JOURNAL OF CRITICAL REVIEWS

ISSN- 2394-5125 VOL 7, ISSUE 12, 2020

WAVE ATOM ,CURVELET AND WAVELET BASED NOISE REDUCTION IN MEDICAL IMAGES

Vivek Kumar Soni
Assistant Professor Computer Sience and Engg
Kalinga University, Raipur (Chhattisgarh) INDIA
vivek.soni@kalingauniversity.ac.in

Abstract— The medical images usually bring different kinds of noise in process of receiving, coding and transmission. In this paper, the wavelet transform, curvelet transform and wave atom were used for denoising of medical images with gaussian noise. At first, we figure the digital implementations of three newly developed all-around representation systems. The paper aims at the analysis of denoising of images using wavelet, curvelet, wave atom transform over medical images. We seek these methods to the problem of reinstating an image from noisy image and compare the effects of denoising on image. The experimental results show the denoising method by wavelet transform is less effective than the other methods. We have used some registered sets of Medical Images like MRI, Ultrasound and quality parameters like MSE, PSNR to know the quality of image.

Keywords- Noise; Denoising; Transform; Wavelets; Curvelets; Wave Atoms; MSE; PSNR.

INTRODUCTION

Enhanced medical images are desired by a surgeon to assist diagnosis and interpretation because medical image qualities are often deteriorated by noise. Medical image processing mainly based on grayscale transform and frequency domain transform. Image reconstruction plays an important role in signal processing. An image is often taint by noise in its accretion or transmission. The aim of denoising is to remove the noise from the corrupted image and retain as much as possible the important information. There are many methods for denoising, such as filtering method and smoothing method which comes under Fourier transform. But Fourier transform based spectral analysis is the dominant analytical tool for frequency
domain persual and cannot provide any information of the spectrum changes with respect to time. Over the last two decades, there has been copious interest in wavelet, curvelet and wave atom methods for noise removal in signals and images. [1][5][7][8][9][14].Reconstruction of medical images corrupted by Gaussian noise using wavelet techniques is effective because of its ability to capture the energy of a signal in few energy transform

merit. But there are some limitations of wavelet while drape the line and curve singularities in the medical image. There are other transforms besides wavelet, namely, curvelet, ridgelet, and wave atom transforms [10]. The paper aims at the analysis of denoising medical images using wavelet, curvelet, wave atom transform.

Gaussian noise- The ensign copy of amplifying noise is Gaussian, additive, free at each pixel and free of the signal intensity, caused mostly by thermal noise, including that which comes from the rearrange noise of capacitors. In color cameras where additional amplification is used , there can be more noise in the channel. Amplifier noise is a most important component of the "read noise" of an image sensor [15].

WAVELET, CURVELET AND WAVE ATOM

A. Wavelet Transform and Denoising

A wavelet is a waveform of an effectively limited duration that has an average value to zero. The transform is the essential transform for time-frequency description of analyzed signal as mentioned in the introduction. It can be used in different signal processing applications, e.g. signal compression, feature extraction, and noise suppression [10]. It gives details regarding the frequency content of a signal. It is the solution to the multi resolution problem. It has the important property of not having a set width sampling window. The transform can be broadly classified into continous wavelet transform, and discrete wavelet transform. The theory of wavelet transform is explained in many publications, a more detailed description of the wavelet transform and its properties can be found, for example, in [11]. The general wavelet denosing procedure is as follows: [3,7]

1. Apply wavelet transform to the noisy image to produce the wavelet coefficients to the level which we can properly distinguish the signal and noise.
2. Select appropriate threshold limit at each level and threshold method (hard or soft thresholding) to best remove the noises.
3. Perform inverse wavelet transform of the thresholded wavelet coefficients to obtain a denoised image.

B. Curvelet Transform and Denoising

Curvelets are a non-adaptive technique for multi-scale object demonstration. Being a conservatory of the wavelet concept, they are becoming accepted in similar fields that are in image processing and scientific computing. The Curvelet transform is a higher dimensional generalization of the Wavelet transform planned to represent images at different scales and different angles [12]. Curvelets can be obtained by partitioning the 2D Fourier plane into dyadic coronae and sub-partitioning those into angular wedges. Curvelets generally obeys a parabolic scaling law at scale 2_{-j} , every element has an effective support of width 2_{-j} and length $2_{-j/2}$. Present are two types of Curvelet Transform such as Curvelet Transorm via USFFT (Unequi-Spaced FFT) and Curvelet Transform via Wrapping. The Unequispaced FFT, wherever the curvelet coefficients are initiate by non-regularly sampling the Fourier coefficients of an image. In Wrapping transform, by means of a series of translations and a wrap around technique, mutually algorithms having the same outcome, but the Wrapping Algorithm gives an earlier computation time than USFFT. The Wrapping translation uses, a decimated rectangular grid aligned with the image axes. For a specified scale, there are essentially two such grids. The latest fast discrete curvelet transforms (FDCTs) [13] which are simpler, faster, and less redundant than existing proposals. The frequency localization of φj implies the following spatial structure: φj(x) is of rapid decay away from a 2_{-j} by $2_{-j/2}$ rectangle with main axis pointing in the vertical direction. In the effective length and width obey the anisotropy scaling relation [12].

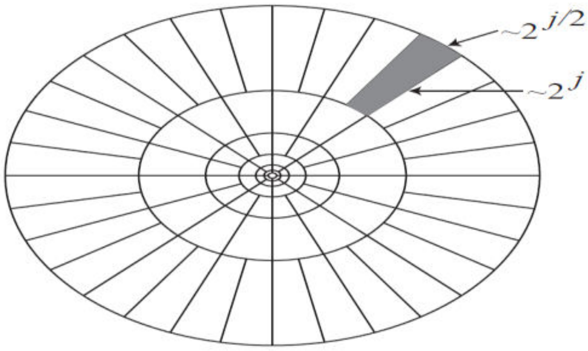


Figure 1. Curvelet tiling of frequency and space.

A. Subband Definitions

We have partitioned the frequency domain into just 3 subbands, indexed by s = 1;2; 3. It is supportive to keep in mind that on a 256 *256 image, the standard discrete wavelet transform would offer 8 subbands, at levels j = 0; … ;7. The Curvelet truss s = 1 corresponds to wavelet subbands j = 0;1;2;3 in a way. Curvelet truss s = 2 corresponds to wavelet subbands j = 4;5 and truss s = 3 corresponds to wavelet subbands j = 6;7. The common rule of succession we are trying to implement in this way is:

Curvelet subband s ↔Wavelet Subbands j ϵ {2s; 2s + 1} (1) so, in an implementation with n = 4096, we would have 5 subbands, with s = 4 corresponding to j = 8; 9 and s = 5 corresponding to j = 10; 11 [13].

B. Subband Filtering

To in fact implement our decomposition into subbands, we use the wavelet transform. We initial decompose the object into its 8 wavelet subbands, after that to form Curvelet truss s, we perform partial reconstruction from those wavelets at levels j϶{2s;2s + 1}.

C. Wave atoms and Denoising

Wave atoms are a recent inclusion to the collection of mathematical transforms of computational eurhythmic analysis. They come either as an orthonormal basis or a tight frame of directional wave packets, and are particularly well suited for representing oscillatory design in images. They also provide a infrequent representation of wave equations, hence the name wave atoms. In continuous frequency, define

$$\Psi_m^0(\omega) = e^{-\frac{i\omega}{2}}[e^{i\alpha_m} g(\varepsilon_m(\omega-\pi(m+1/2))) + e^{-i\alpha_m} g(\varepsilon_{m+1}(\omega+\pi(m+1/2)))] \quad (2)$$

Where $\alpha_m = \frac{\pi}{2}(m+\frac{1}{2})$ and $\varepsilon_m = (-1)^m$ .The function g is a particular real-valued, compactly-supported C bump function chosen such that

$$\sum_m |\Psi_m^0(\omega)|^2 = 1 \quad (3)$$

and such that translates $\{\psi_m(t-k)\}$ k form an orthonormal basis of L^2 (R). Multiscale tilings can be obtained by combining dyadic dilates and translates of Ψ_m^0 on the frequency axis. We need to introduce the subscript j to index scale, and write the basis functions as

$$\psi_{m,n}^j(x) = \psi_m^j(x-2^{-j}n) = 2^{\frac{j}{2}} \Psi_m^0(2^j x-n) \quad (4)$$

To preserve orthonormality of the $\psi_{m,n}^j(x)$ the profile g needs to be asymmetric in addition to all the other properties, in the sense that

ISSN- 2394-5125 VOL 7, ISSUE 12, 2020

$$g(-2\omega-\pi/2) = g(\pi/2+\omega)$$

for $\omega\in[-\pi/3,\pi/3]$, with g supported on $[-7\pi/6,5\pi/6]$, As for characterize, note that the couple (j, m) refers to a point on the wavelet packet tree; the depth at that point is $J - j$, where J is the maximum depth, and m can be interpreted as the number of nodes on the left at the same depth. The conveyance step is now $2^{\,j}$ at scale j, whereas each bump in frequency is supported on an interval 2 of length $2^{\,j} \cdot 2\pi$ .Within the limits of the Fourier uncertainty principle, a given atom $\psi^{j}_{m,n}(x)$ centred in the (x,ω) space at

$$x_{j,n=2^{-j}n}\;,\;\; \omega_{j,n=\pi\, 2^{j}m}$$

More precisely, the admissible values for m are in the interval $[2^{j-1}+1..2^{j+1}+1]$ when $j \geq 2$.The resulting basis of wavelet packets $\psi^{j}_{m,n}(x)$ is orthonormal for L^2 (R). Two-dimensional extraneous basis functions with 4 bumps in the frequency plane can be formed by individually taking products of 1D wavelet packets. Abbreviating $\mu = (j,\boldsymbol{m},\boldsymbol{n})$, where $\boldsymbol{m} = (m_1, m_2)$ and $\boldsymbol{n} = (n_1, n_2)$, the products are

$$\varphi^{+}_{\mu}(x_{1,}x_{2,}) = \psi^{j}_{m1}(x_1\text{-}2^{-j}n_1)\ \psi^{j}_{m2}(x_2\text{-}2^{-j}n_1)$$

And

$$\varphi^{+}_{\mu}(x_{1,}x_{2,}) = \mathrm{H}\psi^{j}_{m1}(x_1\text{-}2^{-j}n_1)\,\mathrm{H}\psi^{j}_{m2}(x_2\text{-}2^{-j}n_1)$$

where H denotes Hilbert transform.Then the recombination.

$$\varphi^{(1)}_{\mu} = \frac{\varphi^{+}_{\mu}+\varphi^{-}_{\mu}}{2}, \quad \varphi^{(2)}_{\mu} = \frac{\varphi^{+}_{\mu}+\varphi^{-}_{\mu}}{2}$$

provides cornerstone functions with two bumps in the frequency plane, symmetric with respect to the origin, hence directional wave packets. Jointly$\varphi^{(1)}_{\mu}$ and $\varphi^{(2)}_{\mu}$ form the wave atom frame and may be denoted jointly as $\varphi^{(1)}_{\mu}$

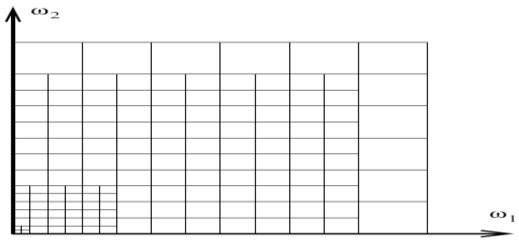


Fig. 2. The wave atom tiling of the frequency plane.[13]

THRESHOLD VALUE ESTIMATION

The aim of the denoising algorithm is to find the best estimate **x** of an unknown signal **x** from a noisy observation of it **y = x + b**. Every element of the noise **b** is assumed to follow an independent normal law specified by a zero-mean and a known variance. [7][16]

In summary, wavelet, curvelet and wave atom denoising involves the following three main steps.

- Transformation: Compute the coefficients by wavelet transform or curvelet transform or wave atom.
- Threshold vale: Apply the proposed scheme to threshold the coefficients using the hard or soft thresholding techniques.
- Inverse transformation: Perform inverse transformation on the thresholded coefficients.

Let Signal be $\{ f_{i,j} ; i, j = 1, 2,\ldots, N\}$, where N is some integer power of 2. It is corrupted by additive noise an signal $g_{i,j}$ can be expressed as

$g_{i,j} = f_{i,j} + \varepsilon_{i,j}$; $i, j = 1, 2, \ldots, N\}$

where $\varepsilon_{i,j}$ are independent and identically distributed (*iid*) as normal $N\ (0,\sigma^2)$ and independent f. Hard thresholding operator with threshold T is defined as

$$D(g_{i,j}, T) = \begin{cases} g_{i,j} & (g_{i,j} \geq T) \\ 0 & (g_{i,j} < T) \end{cases}$$

And soft thresholding operator with threshold T is defined as

$$D(g_{i,j}, T) = \mathrm{sgn}(g_{i,j}) * \max(g_{i,j} - T, 0)\ .$$

The algorithms are evaluated by mean squared error (MSE)[17][18] and peak signal-to-noise ratio (PSNR) as a measure of quality of denoised image. MSE and PSNR are defined respectively as

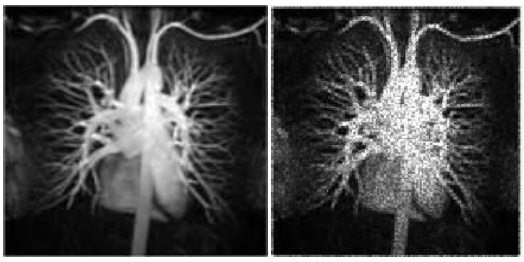
$$\mathrm{MSE} = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}[g(i,j) - p(i,j)]^2,$$

$$\mathrm{PSNR} = 10\log\left(\frac{255^2}{MSE}\right)\mathrm{db},$$

where $g(i, j)$ is original image and $p(i, j)$ is denoised image.

EXPERIMENT

Comparing the effects of three denoising methods, We firstly perform the experiments on the well-known MRI image (Fig. 1a). The image is used as test image with Gaussian noise level of $\sigma = 8$. Experimental results show PSNR and MSE are improved with curvelet transform as compared to wavelet transform and wave atom frame (Table 1). Fig.1b shows the MRI image corrupted with Gaussian noise with $\sigma = 8$. Fig. 1c shows the denoised image with wavelet transform. Fig. 1d shows the denoised image with curvelet transform. Fig. 1e shows the denoised image with wave atom frame.



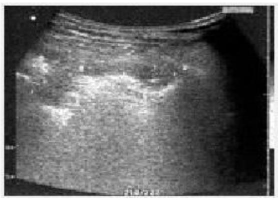
(a) (b)

(c) (d)

ISSN- 2394-5125 VOL 7, ISSUE 12, 2020

(e)

Fig.1 MRI image denoising by using different methods. (a) MRI Image (b) Noisy image, (c) Denoised image by wavelet transform, (d) Denoised image by wave atom, (e) Denoised image by curvelet transform

(a) (b)

(c) (d)

(e)

Fig.2 Ultrasound image denoising by using different methods. (a)Ultrasound Image (b) Noisy image, (c) Denoised image by wavelet transform, (d) Denoised image by wave atom (e) Denoised image by curvelet transform.

TABLE I. PSNR AND MSE OF NOISY AND DENOISED MEDICAL IMAGES

Methods	**MRI Images**		**Ultrasound Images**	
	MSE	**PSNR**	**MSE**	**PSNR**
Noisy	540.02	18.00	2408.32	11.84
Wavelet	61.2	25.45	450.42	18.22
WaveAtom	60.62	25.49	229.23	21.23
Curvelet	52.78	26.12	375.70	20.22

ISSN- 2394-5125 VOL 7, ISSUE 12, 2020

CONCLUSION

In this paper, we make use of curvelet, wavelet and wave

atom transform for denoising of medical images. We performed the experiments on two images, MRI and Ultrasound image. The experimental results show:

- The denoising method by wavelet transform is worst in the three methods.
- When the noise level is low, the effects of denoising by wavelet, curvelet and wave atom transform are equivalent.
- When the noise level is high, the effects of denoising by wavelet transform are not satisfactory, and the effects of denoising by wave atom transform are best.

The problem with hard thresholding is that by thresholding to zero all transform coefficients lower than given threshold causes crease near discontinuities. So we must further study on thresholding.

REFERENCES

[1] M. Antonini, M. Barlaud, P. Mathieu, and I. Daubechies, “Image coding using wavelet transform,” IEEE Trans. Image Processing, vol. 1, no. 2, pp. 205–220, 1992.

[2] Lu, J., Healy, D.M., and Weaver, J.B.: ‘Contrast enhancement of medical images using multi-scale edge repressentation’, Opt. Eng., 1994, 33, (7), pp. 2151–2161

[3] G. Beylkin, R. R. Coifman, V. Rokhlin, “Fast wavelet transforms and numerical algorithms,” Comm. on Pure and Appl. Math., vol. 44, pp. 141-183, 1991.

[4] Fang, Y., and Qi, F.: ‘A method of wavelet image enhancement based on soft threshold’, Comput. Eng. Appl., 2002, 23, pp. 16–19.

[5] I. M. Johnstone and B.W. Silverman, “Wavelet threshold estimators for data with correlated noise,” J. R. Statist. Soc., vol. 59, 1997

[6] Wu, Y., and Shi, P.: ‘Approach on image contrast enhancement based on wavelet transform’, Infrared Laser Eng., 2003, 32, (1), pp. 4–7.

[7] F. Abramovich, T. Sapatinas, and B. W. Silverman, “Wavelet thresholding via a Bayesian approach,” J. R. Statist. Soc., ser. B, vol. 60, pp. 725–749, 1998.

[8] S. Grace Chang, Bin YU, and Martin Vetterli, “Adaptive Wavelet Tresholding for Image Denoising and Compression,” IEEE Transactions on Image Processing, vol. 9, no. 9, pp. 1532-1546, 2000.

[9] J. L. Starck, E. J. Cand_es, and D. L. Donoho, “The curvelet transform for image denoising,” IEEE Transactions on Image Processing, vol .11, pp. 670-684, 2002.

[10] L. Demanet, “Curvelets, Wave Atoms and Wave Equations,” Ph. D.thesis, California Institute of Technology, 2006.

[11] E. Candes, L. Demanet, D. Donoho, and L.Ying, “Fast discrete curvelet transforms,” SIAM Multiscale Model. Simul., vol. 5, no. 3, pp. 861–899, 2006.

[12] Joseph Tan “Curvelet transform for Denoising Advancing Technologies and Intelligence in Healthcare and Clinical Environments Breakthroughs” , IGI Global,June 30,2012,pp.1-382

[13] Priti Naik, Shalini Bhatia ” Image Denoising Using Curvelet Transform”, Proceedings of SPIT-IEEE Colloquium and International Conference, Mumbai, India vol.1,pp.116–120.

[14] Bo Zhang, Jalal M. Fadili, and Jean-Luc Starck, “Wavelets, Ridgelets, and Curvelets for Poisson Noise Removel,” IEEE Transactions on Image Processing, vol. 17, no. 5, pp. 1093-1108, 2008.

[15] Charandeep Singh Bedi “Qualitative and Quantitative Evaluation of Image Denoising Techniques” International Journal of Computer Applications (0975 – 8887)Volume 8– No.14, October 2010, pp. 31–34.

[16] Bo Zhang, Jalal M. Fadili, and Jean-Luc Starck, “Wavelets, Ridgelets, and Curvelets for Poisson Noise Removel,” IEEE

ISSN- 2394-5125 VOL 7, ISSUE 12, 2020

Transactions on Image Processing, vol. 17, no. 5, pp. 1093-1108, 2008

[17] Mukesh C. Motwani “Survey of Image Denoising Techniques” Image Process Technology, Inc. 1776 Back Country Road1776 Reno, NV 89521 USA

[18] Suresh Kumar,” Performance Comparison of Median and Wiener Filter in Image De-noising” International Journal of Computer Applications (0975 – 8887) Volume 12– No.4, Nov 2, 2010