

A STUDY OF CONSTRUCTION OF EFFICIENT, NEARLY ORTHOGONAL EXPERIMENTAL MIXED DESIGNS

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ABSTRACT

Conjoint analysis has been a widely used method for measuring customer preferences since 1970s. This method is based on the idea that customers' decisions depend on all tangible and intangible product features. One of the fundamental steps in performing conjoint analysis is the construction of experimental designs. These designs are expected to be orthogonal and balanced in an ideal case. In practice, it is hard to construct optimal designs and thus constructing of near optimal and efficient designs is carried out. There are several ways to quantify the relative efficiency of experimental designs. The choice of measure will determine which types of experimental designs are favoured as well as the algorithms for choosing efficient designs. In this paper, we propose a simultaneous algorithm which combines two optimality criteria: standard criterion named by A-efficiency, and nonstandard criterion, P-value. The algorithm was implemented as the procedure in MCON software for traditional conjoint analysis.

KEYWORDS: Conjoint analysis, efficient experimental design, optimality criteria, approximate algorithm, MCON procedure.

INTRODUCTION

Attractiveness of the preference measuring techniques and its usage in practice has been rapidly increased in the last few years. The practical significance of these techniques derives from their widespread use for new product or concept development and valuation studies in such diverse areas as marketing, transport and financial services, etc. One method that has become particularly popular in this context is conjoint analysis. Conjoint analysis is a decomposition method which assumes that products/services can "break-down" into their attributive components and which implies the study of joint effects of products' variety attributes on their preference. In conjoint analysis, respondents have to evaluate a set of alternatives that are represented by factorial combinations of the levels of certain attributes. In traditional conjoint approach, the alternatives have to be rank ordered or rated on some graded scale. It is assumed that these preference judgments are based on the overall utility values of the considered profile's levels. These unknown parameters are then estimated from the data. If the data consists of

ranking techniques from linear programming, non-metric versions of ANOVA can be used. Metric conjoint analysis comprises variants of conjoint analyses that use rating scales. Here, the utility values are usually estimated by the least squares procedures. Because of the metric response format and the linear relationship between preference judgments and attributes, it is especially this type of conjoint analysis that can be readily applied to the optimal design theory techniques. The quality of statistical analysis heavily depends on the alternatives presented in the experimental design. An experimental design is a plan for running an experiment. Experiments are performed to study the effects of the factor levels on the dependent variable. The factors of an experimental design are variables that have two or more fixed values or levels of the factors. In conjoint analysis, the factors are the attributes of the hypothetical products or services, and the response is either preference or choice.

Using all combinations of attribute levels, that is, a full factorial design, the number of evaluations required from every respondent soon becomes prohibitively large along with the number of attributes and/or levels increased. To deal with this problem, the application of formal experimental designs was suggested. Green and Rao (1971) and Green and Wind (1973) proposed the use of orthogonal arrays, incomplete block designs and fractional factorial designs of different resolutions to reduce the number of evaluations to be performed. In this reduction process, the goodness of the reduced designs is especially important. This goodness is named efficiency. There are several ways to quantify the relative efficiency of experimental designs. The choice of measure will determine which types of experimental designs are favoured as well as the techniques for choosing efficient designs. Two basic techniques for constructing efficient designs are manual, typically used in surveys with small number of attributes and levels, and computerized search which is based on approximate algorithms. A number of recent papers have searched for, or derived, efficient experimental conjoint designs using approximate algorithms. The most commonly used design criterion in literature has been D-optimality, or the maximization of the determinant of the Fisher information matrix. However, some authors propose use of some other standard as well as non-standard criteria.

CONJOINT EXPERIMENTAL DESIGN

The designs of experiment are fundamental components of conjoint analysis. Experimental designs are used to construct the hypothetical products or services. A simple experimental design is the full-factorial design, which consists of all possible combinations of the factors levels. These combinations in conjoint analysis are referred as profiles or concepts. For example, with six factors, three at two levels and three at three levels (denoted as 2³ 3³), there are 216 possible combinations. In a full-factorial design, all main effects, two-way interactions, and higher order interactions are estimable and uncorrelated. The problem with a full-factorial design is that it is, for more practical situations, too cost-prohibitive and tedious to have subjects rate in all possible combinations. For this reason, researchers often use fractional-factorial designs, which have fewer runs than full-factorial designs. The basic difficulty is how to construct such fractional-

factorial design which can provide worth data. In order to obtain valuable and reliable data, two basic principles must be taken into account: orthogonality and balance. A design is orthogonal if all effects can be estimated independently from all other effects, and it is balanced when each level occurs equally often within each factor, which means the intercept is orthogonal to each effect. In ideal case, experimental design is orthogonal and balanced, hence optimal. This is the case in full-factorial designs. However, orthogonal designs are available for only a relatively small number of very specific problems.

When an orthogonal design is not available, non-orthogonal designs must be used. The measure of experimental design's quality is referred as "efficiency". In efficient experimental designs, both variance and covariance of estimated parameters are minimal. Some orthogonal designs are not always more efficient than the other orthogonal or non-orthogonal designs.

$$\begin{bmatrix} k-1 & 0 & 0 & \dots & 0 & 0 \\ -1 & k-2 & 0 & \dots & 0 & 0 \\ -1 & -1 & k-3 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ -1 & -1 & -1 & \dots & 2 & 0 \\ -1 & -1 & -1 & \dots & -1 & 1 \\ -1 & -1 & -1 & \dots & -1 & -1 \end{bmatrix}$$

Figure 1. Helmert's contrast matrix for an attribute with k levels

Coding

Before the design is used, it must be coded. One standard coding is the binary or dummy variable or (1,0) coding. Another standard coding is effects or deviations from means or (1,0,-1) coding. However, for evaluating design efficiency, an orthogonal coding is the most appropriate. This is because no standard orthogonal coding such as effects or binary is generally correlated, even for orthogonal designs. One of the standard ways to orthogonally coding data is Chakravarty's coding. The other method is called the Helmert's procedure. Helmert's procedure consists of arranging a set of data into a matrix which fulfils the Helmert's characteristics, meaning that the sum of each column is equal to zero. Helmert's contrast matrix is a matrix with $k-1$ number of columns and k number of rows (Figure 1). The diagonal of this matrix from (1,1) to ($k-1$, $k-1$) is filled with a decreasing series of numbers going from $k-1$ down to 1. The supra-diagonal elements are set to zero while the infra-diagonal elements are set to -1 . A matrix following these characteristic automatically has a mean of zero for each of the column.

Optimality criteria

An optimality criterion is a single number that summarizes how good a design is, and it is maximized or minimized by an optimal design. In order to generate an efficient design, specific methodology was developed. Efficient designs can be efficient for one criterion and less efficient for another one. There are some standard criteria for measuring efficiency of experimental design in conjoint analysis (Kuhfeld et al., 1994). Two general types are: Information-based criteria and

distance-based criteria. Consider the linear model where consumers provide utility scores, y_j , for each profile:

$$y_j = \alpha + \beta_1 x_{1j} + \beta_2 x_{2j} + \dots + \beta_m x_{mj} + \varepsilon_j \quad (1)$$

for $j = \{1, \dots, n\}$, where x_{ij} are independent variables. In matrix notation, Equation 1 can be written as $y = \alpha + \beta X + e$, where X is the orthogonally coded design matrix of independent variables. For the distance-based criteria, the candidates are viewed as comprising a point cloud in p -dimensional Euclidean space, where p is the number of parameters in the model. The goal is to choose a subset of this cloud that "covers" the whole cloud as uniformly as possible or that is as broadly "spread" as possible. Two optimality criteria are introduced in this type, U -optimality and S -optimality criteria.

The information-based criteria such as D - and A -optimality are both related to the information matrix $X'X$ for the design. This matrix is important because it is proportional to the inverse of the variance-covariance matrix for the least-squares estimates of the linear parameters of the model. Roughly, a good design should "minimize" the variance $1/(X'X)$, which is the same as "maximizing" the information $X'X$. D - and A -efficiency are different ways of saying how large $(X'X)$ or $1/(X'X)$ are. D -optimality is based on the determinant of the information matrix for the design, which is the same as the reciprocal of the determinant of the variance-covariance matrix for the least-squares estimates of the linear parameters of the model.

$$(X'X) = 1/|(X'X)^{-1}| \quad (2)$$

The determinant is thus a general measure of the size of $1/(X'X)$. D -optimality is the most common criterion for computer-generated optimal designs. A -optimality is based on the sum of the variances of the estimated parameters for the model, which is the same as the sum of the diagonal elements, or trace, of $1/(X'X)$. Like the determinant, the A -optimality criterion is a general measure of the size of $1/(X'X)$. A -optimality is less commonly used than D -optimality as a criterion for computer optimal design. This is partly because it is more computationally difficult to update. Also, A -optimality is not invariant to non-singular recoding of the design matrix; different designs will be optimal with different coding.

For appropriate coded matrix X , measures of efficiency can be scaled to be in interval 0 to 100. For Helmert's coded data (matrix), it is more appropriate to use A -optimality criterion:

$$A-eff = 100 \times \frac{1}{N_D \cdot \text{tr}(X'X)^{-1} / p} \quad (3)$$

When data are coded by Chakravarty's procedure, it is more appropriate to use D optimality criterion:

$$D-eff = 100 \times \frac{1}{N_D |(X'X)^{-1}|^{1/p}} \quad (4)$$

In Equations 3 and 4, p is number of parameters in model. The total number of parameters to be estimated is given by the formula:

COMPUTATIONAL EXPERIMENTS

The power of MCON procedure for constructing experimental designs was tested on numerous examples. Using A-optimality and attribute levels' balance criteria, the efficiency of constructed designs were compared with efficiency of the designs constructed by SPSS® Conjoint procedure.

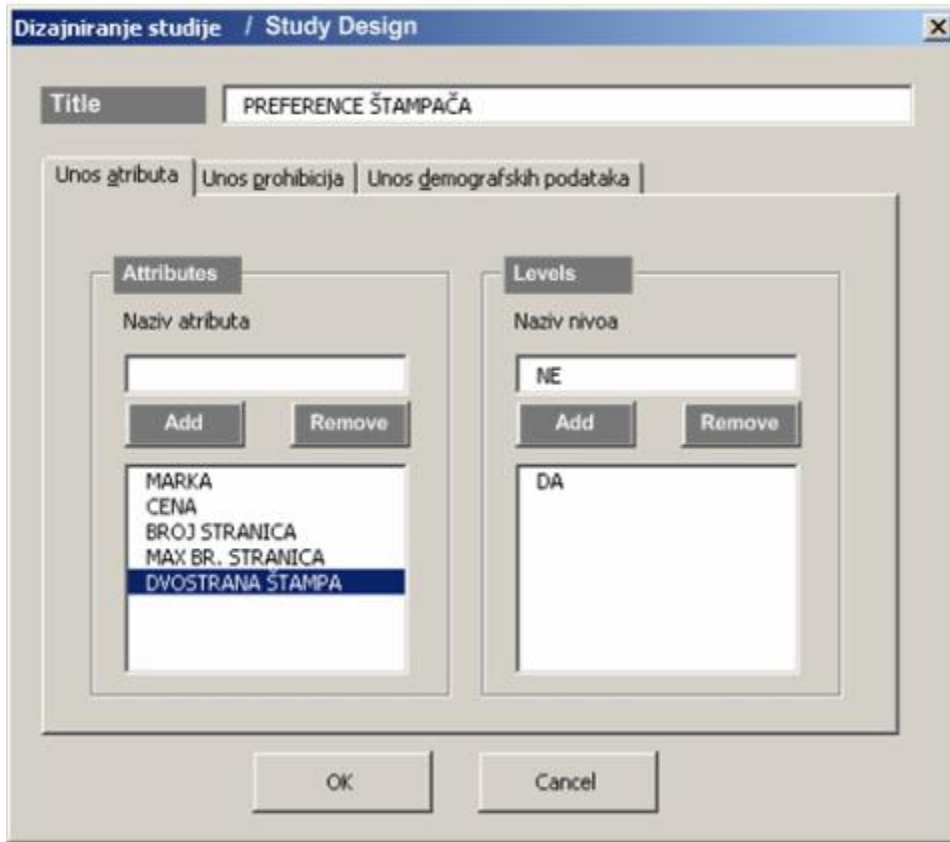


Figure 2. Attributes entry form

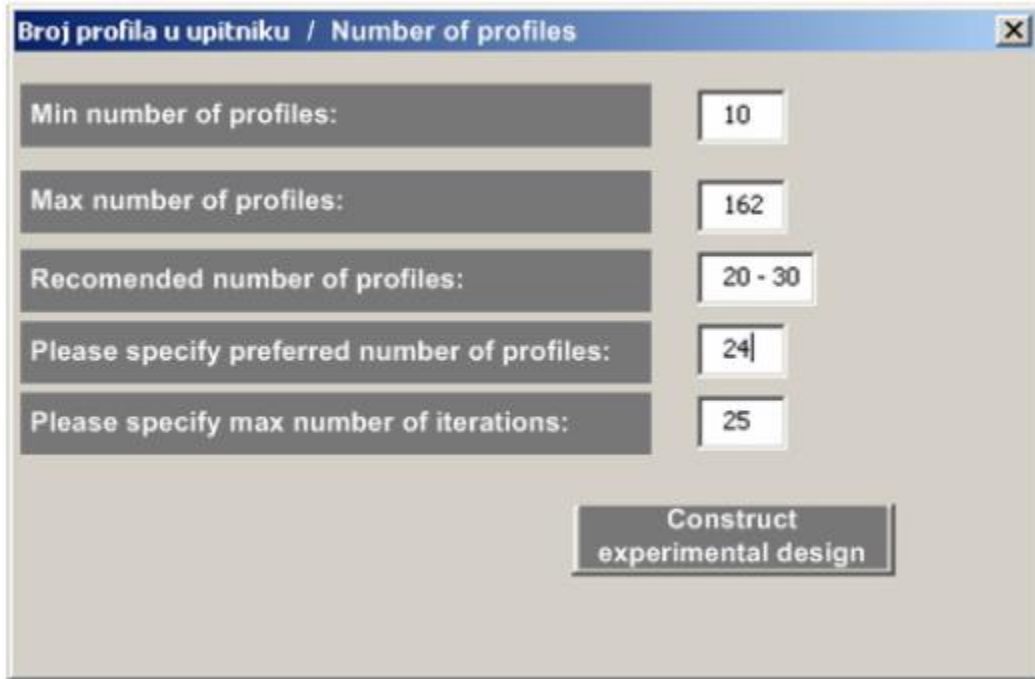


Figure 3. Dialog box for specifying preferred number of profiles to construct.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	A1	A2	A3	A4	A5															
2	-1	1	2	0	-1	-1	2	0	1		67			Anova: Single Factor						
3	-1	1	-1	-1	2	0	2	0	-1		92									
4	-1	1	-1	1	2	0	-1	-1	1		77			SUMMARY						
5	-1	-1	-1	1	-1	-1	-1	1	-1		142			Groups	Count	Sum	Average	Variance		
6	-1	-1	-1	-1	2	0	2	0	-1		146			Column 1	24	0	0	2.086956522		
7	-1	1	2	0	2	0	-1	1	1		57			Column 2	24	0	0	0.695652174		
8	-1	-1	2	0	-1	-1	-1	1	-1		124			Column 3	24	0	0	2.086956522		
9	2	0	-1	-1	-1	-1	-1	-1	-1		54			Column 4	24	0	0	0.695652174		
10	2	0	2	0	2	0	-1	1	-1		4			Column 5	24	0	0	2.086956522		
11	-1	-1	2	0	-1	1	-1	-1	1		119			Column 6	24	0	0	0.695652174		
12	-1	-1	-1	-1	2	0	-1	-1	1		149			Column 7	24	0	0	2.086956522		
13	-1	1	-1	-1	-1	-1	-1	1	-1		106			Column 8	24	0	0	0.695652174		
14	2	0	-1	1	-1	1	-1	-1	1		29			Column 9	24	0	0	1.043479261		
15	-1	1	-1	-1	-1	1	-1	1	1		99									
16	2	0	-1	1	2	0	-1	1	1		21									
17	-1	1	2	0	-1	1	-1	-1	-1		86			ANOVA						
18	2	0	2	0	-1	1	2	0	-1		8			Source of Variation	SS	df	MS	F	P-value	F.crit
19	-1	1	2	0	-1	1	2	0	-1		62			Between Groups	0	0	0	0	1	1.983337938
20	2	0	-1	1	2	0	2	0	1		19			Within Groups	280	207	1.352657005			
21	2	0	-1	1	-1	-1	2	0	1		31									
22	2	0	-1	1	-1	-1	-1	-1	-1		36			Total	280	215				
23	-1	-1	-1	-1	-1	-1	-1	-1	1		161									
24	-1	-1	-1	-1	-1	1	-1	1	1		153									
25	-1	-1	-1	1	-1	1	2	0	-1		134			Aefikasnost:	87.49395					
26																				
27																				

Figure 4. Efficient experimental design constructed by MCON procedure and values of the designs' optimality criteria

CONCLUSION

Conjoint analysis is a technique for measuring consumer preferences for products or services and intentions to buy them. It is also a method for simulating consumers' possible reactions to changes in current products or newly introduced products into an existing competitive market. One of the fundamental problems in performing conjoint analysis is how to construct experimental designs. The purpose of an experimental design is to give a rough overall idea as to the shape of the experimental response surface, while only requiring a relatively small number of runs. These designs are expected to be orthogonal and balanced in an ideal case. In practice, though, it is hard to construct optimal designs and thus constructing of near optimal and efficient designs is carried out. Efficient designs are typically nonorthogonal; however, they are efficient in the sense that the variances and covariances of the parameter estimates are minimized. There are several ways to quantify the relative efficiency of experimental designs. The choice of measure will determine which types of experimental designs are favoured as well as the algorithms for choosing efficient designs.

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