

A HEURISTIC METHOD FOR ONE DIMENSIONAL CUTTING STOCK PROBLEM (1D-CSP) WITH VARIABLE SUSTAINABLE TRIM LOSS

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ABSTRACT: The fundamental of cutting stock problem is to assign cutting plan of required number dm_1, dm_2, \dots, dm_n of order lengths ol_1, ol_2, \dots, ol_n at minimum cost from given stock bars St_1, St_2, \dots, St_m . When this problem was modeled as a linear or an integer programming problem, it required large number of variables, which made the computation unviable. This chapter concerns with a heuristic model, in which we intent to cut two order lengths say αol_i and βol_j at a time where α, β are non-negative number.

The stock lengths St_1, St_2, \dots, St_m are arranged in ascending order with respect to their lengths. Corresponding to each stock length St_k , we assume that the industry may sustain $t_k\%$ trim loss. We have considered the sequence $\{t_k\}$ in non-increasing order, so that the trim loss corresponding to the largest stock length should not exceed the trim loss corresponding to the lower stock length. It has been perceived that this concept of the proposed method has resulted in much less trim loss in comparison to the trim loss computed by other techniques.

KEYWORDS: PDSTL, VPDSTL, VPDCSTL, sustainable trim loss

I. INTRODUCTION

The cutting stock problem involves the cutting of materials of high cost which demands the need of minimization of waste or leftover when consumed to cut the assigned number of orders. This problem arises frequently in the industrial applications to determine how smaller shapes of different requirements can be adjusted to a large stock bar of finite lengths to minimize the leftover.

The fundamental aspects used in the problem of cutting stocks are the generation of patterns, followed by the cutting of the required number of pieces of materials from the measurement of the stock based on the patterns generated. We focus this chapter on the problem of One-Dimensional-Cutting Stock (1D-CSP) in which the type of the patterns may or may not be consistent. For years, the basic two main methodologies in CSP have been in existence, viz., heuristic and linear programming. There are several texts, mathematical simulations and methods available on Cutting stock problem, from which many restraints may arise in its application. Among the various constraints of CSP, a problem which has not fully considered yet is how to control the trim loss by determining the sustainable trim loss and planning the cutting patterns with minimum trim loss. An extensive text has been published on 1D-CSP and we here reflect on some present findings in this field-

In this chapter, we propose a mathematical model to establish the cutting plan which involves the cutting of maximum two order lengths at one instance of the required numbers dm_1, dm_2, \dots, dm_n of the corresponding order lengths ol_1, ol_2, \dots, ol_n respectively from the given m number of stock St_1, St_2, \dots, St_m . Assuming $t\%$ trim loss, we compute a trim loss t_j corresponding to the given stock length St_j . At each stage of the cutting process, the stock utilized to cut the dm_i 's of corresponding ol_i 's is verified with its sustainable trim loss computed, to check whether the used stock is substantial with the scrape it is leaving, since the perception of sustainable trim loss that plays a vital role in monitoring the total trim [7]. In real time, the industry proposes Pre-Defined Sustainable Trim Loss (PDSTL) as a constant value ($\leq 3\%$). The authors are in an opinion that, using a constant value of PDSTL, there is always a greater scrape left with large stock lengths, due to which there is always a possibility of the industry to have greater loss. Therefore, the authors enunciate variations in the PDSTL corresponding to each stock length St_j , such that the PDSTL of the largest stock length should not exceed the PDSTL computed of the shortest stock length. In particular if $t_k\%$ is a sustainable trim loss corresponding to the stock length St_k , then following has been assumed:

$St_1 \leq St_2 \leq \dots \leq St_m$, then

$$t_1 \geq t_2 \geq \dots \geq t_m$$

The use of these variations has resulted in better result. In support of our assertions, we have elaborated one example in which the data has been extracted from [8] and observed that this new technique results with less trim loss than the trim loss obtained in [8].

This chapter is structured as follows: Section 2 describes the preliminaries and notations for the problem; Section 3 briefs a new version of Sustainable Trim Loss. Section 4 elucidates the Mathematical Formulation, Section 5 deals pseudo code, section 6 presents the Computational Results and comparative study of this model with a model being developed by [8], and the succeeding section 7 concludes the study.

II. DEFINITION OF 1DCSP WITH VARIABLE PDSTL (VPDSTL)

PDSTL characterizes a significant role in increasing the performance of production industries. Therefore 1D-CSP with variable PDSTL with respect to each stock length can be defined as an appropriate utilization of stock material with least discard, whereas in the previous work (see [10]), PDSTL was computed against a constant value on all stock lengths. Therefore, VPDSTL play a remarkable role for the determination of the solution with minimum trim loss.

III. NOTATIONS

In 1D-CSP, we assume there is ample number m of stock bars of different lengths St_j , ($j = 1, 2, \dots, m$) accessible, n number of order lengths ol_1, ol_2, \dots, ol_n with corresponding required number of pieces dm_1, dm_2, \dots, dm_n respectively. We demonstrate 1D-CSP anticipated for Transmission tower manufacturing Industry. We recommend cutting plan with cutting at most two order lengths at a time considering the working space restraint and manpower constraint of sorting the scrape.

We are now set to define our problem exclusively with the following notations:

ol_i – the order lengths; $i = 1, 2, \dots, n$ (arranged in descending order with respect to length; $ol_1 \geq ol_2 \geq ol_3 \dots$),

dm_i – required number of pieces of order length ol_i ,

St_j – stock lengths; $j = 1, 2, \dots, m$; the lower bound (u) and upper bound(U) of stock varies from 7mts to 14mts respectively; m depends on the cutting process for a particular dataset; (arranged in ascending order with respect to length; $St_1 \leq St_2 \leq St_3 \dots$).

This model proposes 1D-CSP with cutting at most two order lengths at a time, therefore

ol_{ij} – combinations of order length ol_i and ol_j to be cut from stock St_j ;

$i, j \in BK(n)$ when $BK(n)$ is block of integers; $1, 2, \dots, n$

We generate cutting patterns, which is a combination of demand orders to be cut from appropriate one stock bar which is represented by

p_{ij} – pieces of order length ol_i being cut from the stock St_j ,

The cutting pattern generated should be optimum in order to have minimum waste of materials and it is assumed that all input-data are integers.

$t_{s_k}^p$ % – VPDSTL percent varying for different St_j is used, where $k = 1, 2, \dots, m$, where $t_{s_1}^p \geq t_{s_2}^p \geq \dots \geq t_{s_m}^p$

$t_{s_j}^c$ - computed sustainable trim loss based on VPDSTL by the industry for stock bar where $j = 1, 2, \dots, m$.

t_j – refers the total trim loss corresponding to the stock St_j used to cut the corresponding combinations of order lengths.

It may be possible that some of the required order lengths ol_i s maybe similar to the standard stock lengths St_j , so we classify the order lengths in the following two categories in accordance with their required type of the order lengths:

Category I: Collect all those order lengths which are exactly equal to stock lengths, so that these stock lengths are kept separately with required number of pieces without any trim loss.

Category II: We collect the remaining of the order lengths which vary to the size of the stock lengths which we proceed for cutting plan with combinations in order to minimize the trim loss.

IV. A NEW VERSION OF SUSTAINABLE TRIM LOSS FOR ONE-DIMENSIONAL CUTTING STOCK PROBLEM

Variable pre-defined computed sustainable trim loss $t_{s_j}^c$ is the loss in percent from a given stock length St_j that an industry can sustain when the leftover of the stock after the cutting process is considered as waste, if

not, then there can always be a possibility of limitations of space and manpower, since the leftover may require sorting and searching for reuse.

Therefore, we propose a new concept of computation of VPDCSTL depending on the VPDSTL of the industry. The profit sustainability of the industry is- What is the acceptance limit of the scrape from the used raw material after the cutting process.

Note: The scrape (trim loss t) affordable by the industry is up to 3% for its even and cost-effective functioning.

Here, we presume flexible PDSTL and the variations of the PDSTL should be such that VPDCSTL computed of largest stock length should not exceed the VPDCSTL of the shortest stock length. If keeping a constant PDSTL the scrape generated with the largest stock length will be obviously larger which may lead to have considerably large trim loss. Therefore, this new proposed concept will lead to have trim loss manageable and sustainable.

Computation of the $t_{s_j}^c$:

Variable Pre-defined computed sustainable trim loss is computed depending on $t_{s_j}^p$, the pre-defined sustainability of the industry which is considered to be variable so that the sustainable trim loss of the largest stock length should not exceed the lowest order length. The $t_{s_j}^c$ for stock bar St_j is calculated as

$$t_{s_j}^c = \frac{St_j * t_{s_j}^p}{100} ; j = 1, 2, \dots, m \quad (3.1)$$

which is desired sustainable trim loss corresponding to the stock length St_j exclusively and we call it as variable Pre-defined computed sustainable trim loss.

V. MATHEMATICAL FORMULATION

Transmission tower industry requires angular bars for designing of towers, thereby analyzing the pragmatism of the cutting patterns for the tower industry, a model is been formulated which is as follows:

Objective: To minimize the trim loss t_j when a combination ol_{ij} of order lengths ol_i and ol_j is cut from a given stock St_j . We define the objective function of the model as

$$\min \sum_{j=1}^k t_j (\text{minimization of trim loss}) (k \leq m) \quad (4.1)$$

$$t_j \geq 0 \quad \forall j$$

subject to the conditions which are categorized as follows:

Category I: Choose the order lengths having the size similar to given stock length, since it does not require the process of forming the combinations and cutting pattern, as the stock bars can itself be disbursed for the number of required pieces such that

$$\text{number of } ol_i \text{ cut} = \begin{cases} 0, & ; \quad ol_i \neq St_j \\ dm_i = \text{Number of } St_j & ; \quad ol_i = St_j \end{cases} \quad (4.2)$$

This cut is possible with zero trim loss.

Category II: Consider all other order lengths which are not a part of Category I (cf. (4.2)), to generate cutting patterns with minimal trim loss. At the outset the permutations are obtained by combining the order length ol_i with rest of the order lengths i.e., ol_j where $j = 2, 3, \dots, n$. We consider i being 1, i.e., order length ol_1 being the largest among all order length (arranged in the descending order), is being cut at the initial stages of the cutting process, since it is assumed that smaller lengths can be adjusted easily later in the cutting plan. Therefore, the execution of the cutting plan is as following-

Step 1: We define the combination

$$ol_{1j} = \alpha ol_1 + \beta ol_j \text{ for } 1 \neq j; \alpha, \beta \text{ are non-negative integers; } j = 2, 3, \dots, n; \alpha, \beta \text{ both cannot be zero}$$

Now among the combinations ol_{1j} choose the combination of the order lengths having the maximum length, satisfying the condition $u \leq ol_{1j} \leq U$, where,

- u is the lower bound of the stock bar length which is viable for the supplier to supply.
- U is the upper bound of the stock bar length that cannot be exceeded, due to the limitations transportation.

Therefore,

$$ol_{1j} = \max_j \{ol_{1j} = \alpha ol_1 + \beta ol_j ; j = 2, \dots, r; r < n \text{ or } r = n \text{ when category I does not exist; } i < j; \alpha, \beta \text{ are non-negative integers.} \quad (4.3)$$

Consider

$$S = \{ol_{1j} : u \leq ol_{1j} \leq U\}$$

$$\sum_{j=1}^k p_{ij} = dm_i (\text{demand constraint}) \quad (4.4)$$

where

$$p_{ij} = \begin{cases} 0, \text{ order length } ol_i \text{ not cut from the} \\ \text{stock length } St_j \\ \text{Number of order lengths } ol_i \text{ cut from} \\ \text{the stock length } St_j \text{ (positive integer)} \end{cases}$$

Step2: We now choose the stock length St_r (say) for $j = r$ such that $(St_j - ol_{1j})$ is minimum over α, β and j and also,

$$(St_r - ol_{ij}) \leq t_{s_j}^c \quad (4.5)$$

The first stock length St_r is fixed to be cut. Refer (3.1) for $t_{s_r}^c$.

Upon satisfying (4.5), of the cutting pattern ol_{1j} , get the corresponding order lengths ol_1 and ol_j along with its corresponding required number of pieces dm_1 and dm_j . There are four cases under consideration, of which one is applicable for further cutting plan when stock length St_r is selected with minimum trim loss such that

$$t_r = [St_r - (\max_j \{ ol_{1j} = \alpha ol_1 + \beta ol_j ; j = 2, \dots, n ; \})] \leq t_{s_r}^c \quad (4.6)$$

satisfies.

Case 1: The pieces of order lengths ol_1 and ol_j for some j are completely cut. It is now left with $n - 2$ order lengths which are already arranged in descending order and continues the process with step 1 and procedure continues with any of the cases coming across till all order lengths are cut with minimum trim loss

Case 2: The number of pieces of ol_1 is cut totally. The remaining ol_j 's are already arranged in descending order. We shall now further consider the combinations from the remaining ol_j 's, such that

$$ol_{2j} = \max_j \{ ol_{2j} = \alpha ol_2 + \beta ol_j ; j = 3, \dots, r \}$$

We select the stock length say St_k , to cut the combination ol_{2j} , such that it satisfies (cf.(4.5)) as stated in step 2 and the procedure continues when (4.6) satisfies, which may again come across with any of these cases till all demands are exhausted.

Case3:The number of pieces of ol_j for some j where $j \neq 1$, is cut completely. The process continues the same as the case 2 with ol_1 and remaining ol_j 's when (4.5) and (4.6) are satisfied.

Case 4:Neither the pieces of ol_1 nor ol_j are completely cut. Say dm_1^* and dm_j^* pieces of ol_1 and ol_j remain to cut. For convenience, we denote these numbers by dm_1 and dm_j again. Following the similar process discussed in category II, we trace the same steps till all the order lengths are exhausted.

VI. PSEUDO CODE

Read the order length ol_i and the required number of pieces dm_i and arrange in descending order with respect to ol_i ;

Read the available stock length St_j .

check for $ol_i = St_j$ // cutting process completed for the chosen ol_i for k order length, after which we are left with $n - k$ order lengths, which we again denote by n .

// remove the ol_i for k order length with corresponding dm_i from the order list

do while ($n > 0$)

for $i = 1$ to $n - 1$

for $j = 2$ to n

calculate $ol_{ij} = \alpha ol_i + \beta ol_j$ // α, β ; generate values using loop from 0 to 4

gotoj loop for $j \leq n$

gotoi loop for $i \leq n - 1$

select $ol_{ij} = \max_j (ol_{ij}; u \leq ol_{ij} \leq U)$

check ol_{ij} in St_j list and fetch $ol_{ij} \leq St_j$

then calculate $t_{s_j}^c = \frac{t_{s_j}^p * St_j}{100}$ // $t_{s_j}^p$ is variable
 if $t_j = \min(St_j - ol_{ij} \leq t_{s_j}^c)$

Note: Proceed for cutting plan;

fetch $St_j \geq ol_{ij}$

$totSt += St_j$

check ol_i, ol_j with corresponding dm_i, dm_j analogous to ol_{ij}

Note: i is first element on the list.

if $dm_1 = dm_j$

remove ol_i, ol_j and dm_i, dm_j from the order list.

$n -- 2$

if $dm_1 < dm_j$

remove ol_1 and dm_1 and update $dm_j = dm_j - dm_1$

$n --$

if $dm_1 > dm_j$

remove ol_j and dm_j and update $dm_1 = dm_1 - dm_j$

$n --$

if $dm_1 = dm_j \neq 0$

process continue

if $ol_{ij} \leq u$

select $St_j = u$ to cut ol_{ij}

calculate $gt_j = gt_j + t_j$

go to do loop till all $dm_i = 0$ for all $ol_i \forall i = 1, 2, \dots, n$

print $totSt, gt_j$

compute and print trim loss t_j

VII.COMPUTATIONAL RESULTS AND COMPARATIVE STUDY

An application of the above given algorithm is been executed on the data being used by [135]. This is a factual data set being extracted from Transmission tower industry. For trade with moderate profit the industry considers the leftover to scrape up to 3%. The order lengths and stocks have been considered in centimeters.

We assume that sufficient numbers of stock lengths are available and we compute the variable pre-defined computed sustainable trim loss (VPDCSTL) on VPDSTL (cf.[section 3]), and we obtain, (refer Table: 6.1)

j	Stocklengths St_j (in cms)	$t_{s_k}^p$ %	$t_{s_j}^c$ (in cms)	j	Stocklengths St_j (in cms)	$t_{s_k}^p$ %	$t_{s_j}^c$ (in cms)
1	700	2.8	19.25	8	1050	2	21
2	750	2.8	20.63	9	1100	2	22
3	800	2.8	22	10	1150	1.8	20.13
4	850	2.5	21.25	11	1200	1.8	21
5	900	2.5	22.5	12	1250	1.5	18.75
6	950	2.3	21.38	13	1300	1.3	16.25
7	1000	2.3	22.5	14	1350	1	13.5
				15	1400	1	14

Table 6.1: Details of available stock length and VPDCSTL

The given order lengths with required number of pieces, is mentioned in Table 6.2.

i	Order lengths ol_i (in cms)	Required No. of pieces dm_i (in cms)	i	Order lengths ol_i (in cms)	Required No. of pieces dm_i (in cms)
1	890	36	6	550	39

2	800	13	7	460	21
3	750	24	8	400	47
4	660	16	9	310	40
5	640	23	10	230	32

Table 6.2: Required number of Order lengths

Now we proceed for the cutting patterns for Category I [refer session (4)], we consider, $ol_2 = 800$ and $ol_3 = 750$ which can be cut using the stock bars St_3 & St_2 respectively with their corresponding required number of pieces $dm_2 = 13$ and $dm_3 = 24$. Since cutting of ol_2 and ol_3 is complete, they are removed from the order list, thereby left with $n - 2 = 8, ol_i$'s.

Now dealing with Category II, combinations are made between the order lengths, the first combination satisfying (4.6) is $ol_{(1,8)} = 1350$, having $ol_1 = 890$ and $ol_8 = 230$ with $\alpha = 1$ & $\beta = 2$, this combination can be cut using the stock length $St_{14} = 1350$ with no trim loss. Now we refer (section 4), it satisfies the case 3, i.e., the ol_8 is completely cut and is removed from the order list with ol_1 left with $dm_1 = 20$. Now again the order lengths arranged in the descending order continues with making of the combinations and the next combination satisfying (4.6) is $ol_{1,7} = 1350$ having $ol_1 = 890$ and $ol_7 = 460$ with $\alpha = 1$ & $\beta = 1$. This combination can be cut using $St_{14} = 1350$ and in this case it satisfies case 2 i.e., the required number of ol_1 with $dm_1 = 20$ is completely cut leaving behind ol_7 with $dm_7 = 1$. The process is continued till all dm_i s of ol_i is exhausted. Please refer Table 6.3 for cutting patterns at variable pre-defined sustainable trim loss.

S.No	Order lengths in cm	Pieces to cut	Stock used (in cms)	No. of stock used	Trim loss in cms	Used stock length in cms
1	750	24	750	24	0	18000
	800	13	800	13	0	10400
	890	36	1350	16	0	21600
2	230	32				
	890	20	1350	20	0	27000
5	460	21				
	660	16	1300	16	0	20800
6	640	23				
	640	7	1100	1	0	1100
7	460	1				
	640	6	950	6	0	5700
8	310	40				
	550	39	1350	23	0	31050
9	400	47				
	550	16	950	1	0	950
10	400	1				
	550	15	1100	7	0	7700
11	310	34	950	11	220	10450
	310	1	0	1	40	900
	550	1				
Total trim loss / total stock used					260	155650
Trim loss %						0.1670

Table 6.3 : Cutting patterns at variable PDSTL

Using this method the trim loss generated is 0.167%, it is highly acceptable by the industry. A comparative study of the proposed algorithm was done, using the data being used by [8]. The trim loss computed using [8], refer Table 6.4

For stock length, order length and required number of pieces refer (Table [6.1], [6.2])

S.No	Order Lengths (in cm)	Required no. of pieces cut	Stock used	Trim loss (in cm)	Total stock length (in cms)
1	230	32	1150	480	1150 x 16 = 18400
	660	16			
2	750	24	750	0	750 x 24 = 18000
3	800	13	800	0	800 x 13 = 10400
4	310	36	1200	0	1200 x 36 = 43200
	890	36			
5	310	4	1250	10	1250 x 1 = 1250
6	400	39	950	0	950 x 39 = 37050
	550	39			
7	400	8	800	0	80 x 4 = 3200
8	460	21	1100	0	1100 x 21 = 23100
	640	21			
9	640	2	1300	20	1300 x 1 = 1300
Total trim loss / stock length				510	155900
Total trim loss %					0.33%

Table: 6.4 Cutting pattern according to Power

It is inferred from the above comparative study that the trim loss obtained from our proposed method is less than the trim loss computed by [8], please refer [Table (6.3), (6.4)] for this particular set of data.

CONCLUSION

The study infers that to control the waste, the concept of sustainable trim loss plays a relevant role. In this chapter, the method proposes the use of variable pre-defined sustainable trim loss, which can be made to vary according to the requirement of the stock bar, in order to control the overall trim loss. The use of VPDSTL has given a satisfactory result.

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