C6 and C7 is shown in Figure 3.

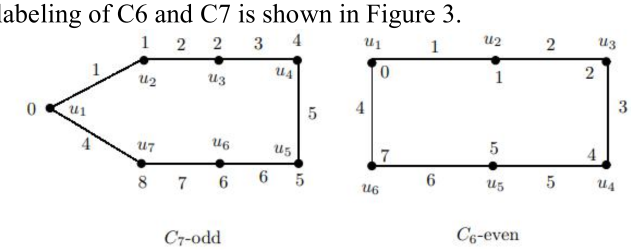

Figure 3: C6, C7

Theorem 3.7 K1,n(n > 4) is not a near mean graph.

Evidence Let V(K1,n)={u, vi:1 ⁇ I ⁇ n} and E(K1,n)={(uvi):1 ⁇ I ⁇ n}. Let V(K1,n)={u, vi:1 ⁇ I ⁇ n} and E(K1,n)={(uvi):1 ⁇ I ⁇ n}. To have the edge mark 1, u and vi are allocated to either 0 and 1(or)0 and 2 for certain i. In any case, 0 must have a certain vertex number.

Suppose if f(u) = 0, then the edge mark q can not be identified. Assume that if f(v1) = 0, then f(u) = 1 or f(u) = 2.

Instance I. The Instance Let f(u) = 1. Let f(u) = 1.

We need the following possibilities to get the edge code q, namely q − 1 and q + 1 or q − 2 and q + 1. If f(u) = 1, it's only true if q is either 2 or 3. But q > 4, so the edge value of q can not be accessed.

The situation of ii. Let 's assume f(u) = 2.

As in case I if f(u) = 2 and if one edge value is q, then either 3 or 4 is the value of q. It is not necessary to achieve an edge value of q in both situations, if q > 4. K1,n(n > 5) is thus not a near-mean table.

The near-mean graph is Remark 3.8 K1,n, n≤ 4. For instance, Figure 4 shows one such close-mean labeling.

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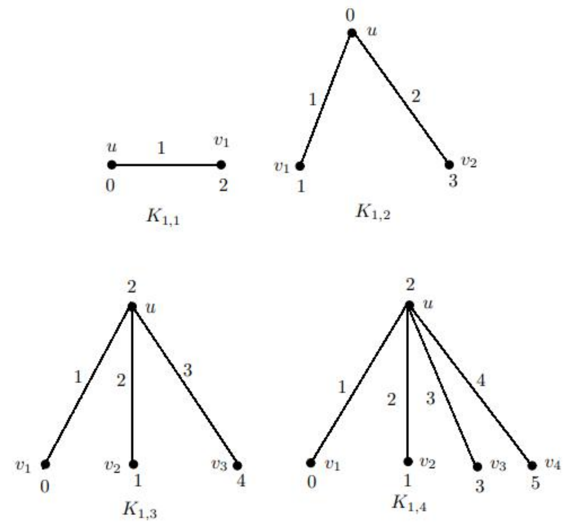

Figure 4: K1,n, n ≤ 4

Theorem 3.9 K2,n admits near mean graph.

Proof Let (V1, V2) be the bipartition of V (K2,n) with V1 = {u1u2} and V2 = {v1, v2, . . ., vn}. E(K2,n) = {(u1vi),(u2vi) : 1 ≤ i ≤ n}.

Define an injective map f : V (K2,n) → {0, 1, 2, . . ., 2n − 1, 2n + 1} by

f(u1) = 1

f(u2) = 2n + 1

f(vi) = 2(i − 1), 1 ≤ i ≤ n.

Then, it can be verified f ∗ (u1vi) = i, 1 ≤ i ≤ n, f ∗ (u2vi) = n+i, 1 ≤ i ≤ n and the edge values are distinct. Hence, K2,n is a near mean graph.

Example 3.10 A near mean labeling of K2,4 is shown in Figure 5.

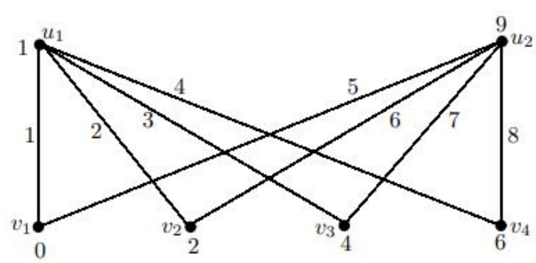

Figure 5: K2,4

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