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AN METHOD TO THE EXPLANATION OF THE CLASSIC PROBLEMS IN THE DISCIPLINE OF EXTERNAL GRAPH THEORY

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Abstract

An Method To The Explanation Of The Classic Problems In The Discipline Of External Graph Theory. Both problems are examples of graph theory being applied in the real world. It is generally agreed that Euler's solution to the problem of Bridges, which he worked on in was the first formal piece of Graph Theory ever produced. A relatively recent development in the field of mathematics is known as graph theory. Nevertheless, what does "Graph Theory" actually entail? The first thing that needs to be done to address that problem and to study some of the ways in which Graph Theory might be utilized is to establish some concepts. These are some of the most well-known unsolved problems in the discipline of graph theory,. As a result of our efforts to find answers to these questions, we have uncovered a variety of new results that are both fascinating and relevant. They are certainly fairly challenging, but they are an example of the joyful reality that combinatoriality can often express the problems that they are working on to folks who aren't in their field!

Keywords: Graph Theory, External Graph Theory , Explanation Of The Classic Problems.

Introduction

The majority of the definitions presented here may be found in several textbooks on graph theory and parametrized complexity theory (). You need to make use of them in order to validate the accuracy of the notations that are used in this book. Graphs Assuming a set X, the following is an illustration of how to express the notation [X] 2, which represents all subsets of X that consist of exactly two elements, given that [X] 2 represents all subsets of X that consist of exactly two members. Pairs that are denoted with the notation (V (G), E(G)) are made up of the finite set V (G), which is also referred to as the vertex set of G, and the set E(G), which is a subset of [V (G)]2 and is also referred to as the edge set of G. It is argued that the pair of variables V (G) and E(G) represent the graph G. In the conventional sense, the cardinalities of V (G) and E(G), respectively, are represented by the variables n(G) and m, respectively (G). Simply write uv to indicate that a pair of E values is not sorted in any particular order. It is to everyone's advantage to steer clear of ambiguities like this (G). If two vertices x and y are next to one another and share the same edge, then the two vertices are referred to collectively as the edge's letter symbol, which is e. This is the case if they are adjacent to one another. In addition to this, the phrase "e is incident to x and y" is frequently utilised alongside this concept. The vertices that are physically near to a given vertex make up that vertex's neighbourhood, which is denoted in the notation by the letter NG (x). Because of this, we refer to a vertex as being x's neighbour if it is physically located in close proximity to x. The degree of a vertex, which is represented by the symbol dG in mathematical notation, is a measurement of the number of edges that are located in close proximity to that vertex (x). In order to draw a conclusion as to whether or not the vertices x and y in the network V x, y represent actual twins, we need to answer the question of whether or not they reside in the same neighbourhood. When there is no possibility of misunderstanding, we shall dismiss the possibility that earlier notations made reference to the backdrop graph. This is because there is no opportunity for misunderstanding. The graph H = (V (H), E(H)) is considered to be a subgraph of the graph G = (V (G), E(G)) in the event that the values of V (H) and E(H) are less than those of V (G) (G). It is claimed that a subgraph H of graph G "spans" (or covers) the rest of graph G if and only if V (H) = V. This is the only condition under which this can be said. It is only true if H is a subgraph of G that the equation V (H) = V is correct (G). It is argued that a particular subgraph H of G is induced if, and only if, E(H) = E(G) [V (H)]. 2. The reason for this is that the expression E(H) = E(G) [V (H)]. 2. Due to the fact that E(H) = E(G) [V (H),] this is the situation. Two, the subgraph of G that is denoted by the induced subgraph G is represented by the variable G[X], where X is the vertex set. We use the phrase "this is the case" whenever X is a subset of the vertex set for G. In addition to this, beginning right now, the induced subgraph of G on V (G) X will be shortened as G X. This change will take effect immediately. If F is a subset of G's edge set, then we'll write G F to refer to the subgraph of G that has a different vertex set than F but an edge set that comes from F. This subgraph will have a different vertex set than G's main graph. In contrast to the subgraph F, this one will have its own special collection of vertices in G. To summarise, we will say that two graphs G and H are isomorphic if and only if there exists a bijection f from the vertex set of one graph to the vertex set of the other graph such that, for any pair of vertices x and y in G, xy is equivalent to E(G) if and only if f(x)f(y) is equivalent to E in both graphs. This is the only condition under which isomorphis The fulfilment of this condition is a precondition for the validity of the claim that two graphs are equivalent (H). And a homomorphism from graph G to graph H is a mapping from graph V (G) to graph V (H), in which case, if xy is an edge of G, then f(x)f(y) is likewise an edge of H. This indicates that a homomorphism exists between the two graphs. In this manner, the definition of a homomorphism from group G to group H is shown. This section will focus on multi-graphs, often known as graphs with a multiset serving as its edge set. On the other hand, the graphs that are going to be discussed further on in this document are going to be simple ones.

Charts that are, to put it bluntly, strange. In the next set of definitions, we are going to discuss a number of fascinating subparagraphs. The abbreviation Kn refers to the full graph on n vertices. This graph is the graph on vertex set V (Kn) = "v1,..., vn" and with edge set [V

(Kn)] if Kn's vertex set has every possible edge in some form or another. 2. There is a subset of a graph's vertex set that is referred to as a clique, and when that clique is induced onto the graph G, it produces a graph that is isomorphic to the graph G. The only way to build a graph with this particular structure is to induce the graph G onto itself. The smallest possible graph with n vertices and no edges linking them is called the empty graph on n vertices. This graph has no edges connecting the vertices. Both of these words originate from the same etymological root. However, there is nothing of interest contained in this graph. An unrelated collection of vertices, when induced on the graph G, results in the production of a graph that is isomorphic to the empty graph. This collection of vertices can also be thought of as a subset of edges to a certain extent. The vertex set V (Pk) = "v1,..., vk" and the edge set "vivi+1: I = 1,..., k 1" make up the graph that we refer to as the route graph on k vertices, which is designated by the letter Pk. The points v1 and vk, which both serve as vertices, denote the beginning and end of the collection of points that is denoted by the letter Pk. When referring to a certain graph, the term "path" denotes an isomorphic subgraph of that graph that is called a path. The k-vertex cycle graph, often known as Ck, is the graph that has the vertex set v1,..., vk and the edge set vivi+1, with the equation I = 1,..., k 1 v1vk describing the edge set. This area contains a graph, which is represented by the letter C in its graphical representation. This particular kind of graph can also be referred to by its abbreviation, which is "ck" for "check" graph. When comparing a graph G to a cycle graph, the term "cycle" refers to the subgraph of G that has the same structure as the cycle graph. In this example, the graph G is being compared to the cycle graph. When a cycle has a specific number of vertices, we'll sometimes refer to it as a "k-cycle," which stands for "k-cycle," to emphasise this fact. Kilo is the unit of measurement that is being utilised, hence the letter "k" stands for "kilo." A graph is considered to have hamiltonian behaviour if it allows for the existence of spanning cycles inside its boundaries. It is considered to be chordal if there are no induced cycles with a magnitude greater than three and acyclic if there are no cycles at all. If there are no cycles at all, then it is said to be acyclic. In a nutshell, a matching in a graph is defined as a group of neighbouring edges that, in the context of a pairwise comparison, are entirely unique from one another. These edges are the only ones in the graph that are not connected to any of the other edges. Now that we have these frameworks, we are able to characterise particular characteristics of graphs. If there is a path inside the graph whose ends correspond to every pair of vertices x and y, then we can say that the graph is linked. However, this is the only condition under which we can make such a claim. All that is necessary in order to classify a graph as linked is this single component. The structure of a tree lends itself well to analysis as a unique variety of connected graph in which cycles are absent. In order for a graph to be considered connected, it is required to contain inside its bounds a spanning tree, which is regarded to represent shared knowledge. There is, however, an exception to this general rule that you should be aware of. The set of vertices in a graph is said to have bipartite qualities if it is possible to partition the set of vertices in the graph into two separate sets. When I equals 1,..., p and j equals 1,..., q, the full bipartite graph Kp,q is the graph with the vertex set consisting of v1,..., vp, w1,..., wq, and the edge set consisting of viwj. This occurs when I equals 1,..., p and j equals 1,..., q. To summarise, we say that a graph G is planar if and only if it can be drawn in the plane in such a way that no two adjacent edges

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intersect anywhere along their length. This is the only condition under which it can be considered planar. To boil things down, G is said to have "representation" on the plane.

Methodology

Definitions

One or more points can be referred to as a "vertex," depending on the context. In diagrams, they are almost often presented in the form of spheres.

To put it another way, an edge, which is denoted by the letter e, is formed in a graph by any two vertices that are not connected to one another. Consider the equation e = a + b with me for a moment. In order to save time when typing, vertex names are frequently reduced to ab rather than the more traditional a, b. Edges are generally represented in paintings as lines that run between two vertices of the canvas. Either "ab" or "ba" will do for the relevant edge, as they both mean the same thing in their own unique way.

The two points (a, b) that constitute the boundary of the edge e equal to a and b are referred to as the "endpoints" of the edge e.

If and only if there is a vertex a such that the edge e has two endpoints at a and b, then the edge e is said to have an incident vertex.

A vertex's neighbour vertices are the vertices that are immediately to its left and right as viewed from that vertex's perspective.

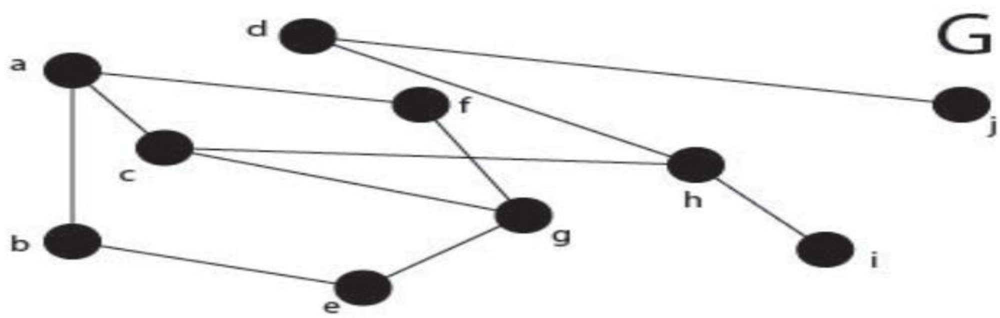
Vertices and edges make up the components of the graph denoted by the notation G = (V, E) (V, E).

• A "walk" is a succession of vertices in which each vertex is adjacent to both the vertex that came before it and the vertex that will come after it in the sequence.

It is essential that no two pathways should touch the same vertices at the same time.

Take the following as an example: • A path is said to be cyclical if it begins and finishes at the same vertex.

The ability to comprehend these definitions is an illustration of how abstract some notions may be. The following graph, G, can assist you in better visualising these principles.



Example 1

1. Please provide a list of the vertices of the graph G V = (a, b, c, d, e, f, g, h, I and j)

2. Compile a list of the edges using the notation G E = "ab, ac, af, be, cg, ch, dj, dh, eg, fg, hi"

3. Make a list of the edges that are incident with the vertex C, including ac, cg, and ch

4. Name the vertices that are neighbouring g, c, e, and f

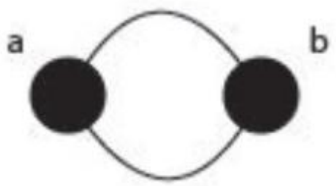
5. Determine how to get from j to b. There are several possible routes; one of them is j, d, h, c, a, and b.

6. Locate a cycle in the letter G There are three cycles in the key of G. They are either the letters a, b, e, g, f, an or the letters a, b, e, g, c, an or the letters a, c, g, f, a. It is important to keep in mind that you can initiate and complete the cycle at any of its vertices.

Rules for Graphs

Graphs might have a broad variety of rules associated with them, depending on the application they're being used in. For instance, graphs can be directed in certain applications such that they only go from point a to point b, but not from point b to point a. (think of it like a one-way street). We are going to concentrate on graphs that adhere to the following guidelines for the purpose of this lesson's graphs:

• Our graphs will not have any duplicate edges. • Our graphs will not have any directed edges (think of it as a street that goes in both directions). This indicates that there will be a single edge running from point A to point B. We will not put up with circumstances such as the following:



• There will never be a case in which the graphs include any self-loops (edges that start and end at the same vertex). You are welcome to have a look at the illustration of a self-loop that is provided below.

Planarity Planarity is a property of a graph that describes its ability to be constructed without any of its edges colliding with one another at any point during the process of making the graph. A graph is said to be non-planar if it cannot be drawn in the form of a planar graph at any point in time. Planar embedding is the name of the representation of the graph that will be used here. It is the representation that we will be employing. In this presentation, there are no edges that link with one another in any way, nor do any of the edges overlap one another. Is the fact that a graph has edges that overlap one another the sole reason why it cannot be considered a planar representation of the data? The fact that this is not the case, despite the fact that it would significantly simplify the job that we perform as mathematicians if it were the case, has unhappily resulted in a setback. This is the case despite the fact that it has produced a setback. It is vital to keep in mind that the only components necessary to define a graph are a set of vertices V and a set of edges E. This is the case regardless of the type of graph being constructed. This is true irrespective of the particular form of graph that is being discussed. When it is possible to construct a graph in which the edges do not cross or interact with one another in any other manner, we refer to the resultant graph as being planar. This is the case when it is possible to construct a graph in which the edges do not interact with one another.

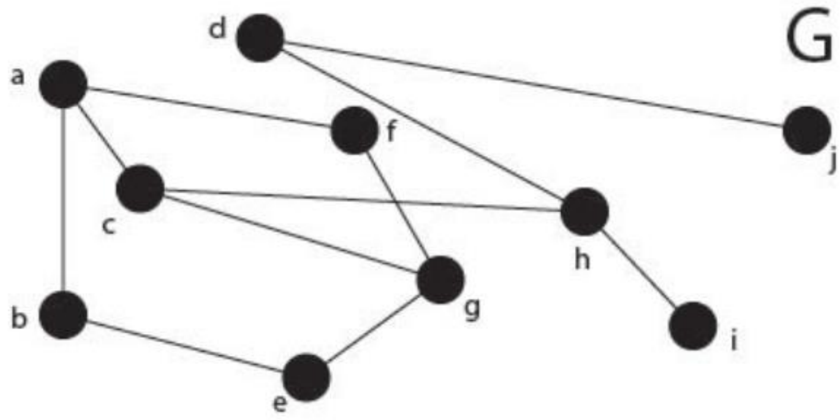
Let's take a look at graph G from earlier, when we were talking about the concepts of graphs, so that we can get a better grasp on what this implies in order to better understand what it requires in order to get a better grasp on what this entails in order to get a better understanding of what this entails in order to get a better grip on what this entails.

It shouldn't come as much of a surprise to anyone that the edges c and h, as well as f and g, intersect with one another. This is true for all four pairs of edges. In consideration of all that has been said up to this point, I would argue that the graph in question is planar. You should be able to recollect the following information about the vertices of the letter G if you carried out the first exercise correctly: V equals a, b, c, d, e, f, g, h, and I j; and E equals a, b, a, c, a, f, b, e, c, g, c, h, d, j, e, g, f, g, h, and I j. Both of these expressions are equivalent to each other. Both of these expressions can be understood in the same way as the other.

Let's have a look at the graph that's been labelled with the letter G0 for the time being: If you look at the vertices and edges of the G0 graph in great detail, what kinds of things do you discover about them?

Both the G0 and the G graphs are similar in every way, with the exception of the amount of vertices and edges that each graph possesses. Since we have established that the only factors that define a graph are its vertices and edges, we are able to reach the conclusion that G and G0 are two different representations of the same graph. This is because we have established that the only factors that define a graph are its vertices and edges. This is due to the fact that we have established that the only components of a graph that are responsible for its definition are its vertices and edges. In addition to this, our picture G0 is able to be categorised as a planar graph because it does not contain any edges that cross each other. This is due to the fact that none of the edges intersect with one another in any way. This is because none of the edges overlap with one another in any way, which is the reason for this property of the shape.

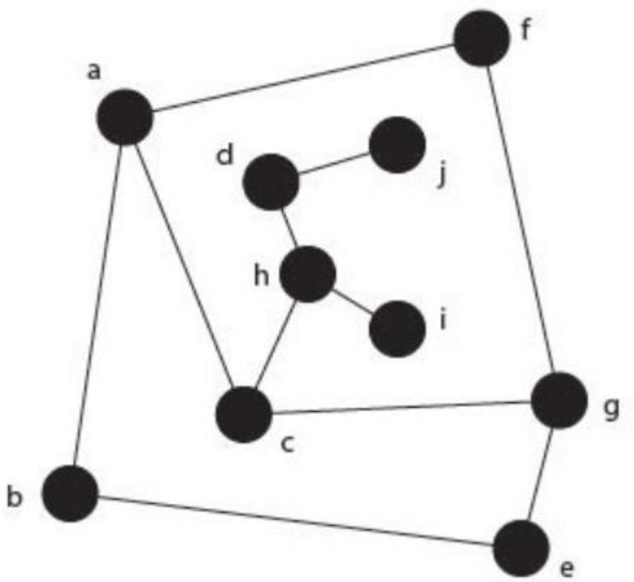
In spite of the fact that our first depiction depicted edges that crossed over one another in a manner that was comparable to that of a network, this demonstrates that G is, in fact, a planar graph. Despite the fact that our first visualisation depicted edges that crossed over one another, this is still the case.



Now, the question that needs to be answered is how we may use this information to figure out whether or not a graph is planar.

The response to that inquiry is that it is possible to traverse the vertices and edges of a graph in a manner that is analogous to walking around the graph until none of the edges cross. This can be done in a number of different ways. This is something that can be done until each and every one of the edges has been travelled. It is possible to do so until all of the edges have been traversed. If a graph is planar, utilising this procedure will result in a successful outcome; but, what will take place in the event that the graph is not planar? Do we not have any other choice but to continuously rearrange the vertices and edges for an unspecified amount of time?

We are not, and there are many different methods that we may analyse a graph in order to determine whether or not it is non-planar. One of these techniques is as follows:



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Conclusion

In contrast, one may use a geometric, a computational, or a technique based on an external graph theory. Algebraic external graph theory is a vast area that includes the study of invariants in external graph theory, applications of linear algebra, and group theory. The study of external graph theory invariants encompasses all three of these subfields. mathematics and theory centred on the perimeter of a graph Linear algebra and external graph theory are the focus of the first subfield of algebraic graph theory. (Citation required) Those who need a citation: The spectrum of the adjacency matrix is analysed; this matrix is commonly thought of as the graph's "external" matrix in graph theory. Graphs are representations of networks that highlight the relationships between nodes. Graph edge theorising The second topic of algebraic graph theory investigates the interplay between graphs and various types of groups, such as automorphism groups and geometric groups. The dissertation addressing the use of graph theory in other contexts falls inside this umbrella. Algebraic characteristics of external graph theory are the subject of study in the third and final subfield of algebraic graph theory. The chromatic polynomial, the Tutte polynomial, and external graph theory invariants are all examples of such characteristics. These properties are crucial in the study of chromatic polynomials. One example is that the chromatic polynomial of a graph can determine the total number of externally correct vertex colorings in the graph. A graph can be used to count this, among many other things. The vast amount of work in this field of algebraic graph theory is a direct result of the efforts made in other areas of graph theory to prove the four colour theorem. Significant progress in the field can be attributed to this effort. However, there are still many open topics in graph theory. A couple of instances of such inquiries include how to discover and characterise graphs that share the same chromatic polynomial.

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