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Production policy for damageable item with dynamic demand and deterioration under inflation and time value of money in an imperfect production process

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Abstract

In this paper, economic production quantity (EPQ) model for deteriorating item is developed with dynamic demand and deterioration i.e. demand and deterioration of the item increase with time. Here rate of production and holding cost are time dependent, unit production cost is a function of both production reliability indicator and production rate. Set-up cost is also partially production rate dependent. The production process produces some imperfect quantities which are instantly reworked at a cost to bring back those units to the perfect ones. The production process ultimately depends on both time and reliability indicator. The model is formulated as optimal control problem and the total profit function with effect of inflation and time-value of money is expressed as finite integrals over the finite planning horizon. The problem is solved using Euler–Lagrange function based on variational calculus and Newton–Raphson method to determine the optimal production reliability indicator η and then corresponding production rate and total profit. Numerical experiment is performed to illustrate the model both numerically and graphically.

Keywords: Inventory; Damageable item; Reliability; Dynamic Deterioration.

1. Introduction:

Items available in the market can be broadly classified into two categories - damageable items and non-damageable items. Damageable items again be classified into two sub-categories - deteriorating and breakable items. Deterioration of units is one of the most crucial factors in inventory problems for deteriorating items. In the literature, there are several articles on the inventory of deteriorating items (Chang 2004a, 2004b, Goyal and Giri 2001, Guchhait et al. 2013, He et al. 2010, Hsu and Lee 2009, Hung 2011, Mahata 2012). To study the development of the models of deteriorating item before the year 2001, one can follow the review article presented by Goyal and Giri (2001). Last few years, models of deteriorating items are extensively studied by the inventory

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researchers. Ouyang et al. (2006, 2009) developed two inventory models for deteriorating items with permissible delay in payment. Some notable research papers of deteriorating items incorporating various types of assumptions are due to He et al. (2010), Hung (2011), Yadav et al. (2011) etc. Most of the above inventory models are developed with constant deterioration. But deterioration also increases with time as stress of units on others causes damage. Also due to nature of these items, rate of deterioration increases with time. According to the author's best knowledge, very few articles have been published incorporating time varying deterioration (Sarkar et al. 2011, Guchhait et al. 2013). Recently, Dye (2012) presented a finite time horizon deteriorating inventory model and solved it using PSO. An overall progress of inventory research for deteriorating item has been presented in the review article of Bakker et al. (2012).

Deteriorating items are deteriorated with time and as a result holding costs of these types of items increase with time to check the increasing deterioration. Normally, seasonal goods like fruits, vegetables, X-mas cake, etc. are deteriorating in nature. Demands of these items normally exist in the market for a finite time and obviously these types of demands are dependent on time. Various types of investigation have already been made by several authors on EOQ and EPQ/EMQ models for time-dependent demand (Chen 1998, Datta and Pal 1991, Dave and Patel 1981, Lee and Hsu 2009, Sarkar et al. 2011). Most of the above models have been formulated with infinite time horizon assuming that demand of the items exists over infinite time. According to this assumption product specification remains unchanged for ever. But in reality, it is observed that unprecedented development of technology leads to rapid change in product specifications with new features. As a result, lifetimes of these types of products are normally finite and demands depend on time obviously.

Now-a-days, in the metropolitan cities, holding-cost increases with time. It increases due to increases in bank interest, etc. Moreover, set-up cost depends partially with the production rate. Only few articles of inventory control problems have been published with variable holding cost (Alfares 2007, Urban 2008) and set-up cost (Darwish 2008, Matsuyama 1995). Till now, none has considered these assumptions in formulating the inventory models for deteriorating items.

At present, it is not possible to ignore the effect of inflation as the economy of any country changes rigorously due to high inflation. Assuming this effects on inventory costs, first impetus was given by Buzacott (1975). Among others, Beirman and Thomas (1977), Datta and Pal (1991), Ray and Chaudhuri (1997) studied some EOQ models with linear time-varying demand taking inflation and time value of money into account. Wee and Law (2004) addressed an inventory problem with finite replacement rate of deteriorating items incorporating the effect of inflation and time value of money. In the same year, Chang (2004a) proposed an inventory model for deteriorating items with trade credit under inflation. In recent years, Sana (2010) and sarkar et al. (2011) presented two inventory models in this direction.

In most of the earlier production models, inventory practitioners considered that all produced units are of perfect quality. However, in reality, all manufactured units are not always of perfect quality and directly affected by the reliability of the production process. Issue of process reliability, quality improvement, and set-up time reduction have been discussed by Porteous (1986). After that, Cheng (1989) and Chung and Hou (2003) presented their models with imperfect production processes. Among others, Sana et al. (2007), Yoo et al. (2009), Sarkar et al. (2010), and Liao and Sheu (2011) developed different types of inventory models incorporating imperfect production process. But, none has considered inventory model with variable production rate, dynamic demand and reliability under the effect of inflation and time-value of money, specially for deteriorating item.

So in brief, the existing literature on damageable items, suffer from at least one of the following limitations.

- Most of the production-inventory models are developed for infinite time horizon. According to this assumption, demand of an item remains unaltered forever. In reality, rapid development of technology leads to change in product specifications with new features which in turn, motivate the customers to go for new product. Hence the inventory models should be developed and analyzed for a finite period of time.
- Normally, no manufacturing process is free from imperfect production. Moreover, with time, malfunctioning of machineries increases. Though the workers are experienced with longer production experience, ultimately their fatigue overcomes their expertiseness. Also, supervision over the production process is slowly loosened as the time passes. Thus, as a result, in a manufacturing system, production of defective items increases with time due to several factors as mentioned above. But this phenomenon has been overlooked by the researchers.
- A little attention has been paid on inventory control problem of deteriorating items considering dynamic deterioration though it is real life phenomena (Sarkar et al. 2011, Guchhait et al. 2013).
- Almost all inventory models are formulated with constant holding and set-up costs. In the changing marketing scenarios, these costs depend on some parameters. Holding cost varies with time, set-up cost depends on production rate.
- In production inventory models, reliability of the process is ignored by most of the inventory investigators.
- Variational principle is a simple technique for the analysis of optimal control problems. Very few researchers have developed the production-inventory models as optimal control problems and solved using this method.
- Moreover, in most of the above models, unit production cost is assumed to be constant. In practice, it varies with production rate, raw material cost, labour charge, wear and tear cost and reliability of the production process.

Till now, no research work is reported taking above features into account in a single model. In this paper, an attempt has been made to remove the above limitations and we have considered an production inventory model with imperfect production process for a deteriorating item over a finite time horizon. The production rate varies with time. Here model has been formulated with dynamic demand and deterioration. Set-up cost is partially production rate dependent and holding cost is also partially time dependent. The unit production cost is a function of production rate and product reliability parameter. The model is formulated as optimal control problem for the maximization of total profit from the planning horizon and optimum profit along with optimum reliability parameter (η) is obtained using Euler-Lagrange equation based on variational principle and Newton-Raphson method. Numerical experiment is performed to illustrate the model. The significance of some parameters on the proposed model are also pointed out. Results are presented in both tabular and graphical forms.

Rest of the paper is organized as follows: Section 2 provides fundamental assumptions and notation. Section 3 describes the formulation of the model. Solution procedure is presented in section 4. Numerical example to illustrate the model is in Section 5. Conclusion of the model is given in Section 6.

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2. Assumptions and notations for the proposed model

The following notations and assumptions are used in developing the model.

i. Inventory system involves only one item.

ii. H is the planning horizon, which is finite.

iii. $q(t)$ is inventory level at any time t with initial and terminal inventory-level zero.

iv. $\dot{q}$ is derivative of $q(t)$ with respect to t.

v. During the time $[0, H]$, the product deteriorates at a rate $\theta(t)$ which depends on time as $(t) = \frac{1}{1+T-t}$, where T is the maximum lifetime of the products at which the total on-hand inventory deteriorates. The physical significance of the deterioration rate is as t tends to T, deterioration rate is 1. At any time $t \in [0, H]$, the deterioration is $\theta(t)q(t)$. The deteriorated units cannot be repaired or replaced.

vi. Demand, $D(t) = a - be^{-ct}$, $where\ a, b, c > 0$, is an increasing function of time t at a decreasing rate.

vii. $U(t)$ is production rate at any time t which depends on the reliability parameter.

viii. η is production reliability parameter which is a decision variable, which is defined as $\frac{Number\ of\ failures}{Total\ units\ of\ operating\ hours}$. The smaller value of η results in higher investment in technology, whereas greater value of η causes smaller investment for the cost of technology.

ix. η_{max} is maximum value of η and $0 < \eta < 1$.

x. η_{min} is minimum value of η and $0 < \eta < 1$.

xi. λ is the variation constant of tool/die costs.

xii. $\phi(\eta)$ is development cost for production system and is of the form (Mettas 2000, Sana 2010): $\phi(\eta) = L_1 + L_2 e^{K(\eta_{max} - \eta)/(\eta - \eta_{min})}$ where L_1 is the fixed cost like labor, energy etc., and is independent of η. $L_2$2 is the cost of technology, resource and design complexity for production when $\eta = \eta_{max}$. K represents the difficulties in increasing reliability, which depends on the design complexity, technology and resource limitations etc.

xiii. Unit production cost, c_u, is a function of production reliability parameter and production rate $U(t)$ and is of the form $c_u(\eta, t) = c_r + \frac{\varphi(\eta)}{U(t)} + \lambda\, U(t)$, where c_r is the fixed material cost. Second term is the development cost which is equally distributed over the production $U(t)$ at any time t and the third term $\lambda U(t)$ is tool/die cost which is proportional to the production rate.

xiv. The amount of defective items produced at time t is $(\delta - \mu\, e^{-\eta t})U(t)$ where δ is the maximum defective rate. In this production system the production of defective items increase with increase of time. Here, the fraction $\delta - \mu\, e^{-\eta t}$ increases with time t and η simultaneously, because almost all manufacturing system undergoes malfunctioning / unsatisfactory performance after some time. During malfunctioning, in long run process, the system shifts in-control state to out of- control state as a result the percent of defective items

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increase with time t. Besides it, the lower value of η decrease the percent of defective items.$w_i = m_{si} c_{pi}$ is the per unit wholesale price for i-th item.

xv. c_R is the rework cost per defective item.

xvi. $c_h(t)$, the holding cost per unit per unit time and is of the form: $c_h(t) = c_{h0} + c_{h1}t$.

xvii. Set-up cost, S_u partially depends on production rate and is of the form: $S_u(U(t)) = S_{u0} + S_{u1}U(t)$.

xviii. $R = d - i$, where d is the interest per unit currency and i is the inflation per unit currency.

xix. c_s is the unit selling price.

3. Model development and analysis:

In this production process, the system produces perfect quality as well as imperfect quality units. Imperfect quality products are reworked at a cost to restore its quality to the original one. The parameter η is an indicator of product reliability. If the production reliability factor η decreases, the system becomes more reliable i.e., smaller value of η provides better quality product.

The inventory level decreases owning to demand and deterioration. Thus, the change of production and inventory level can be represented by the following differential equation:

$$\frac{dq(t)}{dt} = U(t) - D(t) - \theta(t)q(t),$$

i.e., $U(t) = \dot{q}(t) + D(t) + \theta(t)q(t)$, (1)

with $q(0) = 0$ and $q(H) = 0$.

The relevant profit function, incorporating inflation and time value of money during $[0, H]$ is (using Eq. 1)

$$J_H = \int_0^H e^{-Rt}\,[c_s D(t) - (c_u(\eta, t) - c_R(\delta - \mu e^{-\eta t}))U(t) - c_h(t)q(t) - S_u\big(U(t)\big)/H]dt$$

$$= \int_0^H e^{-Rt}\,[c_s D(t) - \big(c_r + c_R(\delta - \mu e^{-\eta t})\big)\big(\dot{q}(t) + D(t) + \theta(t)q(t)\big) - \varphi(\eta) - \lambda\big(\dot{q}(t) + D(t) + \theta(t)q(t)\big)^2 - (c_{h0} + c_{h1}t)q(t) - \{S_{u0} + S_{u1}(\dot{q}(t) + D(t) + \theta(t)q(t))\}/H]dt$$

$$= \int_0^H Z(q, \dot{q}, t)\, dt, \qquad (2)$$

Where

$$Z(q, \dot{q}, t) = e^{-Rt}[c_s D(t) - \big(c_r + c_R(\delta - \mu e^{-\eta t})\big)\big(\dot{q}(t) + D(t) + \theta(t)q(t)\big) - \varphi(\eta) - \lambda\big(\dot{q}(t) + D(t) + \theta(t)q(t)\big)^2 - (c_{h0} + c_{h1}t)q(t) - \{S_{u0} + S_{u1}(\dot{q}(t) + D(t) + \theta(t)q(t))\}/H]$$

(3)

Now, the problem is to find the optimal path of $q(t)$ and $U(t)$ for which J_H is maximized.

Lemma-1.: J_H has a maximum for a path $q = q(t)$ in the interval $[0, H]$.

Proof. J_H depends on the path $q = q(t)$ in between $t = 0$ and $t = H$. Let us take the path p_0 is given by $q = q_0(t)$for which J_H has a maximum value. Also suppose that the class of neighbouring curves p_ρ given by $q = q_\rho(t) = q_0(t) + \rho\xi(t)$, where ρ is a small constant and $\xi(t)\,(> 0, all\ values\ of\ t)$ is an any differentiable function of t. Therefore, the value of J_H for the path p_ρ is given by $J_H(\rho) = \int_0^H Z_\rho\, dt$, where $Z_\rho = Z(q_0(t) + \rho\,\xi(t), \dot{q}_0(t) + \rho\,\dot{\xi}(t), t)$.

For maximum value of $J_H(\rho)$, we have $\frac{d}{d\rho}(J_H(\rho))\Big|_{\rho=0} = 0$ and $\frac{d^2}{d\rho^2}(J_H(\rho))\Big|_{\rho=0} < 0$. Here,

$$\frac{d}{d\rho}(J_H(\rho)) = \int_0^H \left\{\xi(t)\frac{\partial Z_\rho}{\partial q} + \dot{\xi}(t)\frac{\partial Z_\rho}{\partial \dot{q}}\right\} dt$$

$$= \int_0^H \xi(t)\frac{\partial Z_\rho}{\partial q}dt + [\xi(t)\frac{\partial Z_\rho}{\partial \dot{q}}]_0^H - \int_0^H \xi(t)\frac{d}{dt}\left(\frac{\partial Z_\rho}{\partial \dot{q}}\right)dt = [\xi(t)\frac{\partial Z_\rho}{\partial \dot{q}}]_0^H + \int_0^H \xi(t)\left\{\frac{\partial Z_\rho}{\partial q} - \frac{d}{dt}\left(\frac{\partial Z_\rho}{\partial \dot{q}}\right)\right\}dt$$

$$= \int_0^H \xi(t)\left\{\frac{\partial Z_\rho}{\partial q} - \frac{d}{dt}\left(\frac{\partial Z_\rho}{\partial \dot{q}}\right)\right\}dt \qquad (4)$$

As $q(t)$ is fixed at the end points $t = 0$ and $t = H$, so, $\xi(0) = \xi(H) = 0$, therefore, $\frac{d}{d\rho}(J_H(\rho))\Big|_{\rho=0} = 0$ gives $\frac{\partial Z_\rho}{\partial q} - \frac{d}{dt}\left(\frac{\partial Z_\rho}{\partial \dot{q}}\right) = 0$, which is the necessary condition for extreme

value of J_H. Again, $\frac{d^2}{d\rho^2}(J_H(\rho)) = \int_0^H\left\{\xi^2\frac{\partial^2 Z_\rho}{\partial q^2} + 2\xi\dot{\xi}\frac{\partial^2 Z_\rho}{\partial q\partial \dot{q}} + \dot{\xi}^2\frac{\partial^2 Z_\rho}{\partial \dot{q}^2}\right\}dt,$

i.e., $\frac{d}{d\rho}(J_H(\rho))\Big|_{\rho=0} = \int_0^H -2\lambda e^{-Rt}\{\xi^2\theta^2 + 2\xi\dot{\xi}\theta + \dot{\xi}^2\}dt < 0$, (as $2\lambda e^{-Rt} > 0$) (5)

Hence the sufficient condition, $\frac{d^2}{d\rho^2}(J_H(\rho))\Big|_{\rho=0} < 0$ shows that J_H has a maximum in $[0, H]$.

Now, the Euler-Lagrange's equation for the maximum value of J_H is $\frac{\partial Z}{\partial q} - \frac{d}{dt}\left(\frac{\partial Z}{\partial \dot{q}}\right) = 0$ (6) Using (2) and (3), we have $\ddot{q} - R\dot{q} - \frac{R}{1+T-t}q = f(t)$, (7)

$$f(t) = \left(R + \frac{1}{1+T-t}\right)\left\{D + \frac{1}{2\lambda}\left(c_r + c_R(\delta - \mu e^{-\eta t}) + \frac{S_{u1}}{H}\right)\right\} - \dot{D} + \frac{1}{2\lambda}(c_{h0} + c_{h1}t -$$

where $c_R\mu\eta e^{-\eta t})$

Proposed Model: When demand, $D(t) = a - be^{-ct}$, $(a, b, c > 0)$, which increases with time at a decreasing rate, then the solution of the equation (7) is

$$q(t) = [K_1 + K_2t + K_3t^2 + K_4t^3 + K_5t^4 + K_6t^5 + K_7t^6 + K_8log(1 - \frac{t}{1+T})$$

$$+K_9tlog(1 - \frac{t}{1+T})] + e^{Rt}[K_{10} + K_{11}t + K_{12}t^2 + K_{13}t^3 + K_{14}t^4 + K_{15}t^5 + K_{16}t^6 + K_{17}log\left(1 - \frac{t}{1+T}\right) + K_{18}tlog(1 - \frac{t}{1+T})],$$

(8)

$\cong [K_1 + K_{19}t + K_{20}t^2 + K_{21}t^3 + K_{22}t^4 + K_{23}t^5 + K_{24}t^6] + e^{Rt}[K_{10} + K_{25}t + K_{26}t^2 + K_{27}t^3 + K_{28}t^4 + K_{29}t^5 + K_{30}t^6]$
(9)

and the corresponding production rate is

$U(t) = [K_{31} + K_{32}t + K_{33}t^2 + K_{34}t^3 + K_{35}t^4 + K_{36}t^5 + K_{37}t^6] + e^{Rt}[K_{38} + K_{39}t + K_{40}t^2 + K_{41}t^3 + K_{42}t^4 + K_{43}t^5 + K_{44}t^6] - be^{-ct}$
(10)

Where
$M_1 = a + \frac{1}{2\lambda}\left(c_r + \frac{S_{u1}}{H} + c_R\delta\right), M_2 = RM_1 + \frac{1}{2\lambda}\left(c_{h0} + c_{h1}(1+T)\right),\ M_3 = -b(R+c)e^{-c(1+T)}, M_4 = -\frac{1}{2\lambda}(R+\eta)c_R\mu e^{-\eta(1+T)}, M_5 = \frac{M_3}{R+c}, M_6 = \frac{M_4}{R+\eta}, M_7 = -\frac{c_{h1}}{2\lambda} + cM_3 + \eta M_4 + \frac{c^2M_5}{2} + \frac{\eta^2M_6}{2}, M_8 = M_2 + M_3 + M_4 + cM_5 + \eta M_6, M_9 = M_1 + M_5 + M_6, M_{10} = M_3\frac{c^2}{2} + M_4\frac{\eta^2}{2} + M_5\frac{c^3}{6} + M_6\frac{\eta^3}{6}, M_{11} = M_3\frac{c^3}{6} + M_4\frac{\eta^3}{6} + M_5\frac{c^4}{24} + M_6\frac{\eta^4}{24}, M_{12} = M_3\frac{c^4}{24} + M_4\frac{\eta^4}{24} + M_5\frac{c^5}{120} + M_6\frac{\eta^5}{120}, M_{13} = M_3\frac{c^5}{120} + M_4\frac{\eta^5}{120} + M_5\frac{c^6}{720} + M_6\frac{\eta^6}{720}, M_{14} = -\frac{M_7}{R^2} + \frac{M_{10}}{R^3} - \frac{2M_{11}}{R^4} + \frac{6M_{12}}{R^5} - \frac{24M_{13}}{R^6}, M_{15} = \frac{M_7}{R} - \frac{M_{10}}{R^2} + \frac{2M_{11}}{R^3} - \frac{6M_{12}}{R^4} + \frac{24M_{13}}{R^5}, M_{16} = \frac{M_{10}}{2R} - \frac{M_{11}}{R^2} + \frac{3M_{12}}{R^3} - \frac{12M_{13}}{R^4}, M_{17} = \frac{M_{11}}{3R} - \frac{M_{12}}{R^2} + \frac{4M_{13}}{R^3}, M_{18} = \frac{M_{12}}{4R} - \frac{M_{13}}{R^2}, M_{19} = \frac{M_{13}}{5R}, M_{20} = \frac{M_8}{R}, M_{21} = \frac{M_9}{R}, M_{22} = -(M_9 + M_{20}), M_{23} = -M_8 - \frac{RM_9}{2}, M_{24} = -\frac{RM_8}{4} - \frac{R^2M_9}{12}, M_{25} = -\frac{R^2M_8}{18} - \frac{R^3M_9}{72}, M_{26} = -\frac{R^3M_8}{96} - \frac{R^4M_9}{480}, M_{27} = -\frac{R^4M_8}{600} - \frac{R^5M_9}{3600}, M_{28} = -M_{21}, K_1 = M_{28} + (IC_1 + M_{14})(1+T) + M_{15}(1+T)^2 + M_{16}(1+T)^3 + M_{17}(1+T)^4 + M_{18}(1+T)^5 + M_{19}(1+T)^6 + M_{20}(1+T)log(1+T), K_2 = -(IC_1 + M_{14}) - 2M_{15}(1+T) - 3M_{16}(1+T)^2 - 4M_{17}(1+T)^3 - 5M_{18}(1+T)^4 - 6M_{19}(1+T)^5 - M_{20}log(1+T), K_3 = M_{15} + 3M_{16}(1+T) + 6M_{17}(1+T)^2 + 10M_{18}(1+T)^3 + 15M_{19}(1+T)^4, K_4 = -M_{16} - 4M_{17}(1+T) - 10M_{18}(1+T)^2 - 20M_{19}(1+T)^3, K_5 = M_{17} + 5M_{18}(1+T) + 15M_{19}(1+T)^2, K_6 = -M_{18} - 6M_{19}(1+T), K_7 = M_{19}, K_8 = M_{20}(1+T), K_9 = -M_{20}, K_{10} = e^{-R(1+T)}[M_{21} + IC_2(1+T) + M_{23}(1+T)^2 + M_{24}(1+T)^3 + M_{25}(1+T)^4 + M_{26}(1+T)^5 + M_{27}(1+T)^6 + M_{22}(1+T)log(1+T)], K_{11} = e^{-R(1+T)}[-IC_2 - 2M_{23}(1+T) - 3M_{24}(1+T)^2 - 4M_{25}(1+T)^3 - 5M_{26}(1+T)^4 - 6M_{27}(1+T)^5 - M_{22}log(1+T)], K_{12} = e^{-R(1+T)}[M_{23} + 3M_{24}(1+T) + 6M_{25}(1+T)^2 + 10M_{26}(1+T)^3 + 15M_{27}(1+T)^4], K_{13} = -e^{-R(1+T)}[M_{24} + 4M_{25}(1+T) + 10M_{26}(1+T)^2 + 20M_{27}(1+T)^3], K_{14} = e^{-R(1+T)}[M_{25} + 5M_{26}(1+T) + 15M_{27}(1+T)^2], K_{15} = -e^{-R(1+T)}[M_{26} + 6M_{27}(1+T)], K_{16} = M_{27}, K_{17} = e^{-R(1+T)}M_{22}(1+T), K_{18} = -e^{-R(1+T)}M_{22}, K_{19} = K_2 - \frac{K_8}{1+T}, K_{20} = K_3 - \frac{K_8}{(1+T)^2} - \frac{K_9}{1+T}, K_{21} = K_4 - \frac{K_8}{(1+T)^3} - \frac{K_9}{(1+T)^2}, K_{22} = K_5 - \frac{K_8}{(1+T)^4} - \frac{K_9}{(1+T)^3},\ K_{23} = K_6 - \frac{K_8}{(1+T)^5} - \frac{K_9}{(1+T)^4}, K_{24} = K_7 - \frac{K_8}{(1+T)^6} - \frac{K_9}{(1+T)^5}, K_{25} = K_{11} - \frac{K_{17}}{1+T}, K_{26} = K_{12} - \frac{K_{17}}{(1+T)^2} - \frac{K_{18}}{1+T}, K_{27} = K_{13} - \frac{K_{17}}{(1+T)^3} - \frac{K_{18}}{(1+T)^2}, K_{28} = K_{14} - \frac{K_{17}}{(1+T)^4} - \frac{K_{18}}{(1+T)^3},\ K_{29} = K_{15} - \frac{K_{17}}{(1+T)^5} - \frac{K_{18}}{(1+T)^4}, K_{30} = K_{16} - \frac{K_{17}}{(1+T)^6} - \frac{K_{18}}{(1+T)^5},$

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$K_{31} = K_{19} + \frac{K_1}{1+T} + a, K_{32} = 2K_{20} + \frac{K_{19}}{1+T} + \frac{K_1}{(1+T)^2}, K_{33} = 3K_{21} + \frac{K_{20}}{1+T} + \frac{K_{19}}{(1+T)^2} + \frac{K_1}{(1+T)^3}, K_{34} = 4K_{22} + \frac{K_{21}}{1+T} + \frac{K_{20}}{(1+T)^2} + \frac{K_{19}}{(1+T)^3} + \frac{K_1}{(1+T)^4}, K_{35} = 5K_{23} + \frac{K_{22}}{1+T} + \frac{K_{21}}{(1+T)^2} + \frac{K_{20}}{(1+T)^3} + \frac{K_{19}}{(1+T)^4} + \frac{K_1}{(1+T)^5},$

$K_{36} = 6K_{24} + \frac{K_{23}}{1+T} + \frac{K_{22}}{(1+T)^2} + \frac{K_{21}}{(1+T)^3} + \frac{K_{20}}{(1+T)^4} + \frac{K_{19}}{(1+T)^5} + \frac{K_1}{(1+T)^6}, K_{37} = \frac{K_{24}}{1+T} + \frac{K_{23}}{(1+T)^2} + \frac{K_{22}}{(1+T)^3} + \frac{K_{21}}{(1+T)^4} + \frac{K_{20}}{(1+T)^5} + \frac{K_{19}}{(1+T)^6} + \frac{K_1}{(1+T)^7}, K_{38} = RK_{10} + K_{25} + \frac{K_{10}}{1+T},$

$K_{39} = RK_{25} + 2K_{26} + \frac{K_{25}}{1+T} + \frac{K_{10}}{(1+T)^2}, K_{40} = RK_{26} + 3K_{27} + \frac{K_{26}}{1+T} + \frac{K_{25}}{(1+T)^2} + \frac{K_{10}}{(1+T)^3}, K_{41} = RK_{27} + 4K_{28} + \frac{K_{27}}{1+T} + \frac{K_{26}}{(1+T)^2} + \frac{K_{25}}{(1+T)^3} + \frac{K_{10}}{(1+T)^4}, K_{42} = RK_{28} + 5K_{29} + \frac{K_{28}}{1+T} + \frac{K_{27}}{(1+T)^2} + \frac{K_{26}}{(1+T)^3} + \frac{K_{25}}{(1+T)^4} + \frac{K_{10}}{(1+T)^5},$

$K_{43} = RK_{29} + 6K_{30} + \frac{K_{29}}{1+T} + \frac{K_{28}}{(1+T)^2} + \frac{K_{27}}{(1+T)^3} + \frac{K_{26}}{(1+T)^4} + \frac{K_{25}}{(1+T)^5} + \frac{K_{10}}{(1+T)^6}$ and

$K_{44} = RK_{30} + \frac{K_{30}}{1+T} + \frac{K_{29}}{(1+T)^2} + \frac{K_{28}}{(1+T)^3} + \frac{K_{27}}{(1+T)^4} + \frac{K_{26}}{(1+T)^5} + \frac{K_{25}}{(1+T)^6} + \frac{K_{10}}{(1+T)^7}$, where IC_1 and IC_2 are integrating constant.

Using the boundary conditions $q(0) = 0 = q(H)$ in Eq. (8), we can get the value of IC_1 and IC_2 Substituting the value of $q(t)$ and $U(t)$ in Eq. (2) the corresponding profit function reduces to

$$J_H = IH_1 - IH_2 - IH_3 - IH_4 - IH_5 - IH_6 - IH_7 \qquad (11)$$

where $IH_1 = c_s\left[a\frac{1-e^{-RH}}{R} - b\frac{1-e^{-(c+R)H}}{c+R}\right], IH_2 = \left[\varphi(\eta) + \frac{S_{u0}}{H}\right]\frac{1-e^{-RH}}{R}$,

$IH_3 = c_{h0}[K_1\frac{1-e^{-RH}}{R} + K_{19}\{\frac{1}{R^2} - (\frac{H}{R} + \frac{1}{R^2})e^{-RH}\} + K_{20}\{\frac{2}{R^3} - (\frac{H^2}{R} + \frac{2H}{R^2} + \frac{2}{R^3})e^{-RH}\} + K_{21}\{\frac{6}{R^4} - (\frac{H^3}{R} + \frac{3H^2}{R^2} + \frac{6H}{R^3} + \frac{6}{R^4})e^{-RH}\} + K_{22}\{\frac{24}{R^5} - (\frac{H^4}{R} + \frac{4H^3}{R^2} + \frac{12H^2}{R^3} + \frac{24H}{R^4} + \frac{24}{R^5})e^{-RH}\} + K_{23}\{\frac{120}{R^6} - (\frac{H^5}{R} + \frac{5H^4}{R^2} + \frac{20H^3}{R^3} + \frac{60H^2}{R^4} + \frac{120H}{R^5} + \frac{120}{R^6})e^{-RH}\} + K_{24}\{\frac{720}{R^7} - (\frac{H^6}{R} + \frac{6H^5}{R^2} + \frac{30H^4}{R^3} + \frac{120H^3}{R^4} + \frac{360H^2}{R^5} + \frac{720H}{R^6} + \frac{720}{R^7})e^{-RH}\} + K_{10}H + K_{25}\frac{H^2}{2} + K_{26}\frac{H^3}{3} + K_{27}\frac{H^4}{4} + K_{28}\frac{H^5}{5} + K_{29}\frac{H^6}{6} + K_{30}\frac{H^7}{7}$,

$IH_4 = c_{h1}[K_1\{\frac{1}{R^2} - (\frac{H}{R} + \frac{1}{R^2})e^{-RH}\} + K_{19}\{\frac{2}{R^3} - (\frac{H^2}{R} + \frac{2H}{R^2} + \frac{2}{R^3})e^{-RH}\} + K_{20}\{\frac{6}{R^4} - (\frac{H^3}{R} + \frac{3H^2}{R^2} + \frac{6H}{R^3} + \frac{6}{R^4})e^{-RH}\} + K_{21}\{\frac{24}{R^5} - (\frac{H^4}{R} + \frac{4H^3}{R^2} + \frac{12H^2}{R^3} + \frac{24H}{R^4} + \frac{24}{R^5})e^{-RH}\} + K_{22}\{\frac{120}{R^6} - (\frac{H^5}{R} + \frac{5H^4}{R^2} + \frac{20H^3}{R^3} + \frac{60H^2}{R^4} + \frac{120H}{R^5} + \frac{120}{R^6})e^{-RH}\} + K_{23}\{\frac{720}{R^7} - (\frac{H^6}{R} + \frac{6H^5}{R^2} + \frac{30H^4}{R^3} + \frac{120H^3}{R^4} + \frac{360H^2}{R^5} + \frac{720H}{R^6} + \frac{720}{R^7})e^{-RH}\} + K_{24}\{\frac{5040}{R^8} - (\frac{H^7}{R} + \frac{7H^6}{R^2} + \frac{42H^5}{R^3} + \frac{210H^4}{R^4} + \frac{840H^3}{R^5} + \frac{2520H^2}{R^6} + \frac{5040H}{R^7} + \frac{5040}{R^8})e^{-RH}\} + K_{10}\frac{H^2}{2} + K_{25}\frac{H^3}{3} + K_{26}\frac{H^4}{4} + K_{27}\frac{H^5}{5} + K_{28}\frac{H^6}{6} + K_{29}\frac{H^7}{7} + K_{30}\frac{H^8}{8}$,

$IH_5 = (c_r + \frac{S_{u1}}{H} + c_R\delta)[K_{31}\frac{1-e^{-RH}}{R} + K_{32}\{\frac{1}{R^2} - (\frac{H}{R} + \frac{1}{R^2})e^{-RH}\} + K_{33}\{\frac{2}{R^3} - (\frac{H^2}{R} + \frac{2H}{R^2} + \frac{2}{R^3})e^{-RH}\} + K_{34}\{\frac{6}{R^4} - (\frac{H^3}{R} + \frac{3H^2}{R^2} + \frac{6H}{R^3} + \frac{6}{R^4})e^{-RH}\} + K_{35}\{\frac{24}{R^5} - (\frac{H^4}{R} + \frac{4H^3}{R^2} + \frac{12H^2}{R^3} + \frac{24H}{R^4} + \frac{24}{R^5})e^{-RH}\} + K_{36}\{\frac{120}{R^6} - (\frac{H^5}{R} + \frac{5H^4}{R^2} + \frac{20H^3}{R^3} + \frac{60H^2}{R^4} + \frac{120H}{R^5} + \frac{120}{R^6})e^{-RH}\} + K_{37}\{\frac{720}{R^7} - (\frac{H^6}{R} + \frac{6H^5}{R^2} + \frac{30H^4}{R^3} + \frac{120H^3}{R^4} + \frac{360H^2}{R^5} + \frac{720H}{R^6} + \frac{720}{R^7})e^{-RH}\} + K_{38}H + K_{39}\frac{H^2}{2} + K_{40}\frac{H^3}{3} + K_{41}\frac{H^4}{4} + K_{42}\frac{H^5}{5} + K_{43}\frac{H^6}{6} + K_{44}\frac{H^7}{7} - b\frac{1-e^{-(c+R)H}}{c+R}]$,

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$$IH_6 = -c_R\mu\left[K_{31}\frac{1-e^{-(R+\eta)H}}{(R+\eta)} + K_{32}\left\{\frac{1}{(R+\eta)^2} - \left(\frac{H}{(R+\eta)} + \frac{1}{(R+\eta)^2}\right)e^{-(R+\eta)H}\right\} + K_{33}\left\{\frac{2}{(R+\eta)^3} - \left(\frac{H^2}{(R+\eta)} + \frac{2H}{(R+\eta)^2} + \frac{2}{(R+\eta)^3}\right)e^{-(R+\eta)H}\right\} + K_{34}\left\{\frac{6}{(R+\eta)^4} - \left(\frac{H^3}{(R+\eta)} + \frac{3H^2}{(R+\eta)^2} + \frac{6H}{(R+\eta)^3} + \frac{6}{(R+\eta)^4}\right)e^{-(R+\eta)H}\right\} + K_{35}\left\{\frac{24}{(R+\eta)^5} - \left(\frac{H^4}{(R+\eta)} + \frac{4H^3}{(R+\eta)^2} + \frac{12H^2}{(R+\eta)^3} + \frac{24H}{(R+\eta)^4} + \frac{24}{(R+\eta)^5}\right)e^{-(R+\eta)H}\right\} + K_{36}\left\{\frac{120}{(R+\eta)^6} - \left(\frac{H^5}{(R+\eta)} + \frac{5H^4}{(R+\eta)^2} + \frac{20H^3}{(R+\eta)^3} + \frac{60H^2}{(R+\eta)^4} + \frac{120H}{(R+\eta)^5} + \frac{120}{(R+\eta)^6}\right)e^{-(R+\eta)H}\right\} + K_{37}\left\{\frac{720}{(R+\eta)^7} - \left(\frac{H^6}{(R+\eta)} + \frac{6H^5}{(R+\eta)^2} + \frac{30H^4}{(R+\eta)^3} + \frac{120H^3}{(R+\eta)^4} + \frac{360H^2}{(R+\eta)^5} + \frac{720H}{(R+\eta)^6} + \frac{720}{(R+\eta)^7}\right)e^{-(R+\eta)H}\right\} + K_{38}\frac{1-e^{-\eta H}}{\eta} + K_{39}\left\{\frac{1}{\eta^2} - \left(\frac{H}{\eta} + \frac{1}{\eta^2}\right)e^{-\eta H}\right\} + K_{40}\left\{\frac{2}{\eta^3} - \left(\frac{H^2}{\eta} + \frac{2H}{\eta^2} + \frac{2}{\eta^3}\right)e^{-\eta H}\right\} + K_{41}\left\{\frac{6}{\eta^4} - \left(\frac{H^3}{\eta} + \frac{3H^2}{\eta^2} + \frac{6H}{\eta^3} + \frac{6}{\eta^4}\right)e^{-\eta H}\right\} + K_{42}\left\{\frac{24}{\eta^5} - \left(\frac{H^4}{\eta} + \frac{4H^3}{\eta^2} + \frac{12H^2}{\eta^3} + \frac{24H}{\eta^4} + \frac{24}{\eta^5}\right)e^{-\eta H}\right\} + K_{43}\left\{\frac{120}{\eta^6} - \left(\frac{H^5}{\eta} + \frac{5H^4}{\eta^2} + \frac{20H^3}{\eta^3} + \frac{60H^2}{\eta^4} + \frac{120H}{\eta^5} + \frac{120}{\eta^6}\right)e^{-\eta H}\right\} + K_{44}\left\{\frac{720}{\eta^7} - \left(\frac{H^6}{\eta} + \frac{6H^5}{\eta^2} + \frac{30H^4}{\eta^3} + \frac{120H^3}{\eta^4} + \frac{360H^2}{\eta^5} + \frac{720H}{\eta^6} + \frac{720}{\eta^7}\right)e^{-\eta H}\right\} - b\frac{1-e^{-(c+R+\eta)H}}{c+R+\eta}\right],$$

$$IH_7 = \lambda[K_{45}\frac{1-e^{-RH}}{R} + K_{46}\left\{\frac{1}{R^2} - \left(\frac{H}{R} + \frac{1}{R^2}\right)e^{-RH}\right\} + K_{47}\left\{\frac{2}{R^3} - \left(\frac{H^2}{R} + \frac{2H}{R^2} + \frac{2}{R^3}\right)e^{-RH}\right\} + K_{48}\left\{\frac{6}{R^4} - \left(\frac{H^3}{R} + \frac{3H^2}{R^2} + \frac{6H}{R^3} + \frac{6}{R^4}\right)e^{-RH}\right\} + K_{49}\left\{\frac{24}{R^5} - \left(\frac{H^4}{R} + \frac{4H^3}{R^2} + \frac{12H^2}{R^3} + \frac{24H}{R^4} + \frac{24}{R^5}\right)e^{-RH}\right\} + K_{50}\left\{\frac{120}{R^6} - \left(\frac{H^5}{R} + \frac{5H^4}{R^2} + \frac{20H^3}{R^3} + \frac{60H^2}{R^4} + \frac{120H}{R^5} + \frac{120}{R^6}\right)e^{-RH}\right\} + K_{51}\left\{\frac{720}{R^7} - \left(\frac{H^6}{R} + \frac{6H^5}{R^2} + \frac{30H^4}{R^3} + \frac{120H^3}{R^4} + \frac{360H^2}{R^5} + \frac{720H}{R^6} + \frac{720}{R^7}\right)e^{-RH}\right\} + 2\left[K_{52}\frac{e^{RH}-1}{R} + K_{53}\left\{\frac{1}{R^2} + \left(\frac{H}{R} - \frac{1}{R^2}\right)e^{RH}\right\} + K_{54}\left\{-\frac{2}{R^3} + \left(\frac{H^2}{R} - \frac{2H}{R^2} + \frac{2}{R^3}\right)e^{RH}\right\} + K_{55}\left\{\frac{6}{R^4} + \left(\frac{H^3}{R} - \frac{3H^2}{R^2} + \frac{6H}{R^3} - \frac{6}{R^4}\right)e^{RH}\right\} + K_{56}\left\{-\frac{24}{R^5} + \left(\frac{H^4}{R} - \frac{4H^3}{R^2} + \frac{12H^2}{R^3} - \frac{24H}{R^4} + \frac{24}{R^5}\right)e^{RH}\right\} + K_{57}\left\{\frac{120}{R^6} + \left(\frac{H^5}{R} - \frac{5H^4}{R^2} + \frac{20H^3}{R^3} - \frac{60H^2}{R^4} + \frac{120H}{R^5} - \frac{120}{R^6}\right)e^{RH}\right\} + K_{58}\left\{-\frac{720}{R^7} + \left(\frac{H^6}{R} - \frac{6H^5}{R^2} + \frac{30H^4}{R^3} - \frac{120H^3}{R^4} + \frac{360H^2}{R^5} - \frac{720H}{R^6} + \frac{720}{R^7}\right)e^{RH}\right\} + K_{59}H + K_{60}\frac{H^2}{2} + K_{61}\frac{H^3}{3} + K_{62}\frac{H^4}{4} + K_{63}\frac{H^5}{5} + K_{64}\frac{H^6}{6} + K_{65}\frac{H^7}{7}\right] - 2b\left[K_{31}\frac{1-e^{-(R+c)H}}{(R+c)} + K_{32}\left\{\frac{1}{(R+c)^2} - \left(\frac{H}{(R+c)} + \frac{1}{(R+c)^2}\right)e^{-(R+c)H}\right\} + K_{33}\left\{\frac{2}{(R+c)^3} - \left(\frac{H^2}{(R+c)} + \frac{2H}{(R+c)^2} + \frac{2}{(R+c)^3}\right)e^{-(R+c)H}\right\} + K_{34}\left\{\frac{6}{(R+c)^4} - \left(\frac{H^3}{(R+c)} + \frac{3H^2}{(R+c)^2} + \frac{6H}{(R+c)^3} + \frac{6}{(R+c)^4}\right)e^{-(R+c)H}\right\} + K_{35}\left\{\frac{24}{(R+c)^5} - \left(\frac{H^4}{(R+c)} + \frac{4H^3}{(R+c)^2} + \frac{12H^2}{(R+c)^3} + \frac{24H}{(R+c)^4} + \frac{24}{(R+c)^5}\right)e^{-(R+c)H}\right\} + K_{36}\left\{\frac{120}{(R+c)^6} - \left(\frac{H^5}{(R+c)} + \frac{5H^4}{(R+c)^2} + \frac{20H^3}{(R+c)^3} + \frac{60H^2}{(R+c)^4} + \frac{120H}{(R+c)^5} + \frac{120}{(R+c)^6}\right)e^{-(R+c)H}\right\} + K_{37}\left\{\frac{720}{(R+c)^7} - \left(\frac{H^6}{(R+c)} + \frac{6H^5}{(R+c)^2} + \frac{30H^4}{(R+c)^3} + \frac{120H^3}{(R+c)^4} + \frac{360H^2}{(R+c)^5} + \frac{720H}{(R+c)^6} + \frac{720}{(R+c)^7}\right)e^{-(R+c)H}\right\} + K_{38}\frac{1-e^{-cH}}{c} + K_{39}\left\{\frac{1}{c^2} - \left(\frac{H}{c} + \frac{1}{c^2}\right)e^{-cH}\right\} + K_{40}\left\{\frac{2}{c^3} - \left(\frac{H^2}{c} + \frac{2H}{c^2} + \frac{2}{c^3}\right)e^{-cH}\right\} + K_{41}\left\{\frac{6}{c^4} - \left(\frac{H^3}{c} + \frac{3H^2}{c^2} + \frac{6H}{c^3} + \frac{6}{c^4}\right)e^{-cH}\right\} + K_{42}\left\{\frac{24}{c^5} - \left(\frac{H^4}{c} + \frac{4H^3}{c^2} + \frac{12H^2}{c^3} + \frac{24H}{c^4} + \frac{24}{c^5}\right)e^{-cH}\right\} + K_{43}\left\{\frac{120}{c^6} - \left(\frac{H^5}{c} + \frac{5H^4}{c^2} + \frac{20H^3}{c^3} + \frac{60H^2}{c^4} + \frac{120H}{c^5} + \frac{120}{c^6}\right)e^{-cH}\right\} + K_{44}\left\{\frac{720}{c^7} - \left(\frac{H^6}{c} + \frac{6H^5}{c^2} + \frac{30H^4}{c^3} + \frac{120H^3}{c^4} + \frac{360H^2}{c^5} + \frac{720H}{c^6} + \frac{720}{c^7}\right)e^{-cH}\right\}\right] + b^2\frac{1-e^{-(R+2c)H}}{R+2c}]$$

where $K_{45} = {K_{31}}^2, K_{46} = 2K_{31}K_{32}, K_{47} = {K_{32}}^2 + 2K_{31}K_{33}, K_{48} = 2K_{31}K_{34} + 2K_{32}K_{33},$

$K_{49} = {K_{33}}^2 + 2K_{31}K_{35} + 2K_{32}K_{34}, K_{50} = 2K_{31}K_{36} + 2K_{32}K_{35} + 2K_{33}K_{34},$
$K_{51} = {K_{34}}^2 + 2K_{31}K_{37} + 2K_{32}K_{36} + 2K_{33}K_{35}, K_{52} = {K_{38}}^2, K_{53} = 2K_{38}K_{39},$
$K_{54} = {K_{39}}^2 + 2K_{38}K_{40}, K_{55} = 2K_{38}K_{41} + 2K_{39}K_{40}, K_{56} = {K_{40}}^2 + 2K_{38}K_{42} + 2K_{39}K_{41},$
$K_{57} = 2K_{38}K_{43} + 2K_{39}K_{42} + 2K_{40}K_{41}, K_{58} = {K_{41}}^2 + 2K_{38}K_{44} + 2K_{39}K_{43} + 2K_{40}K_{42},$

$K_{59} = K_{31}K_{38}, K_{60} = K_{31}K_{39} + K_{32}K_{38}, K_{61} = K_{31}K_{40} + K_{32}K_{39} + K_{33}K_{38},$

$K_{62} = K_{31}K_{41} + K_{32}K_{40} + K_{33}K_{39} + K_{34}K_{38},$
$K_{63} = K_{31}K_{42} + K_{32}K_{41} + K_{33}K_{40} + K_{34}K_{39} + K_{35}K_{38},$
$K_{64} = K_{31}K_{43} + K_{32}K_{42} + K_{33}K_{41} + K_{34}K_{40} + K_{35}K_{39} + K_{36}K_{38},$

$K_{65} = K_{31}K_{44} + K_{32}K_{43} + K_{33}K_{42} + K_{34}K_{41} + K_{35}K_{40} + K_{36}K_{39} + K_{37}K_{38}.$

5. Solution Methodology:

It has been already proved that there exists a path $q = q(t)$, in the interval $[0, H]$ for which J_H has a maximum. Again, with the known parameters other than the reliability parameter, η the total profit expression, J_H, given by eq. (11) become functions of a single variable η . To find the optimum value of η the first order derivatives of these functions with respect to η, $\frac{dJ_H}{d\eta}$ is made equal to zero. Thus we get a transcendental equation on η, $\frac{dJ_H}{d\eta} = 0$ and are solved using Newton-Raphson method. With this value of η second order derivative of J_H with respect to

η, $\frac{d^2J_H}{d\eta^2}$ is calculated. It is found that $\frac{d^2J_H}{d\eta^2} < 0$ for that value of η. For maximum profit, the appropriate value of η is taken and the corresponding profit is calculated. Again the profit function is optimized using LINGO-13 software and the result obtained is same as that obtained by the previous numerical method. So it can be stated that result obtained following the procedure for a demand pattern is a global optimum.

6. Numerical illustration:

The following parametric values with appropriate units are used to illustrate the model.

Input Data:
$a = 100, b = 32, c = 0.03, \delta = 0.99, \mu = 0.85, T = 1, H = 0.3, c_r = 4, \lambda = 0.02, c_{h0} = 1, c_{h1} = 0.08, s_{u0} = 20, s_{u1} = 0.05, c_R = 6.5, R = 0.07, L_1 = 160, L_2 = 50, K = 0.2, c_s = 25, \eta_{max} = 0.8, \eta_{min} = 0.1.$

With these input data, the optimum value of η and the corresponding profit, J_H for the proposed model is given in Table-1.

Table-1: optimal result

η	J_H
0.3480212	230.33

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With the optimum value of η, pictorial representation of inventory, demand and production rate against time, unit production cost and set-up cost against time and profit and development cost against reliability indicator are presented in Fig-1, -2 and -3 respectively.

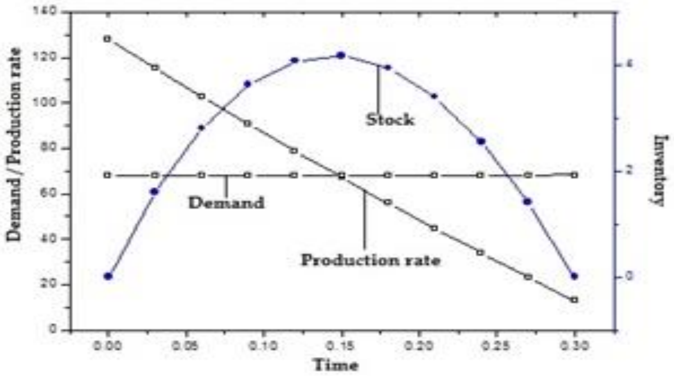

Figure-1: Graph for optimum stock, demand and production rate with respect to time

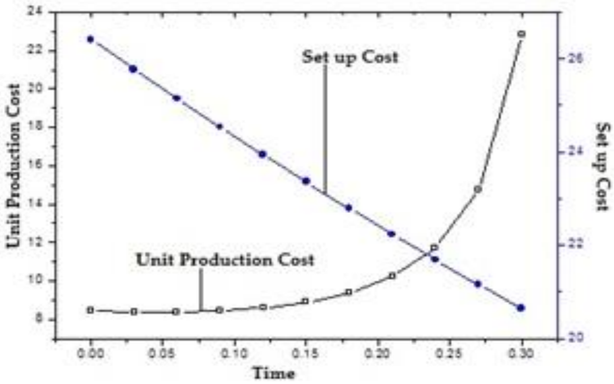

Figure-2: Graph for set up cost and unit production cost with respect to time

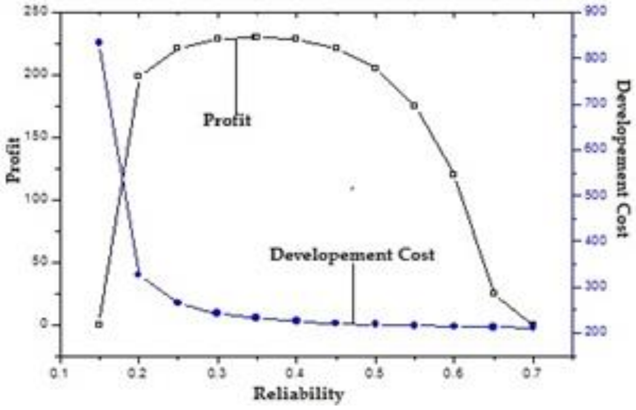

Figure-3: Graph for profit and development cost with respect to reliability indicator

7. Conclusion

For the first time a production inventory model for damageable item is developed where reliability of the production process together with the production rate are controllable. It is observed that an optimum reliability draws maximum profit for an item having particular demand pattern. Also it is found from our findings that minimum unit production cost for an item does not give the maximum profit always. From the present model it can be concluded that optimal control of production rate reduces holding cost as well as damageability which in turn increases profit separately for deteriorating items. The present investigation reveals that process reliability is an important factor which determines the production rate and thus determines the optimal production path, unit production cost and optimal profit for the production-inventory managers. The present model can be extended to the fuzzy and fuzzy-random environment taking constant part of holding cost, set-up cost, etc. as imprecise or fuzzy-stochastic.

References

Alfares, H.K., (2007). Inventory model with stock-level dependent demand rate and variable holding cost. International Journal of Production Economics, 108(1-2), 259-265.

Bakker, M., Riezebos, J. and Teunter, R.H., (2012). Review of inventory systems with deterioration since 2001, European Journal of Operational Research, 221, 275-284.

Beirman, H., Thomas, J., (1977). Inventory decisions under inflationary condition. Decision Science, 8, 151-155.

Buzacott, J.A., (1975). Economic order quantities with inflation. Operations Research Quarterly, 26, 553-558.

Chang, C.T., (2004a). An EOQ model for deteriorating items under inflation when supplier credits linked to order quantity, International Journal of Production Economics, 88, 307-316.

Chang, C.T., (2004b). Inventory models with stock-dependent demand and nonlinear holding costs for deteriorating items, Asia-Pacific Journal of Operational Research, 21, 435-446.

Chen, J.M., (1998). An inventory model for deteriorating items with time-proportional demand and shortages under inflation and time discounting. International Journal of Production Economics, 55(1), 21-30.

Cheng, T.C.E., (1989). An economic production quantity model with flexibility and reliability consideration. European Journal of Operational Research, 39, 174-179.

Chung, K.J., Hou, K.L., (2003). An optimal production run time with imperfect production processes and allowable shortages. Comput. Oper. Res., 30, 483-490.

Darwish, M.A., (2008). EPQ models with varying setup cost. International Journal of Production Economics, 113, 297-306.

Datta, T.K., Pal, A.K., (1991). Effects of inflation and time-value of money on an inventory model with linear time-dependent demand rate and shortages. European Journal of Operational Research, 52(3), 326-333.

Dave, U., Patel, L.K., (1981). Policy inventory model for deteriorating items with time proportional demand. Journal of Operational Research Society 32, 137-142.

Dye, C.Y., (2012). A finite horizon deteriorating inventory model with two-phase pricing and time-varying demand and cost under trade credit financing using particle swarm optimization, Swarm and Evolutionary Computation, 5, 37-53.

Goyal, S.K. and Giri, B.C., (2001). Recent trends in modeling of deteriorating inventory, European Journal of Operational Research, 134(1), 1-16.

Guchhait, P., Maiti, M.K. and Maiti, M., (2013). Two storage inventory model of a deteriorating item with variable demand under partial credit period, Applied Soft Computing, 13, 1-21.

He, Y., Wangc, S.Y. and Lai, K.K., (2010). An optimal production-inventory model for deteriorating items with multiple-market demand, European Journal of Operational Research, 203, 593-600.

Hsu, S.L. and Lee, C.C., (2009). A two-warehouse production model for deteriorating inventory items with time-dependent demands, European Journal of Operational Research, 194, 700-710.

Hung, K.C., (2011). An inventory model with generalized type demand, deterioration and backorder rates, European Journal of Operational Research, 208, 239-242.

Lee, C.C., Hsu, S.L., 2009. A two-warehouse production model for deteriorating inventory items with time-dependent demands. European Journal of Operational Research, 194(3), 700-710.

Liao, G.L., Sheu, S.H., (2011). Economic production quantity model for randomly failing production process with minimal repair and imperfect maintenance. International Journal of Production Economics, 130, 118-124.

Mahata, G.C., (2012). An EPQ-based inventory model for exponentially deteriorating items under retailer partial trade credit policy in supply chain, Experts Systems with Applications, 39, 3537-3550.

Matsuyama, K., (1995). Inventory policy with time-dependent setup cost. International Journal of Production Economics, 42, 149-160.

Mettas, A., (2000). Reliability allocation and optimization for complex systems. In: Proceedings of the annual reliability and maintainability symposium. Institute Electrical and Electronics Engineers, Piscataway, NJ, pp. 216–221.

Ouyang, L.Y., Teng, J.T., Goyal, S.K. and Yang, C.T., (2009). An economic order quantity model for deteriorating items with pertially permissible delay in payments to order quantity, European Journal of Operational Research, 194, 418-431.

Ouyang, L.Y., Wu, K.S. and Yang, C.T., (2006). A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments, Computers & Industrial Engineering, 51, 637-651.

ISSN- 2394-5125 VOL 07, ISSUE 19, 2020

Porteous, E.L., (1986). Optimal lot sizing, process quality improvement and set-up cost reduction. Operation Research 34, 137-144.

Ray, J., Chaudhuri, K.S., (1997). An EOQ model with stock-dependent demand, shortages, inflation and time discounting. International Journal of Production Economics, 53, 171-180.

Sana, S.S., (2010). A production-inventory model in an imperfect production process. European Journal of Operational Research, 200, 451-464.

Sana, S.S., Goyal, S.K., Chaudhuri, K.S., (2007). An imperfect production process in a volume flexible inventory model. International Journal of Production Economics, 105, 548-559.

Sarkar, B., Sana, S.S., Chaudhuri, K.S., (2010). Optimal reliability, production lot size and safety stock in an imperfect production system. Int. J. Math. Oper. Res., 2, 467-490.

Sarkar, B., Sana, S.S. and Chaudhuri, K.S., (2011). An imperfect production process for time varying demand with inflation and time value of money - An EMQ model, Expert System with Applications, 38, 13543-13548.

Urban, T.L., (2008). An extension of inventory models with discretely variable holding costs. International Journal of Production Economics, 114, 399-403.

Wee, H.M., Law, S.T,. (2004). Economic production lot size for deteriorating items taking account of time value of money. Computers and Operations Research, 26, 545-558.

Yadav, D., Pundir, S. and Kumari, R., (2011). A fuzzy multi-item production model with reliability and flexibility under limited storage capacity with deterioration via geometric programming, International Journal of Mathematics in Operational Research, 3(1), 78-98.

Yoo, S.H., Kim, D., Park, M.S., (2009). Economic production quantity model with imperfect-quality items, two-way imperfect inspection and sales return. International Journal of Production Economics, 121, 255-265.