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GENERALISED RAMANUJAN'S DIOPHANTINE EQUATION $x^2 + p = 2^n$

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ABSTRACT

In this paper, generalized Ramanujan's Diophantine equation $x^2 + p = 2^n$ has been obtained for p's having some prime values which are congruent 7 (mod 8). Another form of Ramanujan Problem and Conjecture has also been obtained.

Key words : *Generalised Ramanujan's diophantine equation, Prime number, Congruent.*

Introduction

Famous mathematician Ramanujan (1913) conjectured that the Diophantine equation $x^2 + 7 = 2^n$ has only 5 positive integral solutions given by (x, n) where x is 1, 3, 5, 11, 181 and n is 3, 4, 5, 7, 15 respectively. Since x is odd the given equation can be written as $(2y - 1)^2 = 2^n - 7$, with solutions 1, 2, 3, 6 and 91 for y. This conjecture given by Ramanujan was proved by Nagell.

Mahanthapa, the high school student from Boulder Colorado considered the Diophantine equation $x^3 + 3 = 4^n$. He showed that this equation has only one positive integral solution given by $x = 1$ and $n = 1$. Then we discussed a family of Diophantine equation in general form given by $x^3 + p = 2^n$, where p is an odd integer. He obtained the following results:

(i) For the equation $x^2 + p = 2^n$ to have positive integral solutions, x must be an odd integer and x^2 must be congruent to 1 modulo 8.

(ii) If p is an odd prime greater than and not congruent 7 (mod 8) then the equation has no solution.

(iii)If p is 3 then the equation has a unique solution (1, 2).

Usha Devi [1] discussed the Diophantine equations $x^2 + 19 = 7^n$ and $x^2 + 11 = 3^n$ and showed that these Diophantine equations have integer solutions only for $n = 3$.

In this paper, an attempt has been made to discuss the solution of the. Diophantine equation $x^2 + p = 2^n$ for those values of p which are congruent 7 (mod 8). Another form of Ramanujan Problem has also worked out.

Another form of Ramanujan Problem and Conjecture

The Diophantine equation $x^2 + 7 = 4^n$ has a unique solution given by $x = 3$ and $n = 2$.

Proof : The Ramanujan problem $x^2 + 7 = 2^n$ has a unique solution for an even value of n which is 4 and it can be written as 2^2. Thus if we consider the Diophantine equation $x^2 + 7 = 4^n$, it has unique solution given by $x = 3$ and $n = 2$.

Solution of the Diophantine Equation $x^2 + 23 = 2^n$

Here $p = 23$ is a prime number which is congruent to 7(mod8). Therefore the positive integral solutions of the given equation are possible. If we put $x = 45$, $n = 11$ then the given Diophantine equation is satisfied. Thus (45, 11) is the solution of the equation $x^2 + 23 = 2^n$.

Solution of the Diophantine Equation $x^2 + 31 = 2^n$

Here $p = 31$ is a prime number which is congruent to 7(mod 8). Therefore the positive integral solutions of the given equation are possible. We see that $x = 1$, $n = 5$ satisfy the given Diophantine equation $x^2 + 31 = 2^n$. Thus (1, 5) is the solution of the equation $x^2 + 31 = 2^n$.

We see that $x = 15$, $n = 8$ also satisfy the given Diophantine equation $x^2 + 31 = 2^n$. Thus (15, 8) is also a solution of the equation $x^2 + 31 = 2^n$.

Solution of the Diophantine Equation $x^2 + 47 = 2^n$

Here $p = 47$ is a prime number which is congruent to 7(mod 8). Therefore the positive integral solutions of the given equation are possible. We see that $x = 9$, $n = 7$ satisfy the given Diophantine equation $x^2 + 47 = 2^n$. Thus (9, 7) is the solution of the equation $x^2 + 47 = 2^n$.

Solution of the Diophantine Equation $x^2 + 71 = 2^n$

Here $p = 71$ is a prime number which is congruent to 7(mod 8). Therefore the positive integral solutions of the given equation are possible. We see that $x = 21$, $n = 9$ satisfy the given Diophantine equation $x^2 + 71 = 2^n$. Thus (21, 9) is the solution of the equation $x^2 + 71 = 2n$.

Solution of the Diophantine Equation $x^2 + 79 = 2^n$

Here $p = 79$ is a prime number which is congruent to 7(mod 8). Therefore the positive integral solutions of the given equation are possible. We see that $x = 7$, $n = 7$ satisfy the given Diophantine equation $x^2 + 79 = 2^n$. Thus (7, 7) is the solution of the equation $x^2 + 79 = 2^n$.

Solution of the Diophantine Equation $x^2 + 103 = 2^n$

Here $p = 103$ is a prime number which is congruent to 7(mod 8). Therefore the positive integral solutions of the given equation are possible. We see that $x = 5$, $n = 7$ satisfy the given Diophantine equation $x^2 + 103 = 2n$. Thus (5, 7) is the solution of the equation $x^2 + 103 = 2n$.

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Solution of the Diophantine Equation $x^2 + 127 = 2^n$

Here p = 127 is a prime number which is congruent to 7 (mod 8). Therefore the positive integral solutions of the given equation are possible. We see that x = 1, n = 7 satisfy the given Diophantine equation $x^2 + 127 = 2^n$. Thus (1, 7) is the solution of the equation $x^2 + 127 = 2^n$.

Solution of the Diophantine Equation $x^2 + 151 = 2^n$

Here p = 151 is a prime number which is congruent to 7 (mod 8). Therefore the positive integral solutions of the given equation are possible. We see that x = 19, n = 9 satisfy the given Diophantine equation $x^2 + 151 = 2^n$. Thus (19, 9) is the solution of the equation $x^2 + 151 = 2^n$.

Solution of the Diophantine Equation $x^2 + 199 = 2^n$

Here p = 199 is a prime number which is congruent to 7 (mod 8). Therefore the positive integral solutions of the given equation are possible. We see that x = 43, n = 11 satisfy the given Diophantine equation $x^2 + 199 = 2^n$. Thus (43, 11) is the solution of the equation $x^2 + 199 = 2^n$.

Solution of the Diophantine Equation $x^2 + 223 = 2^n$

Here p = 223 is a prime number which is congruent to 7 (mod 8). Therefore the positive integral solutions of the given equation are possible. We see that x = 17, n = 9 satisfy the given Diophantine equation $x^2 + 223 = 2^n$. Thus (17, 9) is the solution of the equation $x^2 + 223 = 2^n$.

Solution of the Diophantine Equation $x^2 + 271 = 2^n$

Here p = 271 is a prime number which is congruent to 7 (mod 8). Therefore the positive integral solutions of the given equation are possible. We see that x = 89, n = 13 satisfy the given Diophantine equation $x^2 + 271 = 2^n$. Thus (89, 13) is the solution of the equation $x^2 + 271 = 2^n$.

Solution of the Diophantine Equation $x^2 + 367 = 2^n$

Here p = 367 is a prime number which is congruent to 7 (mod 8). Therefore the positive integral solutions of the given equation are possible. We see that x = 41, n = 11 satisfy the given Diophantine equation $x^2 + 367 = 2^n$. Thus (41, 11) is the solution of the equation $x^2 + 367 = 2^n$.

Solution of the Diophantine Equation $x^2 + 431 = 2^n$

Here p = 431 is a prime number which is congruent to 7(mod 8). Therefore the positive integral solutions of the given equation are possible. We see that x = 9, n = 9 satisfy the given Diophantine equation $x^2 + 431 = 2^n$. Thus (9, 9) is the solution of the equation $x^2 + 431 = 22^n$.

Solution of the Diophantine Equation $x^2 + 463 = 2^n$

Here p = 463 is a prime number which is congruent to 7 (mod 8). Therefore the positive integral solutions of the given equation are possible. We see that x = 7, n = 9 satisfy the given Diophantine equation $x^2 + 463 = 2^n$. Thus (7, 9) is the solution of the equation $x^2 + 463 = 2^n$.

Solution of the Diophantine Equation $x^2 + 503 = 2^n$

Here p = 503 is a prime number which is congruent to 7 (mod 8). Therefore the positive integral solutions of the given equation are possible. We see that x = 31, n = 9 satisfy the given Diophantine equation $x^2 + 503 = 2^n$. Thus (3, 9) is the solution of the equation $x^2 + 503 = 2^n$.

Conclusion

In this chapter, the generalized Ramanujan Diophantine equation $x^2 + p = 2^n$ has been solved for p = 23, 31, 47, 71, 79, 103, 127, 151, 199, 223, 271, 367, 431, 463 and 503. This Diophantine equation can further be solved for other values of p. Another form of Ramanujan's Diophantine has also been obtained.

References

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