

An Analysis of Algebraic Graph Theory's Polynomials, Transformations, and Distance Concepts

Gadiya Mahaveer Popatlal¹, Dr. Syed Shahnawaz Ali²,

¹Research Scholar, Dept. of Mathematics

Sri Satya Sai University of Technology and Medical Sciences,
Sehore, Bhopal-Indore Road, Madhya Pradesh, India.

²Research Guide, Dept. of Mathematics

Sri Satya Sai University of Technology and Medical Sciences,
Sehore, Bhopal-Indore Road, Madhya Pradesh, India.

Abstract

We'll start the day by studying functions, specifically the kinds of functions that can be described by expressions with polynomials. In data modelling, the most common types of functions are polynomial functions and rational functions. With a little more research, functions like this can be broken down into subcategories. There are many ways to use these functions, such as in mathematical models of production costs, consumer demands, wildlife management, biological processes, and a wide range of other scientific investigations and research projects. These operations are also used a lot in many different other situations. By using these algorithms and the graphs they create, you can figure out what the data is likely to do in the future. Because it can be done, it shows that it is possible. We look at graph polynomials, graph transformations, and distance-related ideas in the context of algebraic graph theory. With new software that is still being made, it will be possible to figure out the distance polynomials of graphs with up to 200 vertices. The algorithm also finds out what the distance matrix's eigenvalues and eigenvectors are. With just the information about the neighbourhood, the method can make a "distance matrix." The Givens-Householder method is used to figure out the eigenvalues and eigenvectors, and the author's own programmes are used to figure out the characteristic polynomials of the distance matrix. New programmes are tested on a large number of graphs with a lot of vertices to make sure they work as planned. Even though distance polynomials are not usually unique structural invariants, it has been shown that they can be used to tell the difference between certain classes of cyclic isospectral graphs.

Introduction

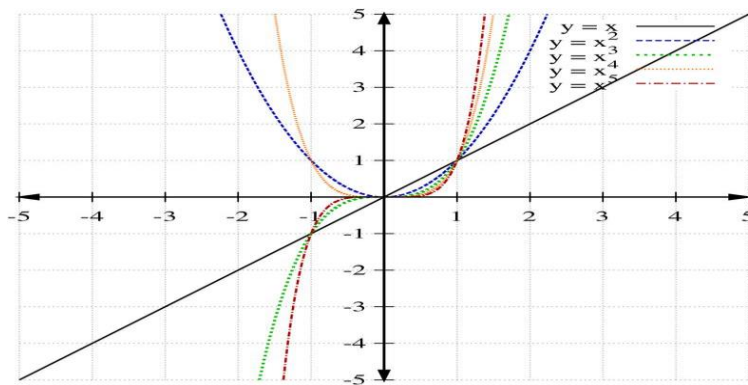
We have now been familiarised with a wide variety of functions, the likes of which include linear functions, constant functions, and quadratic functions, in addition to a great many other kinds of functions. These three functions are all a component of a broader group of functions that are collectively known as the polynomial functions. This bigger group of functions was named after their collective name. There are a great many more functions contained within this more comprehensive group.

A power function is a form of a polynomial that contains the least amount of terms possible. This form is called the simplest form. A power function is a specialised type of polynomial that takes the form where n is a real number, and n is an integer, and is multiplied by x . This type of polynomial is also known as a power factor. The power formula is another name for this particular type of polynomial.

If the value being evaluated is even, the power function will also be referred to as "even," and if the value being evaluated is odd, the power function will also be known to as "odd."

If the value being evaluated is even, then the power function will be referred to as "even."

In the following table, the graphs of the first various power functions are organised according to the sequence in which they are given.



Methodology

Functions of Polynomials and the Graphs of Those Functions: Before we look at polynomials, it would be beneficial for us to become familiar with basic terms.

The term "polynomial of degree n " refers to a function that takes the form $y = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ when the value of n is smaller than zero. The coefficients of the polynomial are indicated by the numerals a_0, a_1, a_2, \dots , and a_n correspondingly. These coefficients are written out in full below. The integer a_0 is used to represent the constant coefficient, which is also

commonly referred to as the constant term. The number a_n , which represents the coefficient with the greatest power, is used to indicate the leading coefficient. An $a_n x^n$ is an expression that is used to denote the leading phrase.

It is essential to keep in mind that a polynomial is typically written as a series of progressively lower powers of the variable, and that the degree of a polynomial is equal to the power of the term that comes first in the equation. Consider, for instance:

$$P = x^4 + 4x^3 + x^2 + 5$$

is an illustration of a polynomial of degree 4, for example. In addition, the phrase $Q = 7x^4$ is an example of a monomial, which is a type of polynomial that only includes a single term.

What Exactly Are Graphs of Polynomials, and How Do They Perform Their Functions?

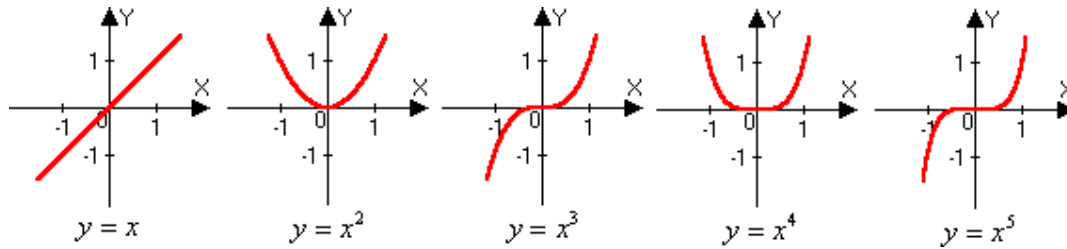
Polynomials of degrees 0 and 1 have linear equations, and the graphs of these polynomials are straight lines because these polynomials have linear equations. These polynomials have linear equations that describe them. Equations that contain polynomials of degree 2 are referred to as quadratic equations, and the graphs of these equations are represented by parabolas. Quadratic equations can be solved by using the quadratic formula. After the degree of the underlying polynomial has passed 2, the graph has the ability to take on a wider variety of forms as the degree of the polynomial continues to climb. On the other hand, the graph of a polynomial function is invariably a curve that is continuous and smooth. This is because polynomials are functions that are composed of several terms (no breaks, gaps, or sharp corners).

Those monomials of the form P multiplied by n as their base form.

represent the simplest possible type of polynomial.

Those monomials that take the form P multiplied by n .

represent the most fundamental kind of polynomial.



According to what is shown in the graphic, the graph of

$y = x^n$ has the same overall configuration as

$$y = x^2$$

when

n is an even number, and it has the same basic form as

If n is an odd number, then y is greater than x^3 . Having said that, because of the degree n

as the number of points increases, the graphs flatten out at the point of origin and grow more steep elsewhere.

Some instances of monomials that can be changed into other forms include the following:

When we are graphing certain polynomial functions, it is feasible for us to make use of the graphs of monomials that we are already familiar with and change them by making use of the methods that we discovered earlier in the process. This is something that we can do when we are graphing monomials. Because we have already completed this procedure, we will have the experience and knowledge necessary to successfully complete this task.

End Behavior of Polynomials:

A description of what happens when x becomes large in either a positive or negative direction is an illustration of the end behaviour of a polynomial. This behaviour can be positive or negative. It's possible that this behaviour will have a beneficial or bad impact. We will use the following notation for the purpose of explaining the behaviour of the

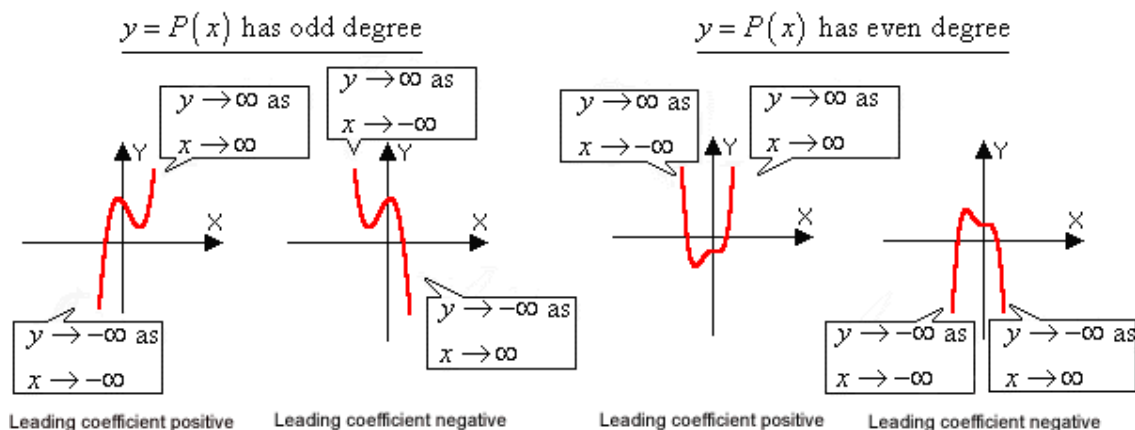
finished product, which is as follows:

This theorem has a number of important repercussions, one of which is that the values of a polynomial are either wholly positive or entirely negative between any two subsequent zeros. This is just one of the crucial implications of this theorem. This is only one of the significant ramifications that may be drawn from this theorem. To put it another way, the graph of a polynomial either falls completely above or completely below the x-axis between two consecutive zeros. In other words, it is either completely above or completely below the x-axis. In other words, it is either entirely above or entirely below. Neither middle ground exists.

In light of this, before we can even begin to sketch the graph of P, we need to first identify all of P's zeros. This is a necessary step before we can ever begin to sketch the graph of P. The third step is to select test points between successive zeros (as well as to the right and left of the zeros), with the intention of determining whether or not P x is positive or negative on each interval that was established by the zeros. This can be done by selecting test points between successive zeros (as well as to the right and left of the zeros).

End Behavior of Polynomials

The polynomial $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ has the same end behavior as the monomial $Q(x) = a_n x^n$, so its end behavior is determined by the degree n and the sign of the leading coefficient a_n .



When graphing polynomial functions, the following instructions should be followed:

1. Locate all of the real zeros of the equation by factoring the polynomial; the real zeros are the x-intercepts of the graph. 2. Determine which of the real zeros are the most significant. You will need to factor the polynomial in order to achieve this objective.

2. You are going to be responsible for creating a table of values for the polynomial as the second section of the examination. Include test points so that it can be determined whether the graph of the polynomial lies above or below the x-axis on the intervals that are dictated by the zeros. Include test points so that it can be determined whether or not test points are included. Include test points so it can be identified whether the zeros or the intervals are the ones that decide the range. The y-intercept is an essential component that must be included in the table that you've built.

3. Determine how the polynomial acts when it reaches its final stage. It is necessary for you to determine how the polynomial will act when it is reduced to its final form.

4. With the help of the graph, draw the intercepts as well as any other points that you discovered in the table. You will be able to build a smooth curve that spans these locations and has the right behaviour at the end if you follow the directions that are provided below and do so in the order that they are presented.

COMPUTATION OF DISTANCE

. It has been discovered that a graph that does not have any directed edges can make use of the D matrix in order to create a real-symmetric matrix for the graph. As a result, tridiagonalizing it utilising the Givens-Householder strategy as the preferred method is something that is both doable and practicable. This will bring about the diagonalization that

we want to see. After it has been transformed into a tridiagonal form, determining the eigenvalues of the changed matrix is a very straightforward process. Eigenvectors can also be generated using this method due to the fact that the unitary transformation that is utilised in this approach and is stored in each iteration of the process. This ensures that Eigenvectors can be generated using this method. The Householder algorithm does not allow the translation of non-symmetrical matrices into the tri-diagonal form, which means that this approach cannot be used for directed graphs. This is the reason why this method cannot be used for directed graphs. Because of this, it is essential to make use of other approaches, despite the fact that these strategies are noticeably less effective than the original ones. The author uses the recursive matrix product approach to construct the characteristic polynomials of the distance matrix. This approach, which was explained in greater detail in earlier publications published by the author, is used to produce the characteristic polynomials. After the generation of the distance matrix, the subroutines for computing the characteristic polynomials of weighted graphs that were constructed earlier receive it as an input, and then proceed to compute those polynomials. In spite of the fact that the weights in this specific case are the distances that exist between particular vertices in the graph, this is nevertheless done. ^{z6} The fact that the matrix product code that was written by the author^{z6} is capable of managing directed graphs in addition to graphs with imaginary edge weights is the one and only benefit that it offers. This is the advantage that it bestows upon its users. This is the most advanced method that has been developed up to this point for the purpose of computing characteristic polynomials, and as a direct consequence of this, it possesses all of these advantages for the purpose of computing distance polynomials as well. This method was developed for the purpose of computing characteristic polynomials. In this view, the method in question is the one that produces the best outcomes. The scaling method that was used could either have been applied to the computation of the distance matrix itself because, for larger graphs, the diagonal elements of A^n that measure the number of self-returning walks (spectral moments) grow exponentially and, as a result, could cause arithmetic overflows; alternatively, the scaling method that was used could have been applied to the computation of the distance matrix itself. Since the vast majority of our focus is currently being directed toward the off-diagonal components, the scaling procedure and the subsequent renormalization are able to

circumvent the issue of overflows. This is possible due to the fact that the off-diagonal components are receiving the majority of our attention. There is also the possibility of excluding totally the computation of the things that are located on the diagonal of A, which is an extra choice. The magnitudes of the coefficients in the distance polynomial are used to decide whether or not the scaling technique should be maintained for the computation of the characteristic polynomials of the D matrix. If they are large enough, the scaling technique should be used. It is possible that the scaling operation will not be performed if the correct response is provided to this question. The implementation of the scaling strategy was not required to be carried out on the great majority of the graphs that were considered for this article. [Case in point:] [Case in point:] One significant exception to this overarching notion is illustrated with a honeycomb lattice network that contains 96 vertices.

CONCLUSION

During our investigation, we came across precise equations for the hub polynomial of a number of different graphs, such as the helm graph, the flower graph, the corona of two graphs, various windmill graphs, and various transformation graphs. These precise equations were found for a number of different graphs. The formulas are presented in this space for your edification and convenience. After that, we proceeded to define an n-wounded spider graph, and after that, we proceeded to identify the hub polynomial for it. Afterwards, we went to move on. By making use of other graphs, one is possible to obtain the hub polynomial of other graphs.

Reference

- [1].Akbari, S., Alikhani, S., and Peng, Y. H., 2010, "Characterization of graphs using domination polynomials", *European J. Combin.*, 31, pp. 1714–1724.
- [2].Grauman, T., Hartke, S. G., Jobson, A., Kinnersley,B., West, D. B., Wiglesworth,L., Worah,P. and Wu, H., 2008, "The hub number of a graph", *Inform. Process. Lett.*, 108(4), pp.226-228.
- [3].Walsh, M., 2006, "The hub number of a graph", *Int. J. Math. Comput. Sci.*, 1(1), pp. 117–124.
- [4].K. Balasubramanian and X. Y. Liu, *J. Comp. Chem.*, 9, 406 (1988).
- [5].W. C. Herndon, unpublished work as referred to in reference (12).

- [6.] N. Anari, S. Oveis Gharan, and C. Vinzant. Log-concave polynomials, entropy, and a deterministic approximation algorithm for counting bases of matroids. FOCS 2018 Proc. (59th Annual IEEE Symposium), pp. 35–46, 201
- [7].M. Aouchiche and P. Hansen. Distance spectra of graphs: a survey. *Linear Algebra Appl.*, 458:301–386, 201
- [8.] J. Borcea and P. Brändén. Pólya–Schur master theorems for circular domains and their boundaries. *Ann. of Math. (2)*, 170(1):465–492, 2000
- [9.] J. Borcea and P. Brändén. The Lee–Yang and Pólya–Schur programs. I. Linear operators preserving stability. *Invent. Math.*, 177(3):541–569, 20
- [10]. A. Bouchet. Representability of Δ -matroids. In: Proc. 6th Hungarian Coll. Combin. 1987, *Colloq. Math. Soc. János Bolyai*, 52:167–182, 198
- [11]. J. Cheeger, B. Kleiner, and A. Schioppa. Infinitesimal structure of differentiability spaces, and metric differentiation. *Anal. Geom. Metric Spaces*, 4(1):104–159, 201
- [12]. H. Lin, Y. Hong, J. Wang, and J. Shu. On the distance spectrum of graphs. *Linear Algebra Appl.*, 439(6):1662–1669, 2013.
- [13]. G. Royle and A.D. Sokal. The Brown–Colbourn conjecture on zeros of reliability polynomials is false. *J. Combin. Th. Ser. B*, 91(2):345–360, 2004.
- [14]. Solanki, R. K., Rajawat, A. S., Gadekar, A. R., & Patil, M. E. (2023). Building a Conversational Chatbot Using Machine Learning: Towards a More Intelligent Healthcare Application. In M. Garcia, M. Lopez Cabrera, & R. de Almeida (Eds.), *Handbook of Research on Instructional Technologies in Health Education and Allied Disciplines* (pp. 285-309). IGI Global. <https://doi.org/10.4018/978-1-6684-7164-7.ch013>
- [15]. S. B. Goyal, A. S. Rajawat, R. K. Solanki, M. A. Majmi Zaaba and Z. A. Long, "Integrating AI With Cyber Security for Smart Industry 4.0 Application," *2023 International Conference on Inventive Computation Technologies (ICICT)*, Lalitpur, Nepal, 2023, pp. 1223-1232, doi: 10.1109/ICICT57646.2023.10134374.
- [16]. Pardeshi, D., Rawat, P., Raj, A., Gadbail, P., Solanki, R. K., & Bhaladhare, D. P. R. (2023). Efficient Approach for Detecting Cardiovascular Disease Using Machine Learning. *Int. J. of Aquatic Science*, 14(1), 308-321
- [17]. Patle, S., Pal, S., Patil, S., Negi, S., Rout, D. D., & Solanki, D. R. K. (2023). Sun-Link Web Portal for Management for Sun Transportation. *Int. J. of Aquatic Science*, 14(1), 299-307.

- [18]. Sayyed, T., Kodwani, S., Dodake, K., Adhayage, M., Solanki, R. K., & Bhaladhare, P. R. B. (2023). Intrusion Detection System. *Int. J. of Aquatic Science*, 14(1), 288-298.
- [19]. Gupta, A., Sevak, H., Gupta, H., & Solanki, R. K. (2023). Swiggy Genie Clone Application. *Int. J. of Aquatic Science*, 14(1), 280-287.
- [20]. Khode, K., Buwa, A., Borole, A., Gajbhiye, H., Gadekar, D. A., & Solanki, D. R. K. (2023). Live Stock Market Prediction Model Using Artificial Neural Network. *Int. J. of Aquatic Science*, 14(1), 333-340.
- [21]. Ahire, S., Gorhe, S., Palod, T., Khalkar, A., Chauhan, D., & Solanki, D. K. (2023). First Copy Logo Detection System. *Int. J. of Aquatic Science*, 14(1), 322-332.
- [22]. Aditi Manoj Tambat; Ramkumar Solanki; Pawan R. Bhaladhare. "Sentiment Analysis-Emotion Recognition". *Int. J. of Aquatic Science*, 14, 1, 2023, 381-390.
- [23]. Arunima Jana; Prarthana Yadav; Omkar Desai; Niraj Pawar; Ram Kumar Solanki; Pawan R. Bhaladhare. "Multi-Disciplinary Approach and Efficient Algorithm for Programming Learning Platform Design". *Int. J. of Aquatic Science*, 14, 1, 2023, 404-425.
- [24]. Manish Patil; Hitesh Bhadane; Devdatt Shewale; Mahesh Lahoti; Dr. Anand Singh Rajawat; Ram Kumar Solanki. "Smart Machine Learning Model Early Prediction of Lifestyle Diseases". *Int. J. of Aquatic Science*, 14, 1, 2023, 271-279.