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Abstract— Primary care, the backbone of the nation's health- care system, is at the risk of collapse. Patients are dissatisfied due to poor access to care, and physicians are unhappy and burning out with an enormous amount of tasks. To improve the primary care access, many healthcare organizations have introduced electronic visits (or e-visits) to provide patient– physician communications through securing messages. In this paper, we introduce an analytical model to study e-visits in primary care clinics. Analytical formulas to evaluate the mean and variance of the patient length of visit in primary care clinics with e-visits are derived. System properties are investigated. In addition, comparisons of different scheduling policies between the office and the e-visits are carried out. The first come first serve, preemptive-resume, and non-preemptive policies are studied and the results show that the first come first serve policy typically leads to the best performance.

***Note to Practitioners*—**The primary care delivery system is under a lot of strain. Due to population growth and aging, and the expanded healthcare insurance coverage, the demand for primary care services has increased substantially in the past years. Patients have difficulty of getting timely access to care, while primary care physicians are facing insurmountable tasks. Electronic visit, or e-visit, as an alternative to the traditional office visit, provides an innovative way of patient–physician communi- cation through securing messages. The successful implementation of e-visit relies on a proper understanding of the impact of e-visit on care access, and an appropriate design and scheduling of workforce and operations. Therefore, the objective of this paper is to develop an analytical model of the primary care delivery with e-visits, using which one can investigate the impact of e-visits on patient accessibility. In particular, the average value and variance of patients' length of visit for their encounters are evaluated. Different policies for physicians to schedule office and e-visit patients are compared. In addition, physicians' nondirect care activities, such as billings and documentations, are also considered in the model.

***Index Terms*— E-visit, length of visit, monotonicity, patient flow, primary care, scheduling policy.**

INTRODUCTION

PRIMARY care, which is the backbone of the nation'shealthcare system, is at a grave risk of collapse and facing a confluence of factors that could spell disaster [1], [2]. The patients are dissatisfied and have difficulty of getting timely access, while the physicians are unhappy with their jobs by facing insurmountable tasks. More patients need access to primary care but less medical students are choosing to enter the field. Recent studies have shown that 62 million people in the U.S. have no or inadequate access to primary care [3], but only 13% of the final-year medical students are planning on primary care careers [4]. The implementation of the AffordableCare Act will likely exacerbate the overcrowding in primary care clinics and the shortage of physicians [5]. Therefore,improving the accessibility of primary care is of significant importance.

The rapid development of information technology has made the delivery of healthcare over a distance possible, which introduces substantial opportunities. Many healthcare organizations have introduced online electronic visit programs,referred to as e-visit (or e-portal, e-service, and so on), to provide the patient–physician communication through securing messages [5]. Recent studies demonstrate that byintroducing e-visits, significant savings can be obtained with improved access to care, and increased provider efficiency and patient satisfaction [6]–[10].

To better understand and implement e-visits, a mathematicalmodel of primary care delivery through both the office and thee-visits is aspired. It can provide the care delivery process a fresh look from an integrated systems' engineering perspec- tive. However, few quantitative models on e-visits are availablein the current literature. How primary care physicians manage their operations in response to the introduction of e-visits is still an open question. Therefore, this paper is devoted todeveloping an analytical tool to investigate e-visit's impact on physician's practice, and identify the conditions that e-visits can improve patient accessibility.

As shown in Fig. 1, the care delivery process is essentiallya service network and patients can get access to care through different venues: Web service, which is usually for patientsto inquire some standard questions about simple diseases through

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an online questionnaire program; e-visits, mainly for the patients with low-acuity complaints and ongoing care of chronic diseases to communicate with physicians; office visits, traditional face-to-face encounters; urgent care, for after hour visits or walk-in for a quick treatment, where scheduling is not required; emergency department, for night and emergent visits. After finishing the online programs, a patient may still seek communication with his/her primary care physician through an e-visit if the online evaluation is not sufficient or satisfactory. In addition, the support staff will review the Web service results and, if needed, forward those complex inquiries to the patient's primary care physician for a follow-up e-visit. Therefore, patients can transfer from Web service to e-visit. Similarly, after e-visit, a patient might still need an office visit according to his/her health status and the complication of the disease. In the case of long queues or extended waiting time for office visits, or during after hours, patients may seek care services through other channels such as urgent care units, and if not available, emergency departments for prompt treatment.

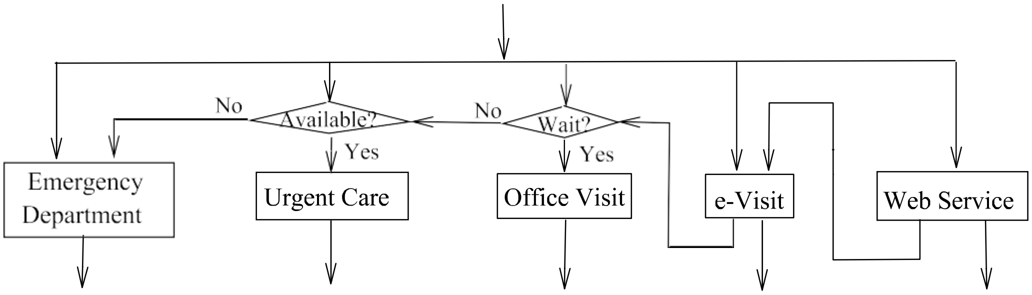

Fig. 1. Patient flow in primary care.

Although electronic communication is desirable, the method to adopt it is still unsettled [11]. In particular, questions such as how is the workflow in primary care clinics affected by the use of e-visits and what is the impact of e-visits on resources to deliver proper care arise naturally. To answer these questions, the key is to evaluate the efficiency of primary care operations with e-visits and to determine the optimal scheduling policy coordinating office and e-visits. Unfortunately, the current literature lacks effective methods to address these issues. Computer simulation, as a prevailing tool to study healthcare delivery, such as primary care delivery, is often case study-based, and typically suffers from long model development and simulation times. To the best of our knowledge, only one analytical study exists, which analyzes primary care operations with e-visits and the focus is on identifying the incentives that drive the implementation of e-visits [12]. In addition, no effective method is available to address the unavailability of primary care providers due to other tasks on top of meeting with patients. Therefore, developing a novel method to model primary care delivery with e-visits, analyze its performance, and design the optimal operating policy is critically aspired, which is the goal of this paper.

The remainder of this paper is structured as follows. Section II reviews the related literature. Section III introduces the assumptions and formulates the problem. Performance analysis formulas are provided in Section IV. System monotonic properties for the mean and variance of the patient length of visit are discussed in Sections V and VI, respectively. Section VII devotes to the investigation of the scheduling policies coordinating office and e-visit services. Finally, conclusions are presented and the avenues for future research are highlighted in Section VIII. All the proofs are sketched in the Appendix.

LITERATURE REVIEW

Redesigning primary care clinics to improve the operational efficiency has been studied for decades. Most of the research addresses issues such as teamwork [13], [14], electronic health record and information systems [15], [16], medical homes [17], [18], payment systems [19], [20], and advanced access [21], [22]. For more details, see [23]–[25].

E-visit, as a novel alternative to the traditional office visit, has aroused growing attention in recent years. Many healthcare organizations, such as Henry Ford Health System, Mayo Clinic, Kaiser Permanente Health Plan, and the University of Pittsburgh Medical Center, have initiated e-visit programs [5], [6]–[10]. Most e-visit studies focus on investigating the effectiveness and patients/providers' experience of implementing e-visits. It is reported that the quality of care and the patient outcomes using e-visits are equivalent to those achieved with office visits [8], [9]. Implementing e-visits can free up extra office appointments for the patients with urgent and complicated issues, reduce urgent care and emergency room visits and inpatient hospital admissions, improve care for the senior population with chronic diseases, and substantially reduce the cost of care [6]–[10]. Additional studies investigate the issues such as billing and reimbursement, information system structures, legal and regulatory issues, financial return, and system implementation, and training [5]. As a quantitative analysis of e-visits, a patient health dynamics model is developed in [12] under the alternative primary care delivery mode, which includes the usage of e-visits and nonphysician providers. This paper quantifies the overall impact of adopting e-visits on physician's choices and expected earnings and patients' expected health outcomes. In a follow-up study based on these results, it is argued that e-visits provide a gateway for transforming traditional primary care delivery [26].

Discrete-event simulation has been used prevailingly to study primary care delivery (see [25], [27]–[31]), and questions pivoting around appointment scheduling, patient arrival, staffing allocation, and equipment maintenance in primary care clinic settings have been explored extensively. Analytical models, on the other hand, have less frequently been used to study primary care operations. Reviews of such models can be found in [30]–[33]. For instance, queuing models are introduced to determine the bed capacity and evaluate the patient cycle times in urgent care and maternity facilities in [34] and [35], respectively. Markov chain models are used to study the workflows in computed tomography test centers, gastroenterology clinics, and in-room care delivery systems [36]–[38]. A recursive procedure to address the limited availability of care providers with an application in a mammography imaging center is presented in [39]. The issues of outpatient appointment scheduling are studied in [40]–[42]. However, all these

papers lack the specificity to address e-visit issues, and when evaluating the providers' productivity, the scenario that care providers may not be available for clinical service due to other duties is overlooked.

In spite of these efforts, no analytical study on patient flow and operations management has been carried out for primary care clinics with e-visits, and this paper intends to contribute to this end.

II. SYSTEM DESCRIPTION AND MODELING

As the focus of this paper is on studying e-visit and its impact on primary care physicians' operations, only Web services, e-visits, and office visits in a primary care delivery system are considered (see Fig. 2). The majority of patients in primary care clinics are associated with their dedicated primary care physician. Therefore, we consider a model with all the services linked with one physician. The following assumptions address the patients, the services, and their interactions.

1) The patients associated with the same primary care physician access care services with the following Poisson arrival rates: λ_{ws} for Web services, λ_{ev} for e-visits, and λ_{ov} for office visits.
2) The primary care physician's service times for e-visits and office visits are described by probability distributions with service rates μ_{ev} and μ_{ov}, coefficients of variation (CVs) cv_{ev} and cv_{ov}, as well as the third moments (or skewness) $E(S^3\ )$ and $E(S^3\ )$, correspondingly.
3) After Web service, a patient has the probability β_{ev} to seek an e-visit for further inquiries. After e-visit, a patient may need to go for an office visit with the probability β_{ov}.
4) The physician also deals with billings and documentations intermediately between patient visits. When no patient is waiting, he/she works on nondirect care related tasks, and the duration of tasks follows a probability distribution with the vacation rate μ_v , the CV μ_v , and the third moment $E(S^3)$. The physician will return to serve patients only after finishing an ongoing activity.
5) The following scheduling policies for coordinating office and e-visits are proposed: 1) non-preemptive, i.e., an ongoing e-visit service will not be interrupted even if an

office visit patient arrives; 2) preemptive-resume, i.e., the current e-visit service can be interrupted if an office visit patient arrives, and the e-visit will resume afterward (in both the policies, office visit has a higher priority); and

3) first come first serve, i.e., the service will be carried out without priority but only based on who comes earlier.

In an appropriately defined state space, the system with assumptions 1)–5) forms a stationary random process. Note that a patient who finishes an e-visit still needs to go through the regular scheduling process for a subsequent office visit. Thus, from a physician's point of view, the combined arrival process can still be modeled as a stationary Poisson process. To quantify the system performance, an extensively used measure is the patient length of visit, which characterizes the duration of an episode of clinic stay [43]. However, a desired mean time performance alone cannot guarantee patient satisfaction—a large variation implies that some patients still wait for an extremely long time and even the mean waiting time is moderate. Moreover, unexpected variations may also impact the clinical outcome and patient safety [43]. Therefore, evaluating the variability of the patient length of visit is also important. Let T_i and Var_i denote the mean and variance of patient length of visit for the type i service, and i ev, ov, representing the e-visit and the office visit. In the framework of 1)–5), T_i and Var_i are the functions of all system parameters

$$T_i = f_{T,i}(\mathrm{L}, \mathrm{M}, \mathrm{B}, \mathrm{CV}), \quad i = \mathrm{ev}, \mathrm{ov} \tag{1}$$

$$\mathrm{Var}_i = f_{\mathrm{Var},i}(\mathrm{L}, \mathrm{M}, \mathrm{B}, \mathrm{CV}, \mathrm{E}), \quad i = \mathrm{ev}, \mathrm{ov} \tag{2}$$

where

$$\mathrm{L} = [\lambda_{ws}, \lambda_{ev}, \lambda_{ov}]\mathrm{M} = [\mu_{ev}, \mu_{ov}, \mu_v]\mathrm{B} = [\beta_{ev}, \beta_{ov}]$$

$$\mathrm{CV} \quad cv_{ev}, cv_{ov}, cv \qquad = [\qquad] \qquad =$$

$$\mathrm{E} = \ E\ S^3\ , E\ S^3\ , E(S^3\ .$$

$$_{ev} \quad _{ov} \quad _{v}$$

Remark 1: In addition to serving office and e-visit patients, physicians work on other tasks not directly encountering patients, such as documentation, paperwork, and dealing with insurance and billings. As summarized in [44], these nondirect care activities have become a significant part of physicians' workload. Assumption 4) implies that the physician works on these activities whenever no patients are waiting. Whena new patient arrives, he/she will go to serve that patient after finishing the current activity.

The problem addressed in this paper is: under assumptions 1)–5), develop a method to evaluate the mean and variance of the patient length of visit, and investigate system properties and the impact of different scheduling policies between the office and the e-visits.

The solutions to this problem are presented in Sections IV–VII.

PERFORMANCE EVALUATION

A. Average Length of Visit

Consider the primary care physician's operations described in Section III. For e-visit patients, the arrival includes the

patients directly seeking e-visits and those coming to e-visits after Web services, which is characterized by the transition probability β_{ev}. Thus, the effective arrival rate for e-visits is

the variability. In fact, ω_i represents the ratio between the second and first moments, multiplied by a factor of 0.5. Inthe case of exponential distributions, $\omega_i = \tau_i$, this variable

Proof: By plugging in $\overline{\delta}_i = 1$, $\omega_i = (1/\mu_i)$, and $E(S_i^3) = (6/\mu_i^3)$, $i =$ ev, ov, v, (19)–(23) can be obtained after severalsteps of algebraic operations. ■

Building upon these system performance evaluation for- mulas, system properties like monotonicity can be studied. Then, questions such as how do system parameters impact performance measures and what are the directions to improve system performance can be answered. In Sections V and VI, the properties of the mean and variance of lengths of visit are discussed, and different scheduling policies are compared.

III. PROPERTY OF AVERAGE LENGTH OF VISIT

In this section, we investigate the impact of routing probabilities on e-visit and office visit patients' average lengths of visit. Since β_{ev} and β_{ov} are the probabilities that patients continue to seek e-visits and office visits after Web services and e-visits, the monotonicity of T_i, $i =$ ev, ov with respect to β_{ev} and β_{ov} could provide insights on how e-visits impact patient access to primary care.

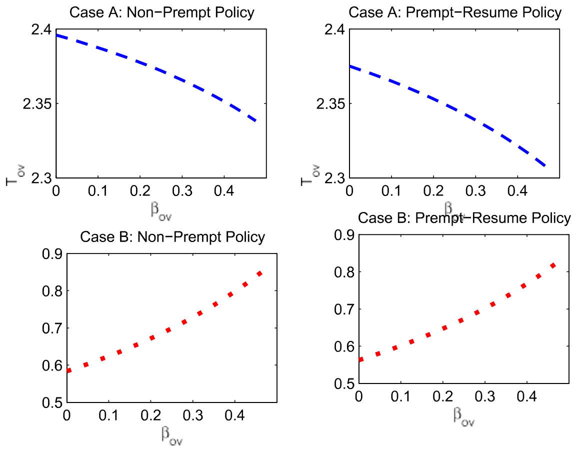

Fig. 2. Network model for primary care patient flow.

A. Monotonicity of T_{ov} With Respect to β_{ov}

Proposition 1: Under assumptions 1)–5), T_{ov} is monotoni- cally increasing with respect to β_{ov}, i.e., $(\partial T_{\text{ov}}/\partial \beta_{\text{ov}}) > 0$, if and only if

$$\begin{cases} \omega_{\text{ov}} > (\omega_v \quad \omega_{\text{ev}})\rho_{\text{ev}}, & \text{non-preemptive policy} \\ \omega_{\text{ov}} > \omega_v \rho_{\text{ev}}, & \text{preemptive-resume policy} \\ \text{without condition}, & \text{first come first serve policy.} \end{cases}$$

Intuitively, if the routing probability of seeking office visits after e-visits, β_{ov}, is increasing, the physician's workload with office visit patients is increasing. Under the non-preemptive policy, when $\omega_{\text{ov}} > (\omega_v\ \omega_{\text{ev}})\rho_{\text{ev}}$, the office visit patient's length of visit will increase with respect to β_{ov} and will be nonincreasing vice versa. Such a condition suggests that, roughly, the moment ratio of the office service is larger than that of the difference between vacation and e-visit.

In practice, this type of condition typically holds, sinceboth the e-visit and the vacation have lower priorities thanthe office visit and usually take a shorter time compared with the office visit. The difference will be even smaller consideringthe discount factor $\rho_{ev} < 1$. In particular, when service and vacation times are exponentially distributed, this condition is simplified to $\tau_{ov} > (\tau_{v} \quad \tau_{ev})\rho_{ev}$, which again holds most of the time.

Under the preemptive-resume policy, the condition becomes more strict, where $\omega_{ov} > \omega_{v}\ \rho_{ev}$ (in the exponential case, $\tau_{ov} > \tau_{v}\ \rho_{ev}$) is required. The reason is that under the preemptive- resume policy, the physician will stop working on e-visit patients and immediately serve an incoming office visit patient. Then, the service time and the variability of e-visits will not play a significant role in the waiting time of office visit patients compared with the non-preemptive case, where the physician has to finish any ongoing e-visit service before moving to office visit patients. However, since vacations usually take a shorter time and $\rho_{ev} < 1$, this condition is typically satisfied, so that the monotone increasing property holds.

An illustration of such monotonicity property in exponential scenarios is shown in Fig. 3, in which the parameters are selected as follows:

$$\beta_{ov} \in [0, 0.5), \quad \beta_{ev} = 0.5 \qquad (24)\text{Case A:} \quad \tau_{v} = 30\tau_{ev} \underset{\tau}{=} 10\tau_{ov} \qquad (25)$$

$$\text{Case B:} \quad \tau_{v} = \tau_{ev} = \frac{3}{ov}. \qquad (26)$$

The reason to include the seldom occurring Case A is to show the decreasing monotonicity. As one can see, when office visits take a longer time, which meets the requirement B). However, if vacation (or nondirect care) takes an extremely long time than office and e-visits, T_{ov} could decrease with respect to β_{ov} (Case A). In a sense, waiting for short office visits is better than for long vacations.

When the first come first serve policy is applied, the office visit patient's length of visit is monotonically increasing with respect to β_{ov} without any condition. In this case, both the office and the e-visits are treated with equal priority. Increasing physician's workload [ρ_{ov} and ρ in (11) and (13)] will lead to a longer patient length of visit.

Therefore, in most of the practical cases, if more patients need to seek additional office visits after e-visits, the accessi- bility to office visits can be further impaired. Thus, the method to implement e-visits to limit this routing probability is of importance, and will be part of future work.

B. Monotonicity of T_{ov} With Respect to β_{ev}

Proposition 2: Under assumptions 1)–5), T_{ov} is monotoni- cally increasing with respect to β_{ev}, i.e., $(\partial T_{ov}/\partial \beta_{ev}) > 0$, if and only if

$$\beta_{ov}\mu_{ev}\omega_{ov} > (\mu_{ov} \quad \lambda_{ov})(\omega_{v} \quad \omega_{ev}),$$

non-preemptive policy

$$\beta_{ov}\mu_{ev}\omega_{ov} > (\mu_{ov} \quad \lambda_{ov})\omega_{v},$$

preemptive-resume policy without condition, first come first serve policy. *Proof:* See the Appendix.

Again, the increasing monotonicity exists without any condition under the first come first serve policy. For non- preemptive and preemptive-resume policies, the necessary and sufficient conditions become more complex.

When β_{ev} is increasing, i.e., more patients continue to seek e-visits after Web services, which leads to an increase in the number of patients to further come to the office visit(as $\beta_{ov} > 0$ and is kept constant). Since β_{ev} mainly affects the arrival of e-visits, only when β_{ov} is large enough, the increase of follow-up office visits can exert a significant effect (which explains the conditions with the factor β_{ov} on the left-hand side of the inequalities in Proposition 2, required for both the policies).

For the non-preemptive policy, if the physician spends more time, which also has a higher variability on office and

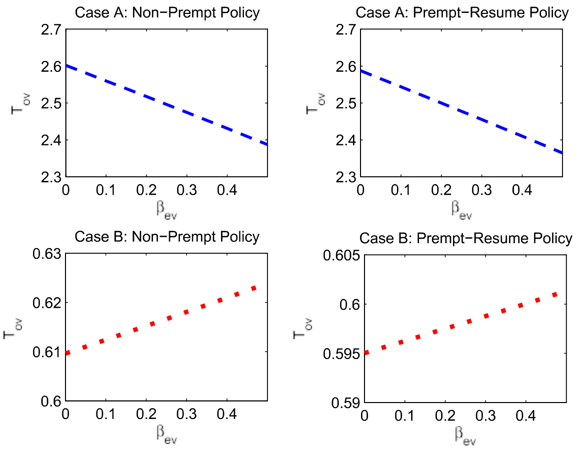

Fig. 4. Monotonicity of T_{ov} with respect to β_{ev}.

e-visits than vacations, a longer length of visit can be observed (which explains the condition regarding the ω_{ov} and ω_v ω_{ev} factors in Proposition 2 for the non-preemptive policy). For the preemptive-resume policy, additional e-visit patients will not significantly impact office visits, since the physician will stop working on any e-visit and immediately work on the coming office visit patient. Thus, the condition in Proposition 2 for the preemptive-resume policy becomes stricter, where the ω_v ω_{ev} term changes to ω_v.

Note that these conditions are necessary and sufficient, which indicates that if these conditions are not met, T_{ov} will be monotone nonincreasing with respect to β_{ev}. Fig. 4 shows such properties in exponential cases. System parameters are selected as in (25) and (26), but (24) is replaced by (27) to represent the scenario that the Web service has a higher referralratio than e-visits

$$\beta_{\text{ev}} \in [0, 0.95), \quad \beta_{\text{ov}} = 0.1. \qquad (27)$$

As exhibited in Fig. 4, when the vacation time is much longer, waiting for more office visits could be even beneficial, so that the decreasing monotonicity can be observed.

C. Monotonicity of T_{ev}

Unlike T_{ov}, the monotonicity of T_{ev} is consistent for the non-preemptive, preemptive-resume, and first come first servepolicies.

Proposition 3: Under assumptions 1)–5), T_{ev} is monotoni- cally increasing with respect to β_{ev} and β_{ov}, i.e., $(\partial T_{\text{ev}}/\partial \beta_i) > 0$, i ov, ev.

Proof: See the Appendix. ■

Proposition 3 articulates that the length of visit of e-visit patients is always monotonically increasing with respect to β_{ev} and β_{ov}, no matter which policy is implemented. Larger β_{ov} and β_{ev} increase the effective arrivals, resulting in more patients waiting in line. In close, under all the policies, a newly arrived e-visit patient needs to wait until all the types of patients in line are finished. Thus, the increase of average length of visit can be foreseen.

are with or without priority. For the first come first policy, patient types are not differentiated, and increasing either β_{ov} or β_{ev} increases the total patient arrival, so does the server intensity ρ_{ov}, ρ_{ev}, and ρ. In addition, the effect of vacation on patient length of visit is independent of server intensity, which is elucidated in (10) and (11) (where the terms related to ω_v or τ_v are independent of ρ_{ev}, ρ_{ov}, and ρ). Therefore, it is straightforward that the increasing monotonicity holds for the lengths of visit of both the office and e-visit patients unconditionally.

For the policies with priorities, the results differ for the office and e-visit patients. As the e-visit patients have a lower priority, their waiting incorporates the waiting for all the patients in line and the waiting for the physician to return from a vacation. Larger β_{ev} or β_{ov} increases the overall patient arrival, and thus the overall number of patients waiting in line. Therefore, the monotonicity of their length of visit holds naturally without conditions.

On the other hand, office visit patients are mainly waiting for other office visit patients in line and the physician returning from a vacation. There exists a tradeoff between waiting for more office and e-visits due to the increase of β_{ov} or β_{ev} and waiting for potentially fewer vacations. Therefore, conditions are required to ensure the monotone increasing of the length of visit for office visits. In extreme cases, if vacations are very long or suffer large variations (ω_v ω_{ev} or ω_v ω_{ov}), then having more office and e-visit arrivals could be beneficial (i.e., T_{ov} is monotonically decreasing with respect to β_{ov} and β_{ev}). Moreover, for T_{ov} to be monotonically increasing with β_{ev}, as β_{ev} mainly affects e-visits and its impact on office visit is through β_{ov}, additional conditions on β_{ov} are required.

The conditions for the preemptive-resume policy are always stricter than that for the non-preemptive policy. In the former case, physicians will stop the ongoing e-visit, and thus, only significant changes in e-visits will impose effects on office visits, while in the latter case, physicians will finish the current e-visit service, and any change in e-visits may immediately impact office visits.

In summary, in practical cases, office visits have a higher demand and take a longer time, and then both T_{ov} and T_{ev} are monotonically increasing with respect to β_{ov} and β_{ev}.

IV. PROPERTY OF VARIANCE OF LENGTH OF VISIT

A. Monotonicity of Var_{ov}

First, we investigate the monotonicity of variance of length of visit Var_{ov} with respect to β_{ov}. The increasing monotonicity holds under a sufficient but not necessary condition.

$$\omega_{\text{ov}} \geq \rho_{\text{ev}}\omega_v \text{ and } \mu_{\text{ov}} E\ S^3 \geq \rho_{\text{ev}}\mu_v E\ S^3,$$

preemptive-resume policy
without condition, first come first serve policy.

Proof: See the Appendix. ■

The sufficient conditions for the variance of length of visit are much more complex compared with that of the average length of visit, since the third moments are involved. These conditions indicate that when the office visit has a longer service time and a larger variance, and the vacation (i.e., nondirect care activity) has a smaller moment ratio, then more patients seeking office visits after e-visits will lead to a larger variability in the patient flow. Similar to the T_{ov} case, the sufficient conditions under the preemptive-resume policy are stricter than those under the non-preemptive policy. Under the first come first serve policy, fortunately, the monotonicity is straightforward that the variance of length of visit for office visit patients is always increasing when more patients shift to office visits.

It can be noticed that the characteristic of vacation plays an important role in determining the monotonicity of the system performance indices—as long as all the three distribution

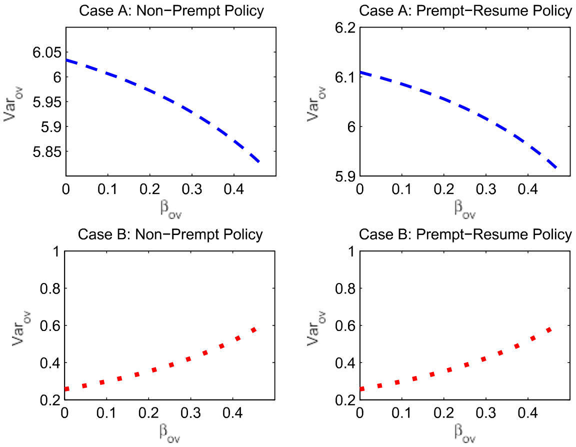

Fig. 5. Monotonicity of $\mathrm{Var}_{\mathrm{ov}}$ with respect to β_{ov}.

if

$$\beta_{\mathrm{ov}}\mu_{\mathrm{ev}}\omega_{\mathrm{ov}} > (\mu_{\mathrm{ov}} - \lambda_{\mathrm{ov}})|\omega_{\mathrm{ev}} - \omega_{v}| \quad S \quad \geq$$

$$\mathrm{ev} \quad S \; ,$$

moments of vacation are small enough, the monotonicity holds. One other observation is that the length of visit variation is affected by multiple factors comprising the first,

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