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# INTERVAL LOAD FLOW SOLUTION FOR ILL-CONDITIONED POWER SYSTEMS

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**ABSTRACT :** *This paper presents a methodology to solve load flow problem, wherein the input data are uncertain due to measurement errors. In order to deal with these here the uncertainties, interval mathematics (IM) tool has been applied to obtain the interval load flow solution. The data arrangement to account for uncertainties, the load and generator data at the buses have been treated as interval numbers and subsequently, the interval arithmetic has been used to compute the solution. This approach arranges the interval input data and compute interval load flow solution using Interval Newton's method. The proposed method also, compares with known probabilistic method based on Monte Carlo simulation for the defined range. This methodology is programmed and successfully demonstrated for defined networks having 11and 13 bus ill conditioned and IEEE 30 well-conditioned bus system data*.

**Keywords:** Uncertainty, Power flow, Interval arithmetic, Interval Newton's method**.**

## 1. INTRODUCTION :

Load flow calculation is one of the fundamental tools for power system operation analysis and planning, by allowing the simulation of the system steady state operation for a specific set of generation and load values. The most common approach to solve the load flow problem is the use of deterministic values for the input variables.Available conventional methodology does not answer the presence of uncertainties in the mathematical modeling of power systems .The consideration of uncertainties in the future system operation is a key aspect in current planning methodologies. There is a need for tools that incorporate uncertainty in some system variables has been widely recognized by researchers focused on system planning.

Common uncertainties can be categorized as environmental, regulatory and technological. The sources of uncertainty happen to be (i) The type of the assumed mathematical model, (ii) Representation of various physical components (iii) Values of the parameters may be error (iv) Introduction of noise at the inputs and (v) Numerical modeling using finite arithmetic. Qualitative and quantitative aspects govern the classification of uncertainties. Qualitative uncertainty is generally expressed verbally like 'near to', 'smaller to' etc., whereas the quantitative uncertainty is quantifiable in numerical terms following the Interval arithmetic. Probabilistic methods, Fuzzy logic and Interval arithmetic are the ways to determine the solutions to any systems with uncertainty.

Conventional methodologies available in the literature propose the use of probabilistic methods for these type of studies. Which accounts for the variability and stochastic nature of the input data. In particularly, Uncertainty propagation studies based on sample-based methods, such as Monte carlo's require several model runs that sample various combinations of inputs values. Since the number of required model runs may be rather large, the needed computation resources for these types of studies could be prohibitively expensive.

Probabilistic methods are useful tool, especially for planning studies. However, as discussed in [1-2],these present various shortcomings due mainly to non-normal probability distribution and the

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statistical dependence of the input data, as well as the problems associated with accurately identifying probability distribution for some input data, as in the case for example of power generated by wind or solar generators. These could lead to complex computation that may limit the use of these methods in practical applications, especially in the study of large networks.

The second family of the load flow algorithms incorporating uncertainty has been developed more recently and utilized fuzzy sets for its for modelling [3-7]. This is a qualitatively different way of expressing uncertainty. It represents impressive, or vargue, knowledge rather than uncertainty related to the frequency of occurrence. One inherent advantage of this approach is the ability to easily incorporate experts knowledge about the system under study. With this approach, inputs variables are represented as fuzzy numbers(FNs), which are special type of fuzzy sets , Although the calculation in fuzzy analysis are somewhat simpler than the in a probabilistic case (convolution is not needed), it is still far too complex to be applied directly to the full system model. Therefore, again a linearized model of the system is used and results obtained are approximate.

In recent years the uncertain variable in the load flows are being represented using Interval numbers. By considering Interval arithmetic the obtained solution for the load flow can be attributed to the every punctual or instant value of the problem with uniform validity. This attractive feature of the Interval arithmetic made many researchers to put their efforts in solving the uncertain electrical power flow problems using Interval arithmetic. The contributions made by Barboza, Zian Wang etc., [8-12] are significant in International literature.

In their contributions Zian Wang and F.L. Alvarado [8] suggested a method for solving the load flow using Interval arithmetic taking the uncertainty at the nodal values. It is stated in their article that the required solution to the of non-linear equations can be obtained by Interval Newton operator, Krawczyk operator or Hansen-Sengupta operator. In their series of research articles [10-13] Barboza and others presented their methodology for solving the uncertain power flow problems. Also in the literature Interval mathematics has been applied to the load flow analysis [10-12] by considered Krawczyk's method to solve the non-linear equations. It is mentioned that the existing problem of excessive conservatism in solving the Interval linear equations could be overcome by Krawczyk's method. In these methods the linearized power flow equations should be preconditioned by an M-matrix in order to guarantee convergence.

In another paper [8] the set of non-linear equations were solved by Gauss-Seidal method. Preconditioning is required and no guarantee of convergence if Interval input is too large, hence this method cannot give exact solution.

In the article [15] Fast Decoupled power flow using Interval arithmetic has been used to obtain the solution to the power flow with uncertainty. Linearization is done by Interval gauss elimination method. In particularly the use of the Interval Gauss elimination in the power flow process leads to realistic solution bounds only for certain special classes of matrices. This solution show excessive conservatism.

These issues are addressed by proposinga methodology to solve load flow problems in which the load data are uncertain due to measurement errors. In order to deal with those uncertainties the application of interval arithmetic is proposed. The proposed algorithm uses interval Newton's method to solve the nonlinear system of equations generated.

## 2. Interval Newton's method Under Uncertainty:

All loads and generator bus data in the electric system are provided by measurement instruments which frequently are inaccurate. Moreover, the specified variables like real power at PVbuses also can be uncertain since their values are obtained via measurement equipment. This uncertainty in the input data can be enlarged due to both rounding and truncating processes that occur in numerical computation. As a consequence the actual error presented in the final results cannot be easily evaluated. In order to rigorously control and automatically handle these numerical errors. A technique of Interval Newton's method for solving power flow equations using Interval Arithmetic for a more reliable load and generator modelling.

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**Interval Newton's Power Flow Proposed Algorithm:**

**Mathematical Modelling:**

$P\,i = |Vi| \sum_{k=1}^{n} |V_k||y_{ik}| \cos(\,\theta ik + \delta\,k - \delta\,i\,)$ i = 1, 2, ….n ----------(1)

$Q\,i = -|Vi| \sum_{k=1}^{n} |V_k||y_{ik}| \sin(\,\theta ik + \delta\,k - \delta\,i\,)$ i = 1, 2, ….n ----------(2)

Conventional Power flow equations (1) and (2) become interval power flow equations as below by introducing uncertainty in the parameters.

$[Pical^{-}, Pical^{+}] = [\widetilde{V}i] \sum_{k=1}^{n} \{[\widetilde{V}k]\,|Yik|\,\mathrm{Cos}\,(\theta ik + \tilde{\delta}k - \tilde{\delta}i)\}$

$i = 1 \ldots\ldots n\,; i \neq slack$ --------- (3)

$[Qical^{-}, Qical^{+}] = [\widetilde{V}i] \sum_{k=1}^{n} \{[\widetilde{V}k]\,|Yik|\,\sin\,(\theta ik + \tilde{\delta}k - \tilde{\delta}i)\}$

$i = 1 \ldots\ldots n\,; i \neq PV\; i \neq slack$ --------- (4)

Where, nis the number of buses

$[\theta ik^{-}, \theta ik^{+}] = [\theta i^{-}, \theta i^{+}] - [\theta k^{-}, \theta k^{+}]$

The interval power flow method essentially means a procedure to find a solution for the following interval power equations i.e., solving (3), (4) for $[Vi^{-}, Vi^{+}]$ (i ∈PQ buses) and $[\delta i^{-}, \delta i^{+}]$ (i ≠slack) for given $[Pi^{-}, Pi^{+}]$ (p ≠slack), $[Qi^{-}, Qi^{+}]$ (i ∈PQ buses) and $[Vi^{-}, Vi^{+}]$ (i ∈PV buses).

$\widetilde{V}_i$ and $\widetilde{\delta}_i$. i.e. $\widetilde{V}i$or $Vio = [Vi^{-} + Vi^{+}]/2$ and $\widetilde{\delta}\,i$ or $\delta i_0 = [\delta i^{-}, +\delta i^{+}]/2$

It must be noted here that equation (3) and (4) differs from the standard load flow equations in polar coordinates. Since the active and reactive power at all the PQ buses and active power and voltage magnitude at all the PV buses are intervals.

**Solving interval power flow equations:**

**Initialization of iterative process:**

The interval Newton's method is run after convergence of deterministic or punctual power flow. Its initialization is carried out based on deterministic or punctual voltage profile and on definition of load variations as follows:

Note : The real and reactive powers are given by Pisp = $Pg_i - Pd_i$and Qisp = $Qg_i - Qd_i$, respectively, where $Pg_i$ and $Qg_i$ are the generated real and reactive powers at bus i, and $Pd_i$ and $Qd_i$are the real and reactive power loads at bus i, respectively.

Let $[Pisp^{-}, Pisp^{+}] = [Pisp(1-e), Pisp(1+e)]$ and

$[Qisp^{-}, Qisp^{+}] = [Qisp(1-e), Qisp(1-e)]$

Where Pisp and Qisp are at respective bus i. Obtained from given bus data for the given test system. e is the error or percentage of uncertainty.In operation of actual power system, the influence of parameter uncertainty of electric lines and transformer factor often small enough to be neglected.

1. Interval voltages are initialized by using the deterministic or punctual voltage profile as midpoint and the largest load data variation factor i.e. uncertain error as radius of interval.

Thus $[Vi^{-}, Vi^{+}] = [Vio(1-e), Vio(1+e)]$ and

$[\delta i^{-}, \delta i^{+}] = [\delta io(1-e), \delta io(1-e)]$

Where Vio and $\delta$io are obtained from deterministic or punctual load flow in order to ensure a good initial condition for convergence of iterative process.

**Calculation of Interval power mismatches:**

2. Compute the elements of the load flow Jacobian matrix (JPolar) at $\widetilde{V}_i$and $\widetilde{\delta}_i$.

3. Calculation of the real (Pical) and reactive power (Qical) at each bus using midpoint value of voltage and phase angle using equation (3) and (4), and checking if MVAR of generator buses are within the limits, otherwise update the voltage magnitude at these buses by ±5 %.

Solve the following equations for $[Vi^{-}, Vi^{+}]$ ,$[\delta i^{-}, \delta i^{+}]$

$$\begin{bmatrix} \Delta P^{-}, \Delta P^{+} \\ \Delta Q^{-}, \Delta Q^{+} \end{bmatrix} = (\mathrm{JPolar}) \begin{bmatrix} [\Delta\delta^{-}, \Delta\delta^{+}] \\ [\Delta V^{-}, \Delta V^{+}] \end{bmatrix} \qquad \text{---------- (5)}$$

Where$\begin{bmatrix} \Delta P^{-}, \Delta P^{+} \\ \Delta Q^{-}, \Delta Q^{+} \end{bmatrix} = \begin{bmatrix} [Pisp^{-} - Pical^{-}, Pisp^{+} - Pical^{+}] \\ [Qisp^{-} - Qical^{-}, Qisp^{+} - Qical^{+}] \end{bmatrix}$

AndJPolaris the standard [51] load flow Jacobian matrix

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$$\text{i.e.}\begin{bmatrix} \frac{\partial P}{\partial \delta} & \cdots & \frac{\partial P}{\partial |V|} \\ \vdots & \ddots & \vdots \\ \frac{\partial Q}{\partial \delta} & \cdots & \frac{\partial Q}{\partial |V|} \end{bmatrix} \text{or} \begin{bmatrix} H & \cdots & N \\ \vdots & \ddots & \vdots \\ J & \cdots & L \end{bmatrix}$$

where, $\frac{\partial P}{\partial \delta}, \frac{\partial P}{\partial |V|}, \frac{\partial Q}{\partial \delta}, \frac{\partial Q}{\partial |V|}$ are the elements of the standard load flow Jacobian matrix in polar coordinates [38,39] and these elements are calculated at $\tilde{V}_i$ and $\tilde{\delta}_i$.

$$\frac{\partial Pi}{\partial \delta k} = \sum_{\substack{k=1 \\ K \neq i}}^{n} \{|\tilde{V}i||\tilde{V}k||Yik|\sin(\theta ik + \tilde{\delta}k - \tilde{\delta}i)\}$$

$$\frac{\partial Pi}{\partial |Vk|} = 2|\tilde{V}i||Yii|\cos(\theta ii) + \sum_{\substack{k=1 \\ K \neq i}}^{n} \{|\tilde{V}k|\,|Yik|\cos(\theta ik + \tilde{\delta}k - \tilde{\delta}i)\}$$

$$\frac{\partial Qi}{\partial \delta k} = -\sum_{\substack{k=1 \\ K \neq i}}^{n} \{|\tilde{V}i||\tilde{V}k|\,|Yik|\cos(\theta ik + \tilde{\delta}k - \tilde{\delta}i)\}$$

$$\frac{\partial Qi}{\partial |Vk|} = 2|\tilde{V}i||Yii|\sin(\theta ii) + \sum_{\substack{k=1 \\ K \neq i}}^{n} \{|\tilde{V}k|\,|Yik|\sin(\theta ik + \tilde{\delta}k - \tilde{\delta}i)\}$$

In order to solve the above equation (5) i.e. Linear equation can solved as to give Newton's operator [40] refers to the interval Newton's method.
Mismatch in voltage and voltage angle can be find out as
It result iteration in

$$\begin{bmatrix} [\Delta\delta^{(k)}] \\ [\Delta V^{(k)}] \end{bmatrix} = -J^{(k)} \backslash \begin{bmatrix} [\Delta P^{(k)}] \\ [\Delta Q^{(k)}] \end{bmatrix}$$

$$\begin{bmatrix} [\Delta\delta^{(k)^-}, \Delta\delta^{(k)^+}] \\ [\Delta V^{(k)^-}, \Delta V^{(k)^+}] \end{bmatrix} = -J^{(k)} \backslash \begin{bmatrix} [\Delta P^{(k)^-}, \Delta P^{(k)^+}] \\ [\Delta Q^{(k)^-}, \Delta Q^{(k)^+}] \end{bmatrix}$$

$$N(\tilde{\delta}^{(k)}, \delta^{(k)}) = \tilde{\delta}^{(k)} + \Delta\delta^{(k)}$$

$$\delta^{(k)+1)} = \delta^{(k)} \cap N(\tilde{\delta}^{(k)}, \delta^{(k)}) \qquad \text{---------} (6)$$

$$N(\tilde{V}^{(k)}, V^{(k)}) = \tilde{V}^{(k)} + \Delta V^{(k)}$$

$$V^{(k)+1)} = V^{(k)} \cap N(\tilde{V}^{(k)}, V^{(k)}) \qquad \text{---------} (7)$$

Where, N (.) is Newton operator. It must be noted here that interval addition and multiplication is quite different from the usual addition and multiplication[41].

4. After calculating the Newton's operator, a new interval voltage solution is obtained as$[Vi^-, Vi^+] = [\tilde{V}i + \Delta V^-, \tilde{V}i + \Delta V^+]$and$[\delta i^-, \delta i^+] = [\tilde{\delta} + \Delta\delta^-, \tilde{\delta} + \Delta\delta^+]$.

5. To check convergence of the proposed method, the difference between radii at iteration (k + 1) and radii at iteration (k) is calculated. If the difference is greater than a specified tolerance, denoted by $\varepsilon$, then interval Newton's method must be employed to calculate new interval voltages. Otherwise, the iterative process is stopped.

## 3.System studies:

A power flow program has been written to implement the ideas of interval Newton's method for finding the well-conditioned power system load flow solutions. Extensive numerical simulations have been carried out on the 30 IEEE bus well-conditioned systems and 11 IEEE bus and 13 IEEE ill conditioned systems , with a convergence accuracy of $10^{-3}$ on a MVA base of 100 or equivalently $10^{-1}$ MVA for both power residuals $\Delta P$ and $\Delta Q$.

## 4.Simulation Results :

**Two types of tests were performed:**

**Case 1.**We consider the uncertainty of load and generator measurement error of on voltage profile at all the buses and compare proposed IA method with the punctual mid value. Studied with different degree of uncertainty i.e. 10% and 20%.

**Case 2.**The analysis of the behaviour of the voltage profile under load and generator is compared with a known probabilistic method based on Monte Carlo simulation. Uncertainty of 10% in the load and generator value of active and reactive power data was considered

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Case 1: IEEE – 30 bus system simulation results:

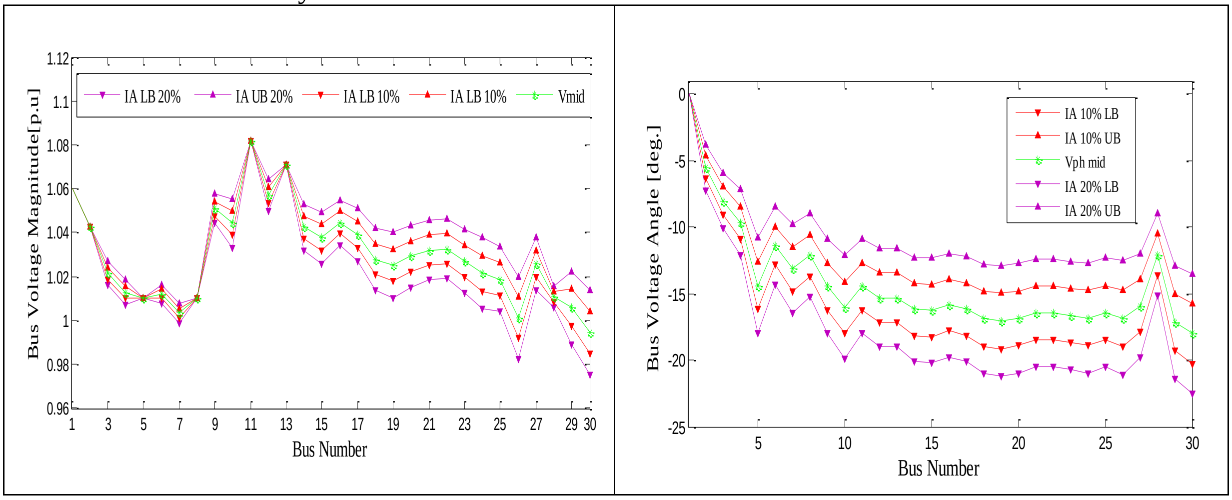


Fig .1. Bus voltage magnitude and voltage angle of Punctual mid value in comparison with Interval for IEEE 30 bus system

Case 2: IEEE – 30 bus system simulation results:

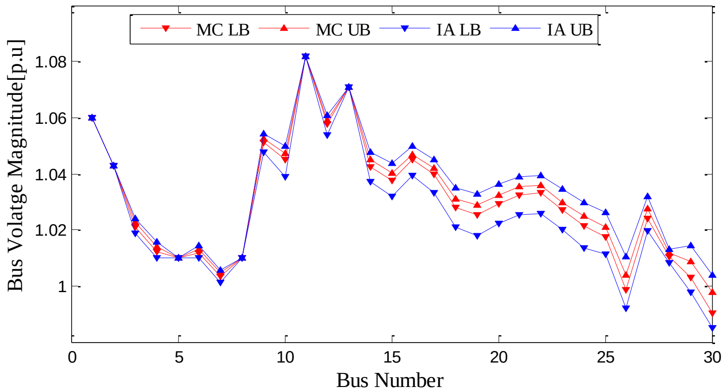


Fig .2.Bus voltage magnitude of Monte Carlo in comparison with Interval for IEEE 30 bus system

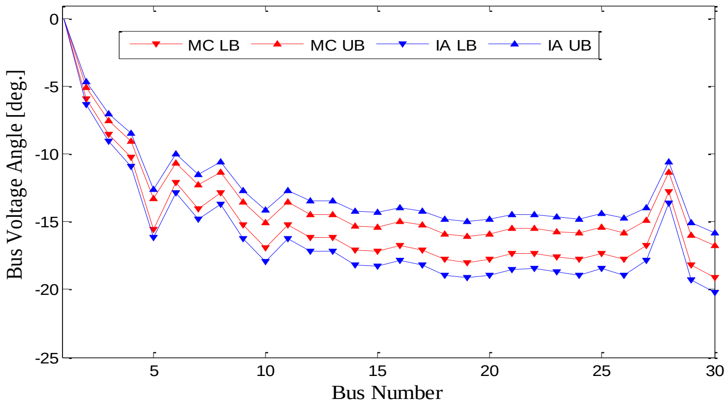


Fig.3.Bus voltage angle of Monte Carlo in comparison with Interval for IEEE 30 bus system

**4.1 ILL conditioned systems study:**

In the load flow calculations, we sometimes encounter the same kind of problem, when dea1ing with a 1inearized load flow model.

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This model has the fo1lowing form:

$$J \Delta x = F \qquad \text{------------ (8)}$$

Where J is the load flow Jacobian matrix, Δx is the bus voltage state correction vector,
F is the power mismatch function vector.
In some power systems, the load flow Jacobian matrix J in equation (5.8) is very ill-conditioned. This causes the instability and or divergence of the load flow solutions. We define this kind of problem as an ill-conditioned load flow problem.
The features which cause the instability and/or divergence in the power systems load flow calculations are the following:
1. Bad choice of the slack bus
2. Large number of radial lines
3. Heavily loaded network.
4. Existence of negative line reactance
5. Lines with high *RIX* ratios
6. Atypical circuit parameters

The analysis of the behaviour of the voltage profile under load and generator is compared with a known probabilistic method based on Monte Carlo simulation. Uncertainty of 10% in the load and generator value of active and reactive power data was considered.

**IEEE-11 ILL buses system:**

Table 1: Interval complex voltages and Monte Carlo simulation - IEEE-11 ILL bus system

| | | **Punctual** | | **Interval(I.A)** | | **Monte Carlo simulation(M.C)** | |
|---|---|---|---|---|---|---|---|
| **Bus** | **Type** | **Voltage (p.u)** | **Phase angle (°)** | **Voltage (p.u)** | **Phase angle(°)** | **Voltage (p.u)** | **Phase angle (°)** |
| 1 | Slack | 1.024 | 0 | [1.0240, 1.0241] | [ 0.0000, 0.0000] | [1.0240, 1.0240] | [0, 0] |
| 2 | PV | 1.01 | -0.232 | [ 1.0100, 1.0101] | [ -0.2574, -0.2084] | [1.0100, 1.0100] | [-0.2492, -0.2168] |
| 3 | PQ | 1.0088 | -0.437 | [ 1.0085, 1.0090] | [ -0.4825, -0.3916] | [1.0088, 1.0088] | [ -0.4641, -0.4085] |
| 4 | PV | 1.01 | -0.385 | [ 1.0100, 1.0101] | [ -0.4272, -0.3443] | [1.0100, 1.0100] | [ -0.4143, -0.3581] |
| 5 | PQ | 1.0071 | -0.534 | [ 1.0068, 1.0075] | [ -0.5933, -0.4753] | [1.0070, 1.0072] | [-0.5741, -0.4951] |
| 6 | PQ | 1.0099 | -0.394 | [ 1.0098, 1.0099] | [ -0.4370, -0.3523] | [1.0099, 1.0099] | [-0.4235, -0.3670] |
| 7 | PV | 0.99 | -1.053 | [ 0.9900, 0.9901] | [ -1.2683, -0.8379] | [0.9900, 0.9900] | [-1.2382, -0.8631] |
| 8 | PV | 0.99 | -3.352 | [ 0.9900, 0.9901] | [ -3.8149, -2.8900] | [0.9900, 0.9900] | [-3.7610, -2.9313] |
| 9 | PQ | 0.9892 | -3.457 | [ 0.9890, 0.9893] | [ -3.9316, -2.9841] | [0.9891, 0.9892] | [-3.8729, -3.0265] |
| 10 | PV | 0.99 | -4.151 | [ 0.9900, 0.9901] | [ -4.6938, -3.6082] | [0.9900, 0.9900] | [-4.6399, -3.6545] |
| 11 | PQ | 0.988 | -4.38 | [ 0.9878, 0.9883] | [ -4.9551, -3.8191] | [0.9879, 0.9880] | [-4.8999, -3.8671] |

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**IEEE-13 ILL buses system:**

Table 2: Interval complex voltages and Monte Carlo simulation - IEEE-13 ILL bus system

| | | Punctual | | Interval(I.A) | | Monte Carlo simulation(M.C) | |
|---|---|---|---|---|---|---|---|
| Bus | Type | Voltage(p.u) | Phase angle(°) | Voltage (p.u) | Phase angle(°) | Voltage (p.u) | Phase angle(°) |
| 1 | 1 | 1 | 0 | [ 1.0000, 1.0000] | [ 0.0000, 0.0000] | [1.0000 1.0000] | 0 0 |
| 2 | 0 | 1.006 | 1.574 | [ 1.0061, 1.0076] | [ 1.3644, 1.7836] | [1.0063 1.0074] | [1.4183, 1.7370] |
| 3 | 0 | 1.038 | 2.5211 | [ 1.0379, 1.0391] | [ 2.1574, 2.8848] | [1.0380, 1.0389] | [2.2336, 2.8070] |
| 4 | 0 | 1.021 | 2.5674 | [ 1.0214, 1.0219] | [ 2.2046, 2.9302] | [1.0215, 1.0218] | [2.2807, 2.8526] |
| 5 | 2 | 1 | 2.6188 | [ 1.0000, 1.0000] | [ 2.2564, 2.9811] | [1.0000, 1.0000] | [2.3324, 2.9036] |
| 6 | 2 | 1.037 | 9.8607 | [ 1.0370, 1.0371] | [ 8.5352, 11.1861] | [1.0370, 1.0370] | [8.8759, 10.8918] |
| 7 | 0 | 1.063 | 9.0877 | [ 1.0626, 1.0633] | [ 7.8243, 10.3510] | [1.0628, 1.0632] | [8.1247, 10.0747] |
| 8 | 2 | 1.1 | 8.2385 | [ 1.1000, 1.1001] | [ 7.0562, 9.4209] | [1.1000, 1.1000] | [7.3041, 9.1680] |
| 9 | 2 | 0.943 | 14.3711 | [ 0.9430, 0.9431] | [ 12.4906, 16.2515] | [0.9430, 0.9430] | [12.8023, 15.9020] |
| 10 | 2 | 1.1 | 8.3575 | [ 1.1000, 1.1001] | [ 7.0365, 9.6785] | [1.1000, 1.1000] | [7.3085, 9.3491] |
| 11 | 0 | 1.051 | 12.0082 | [ 1.0409, 1.0611] | [ 10.2839, 13.7325] | [1.0464, 1.0577] | [10.5942, 13.3704] |
| 12 | 0 | 1.084 | 8.2217 | [ 1.0840, 1.0851] | [ 7.0160, 9.4273] | [1.0841, 1.0849] | [7.2704, 9.1616] |
| 13 | 0 | 1.084 | 5.4418 | [ 1.0730, 1.0951] | [ 4.6557, 6.2278] | [1.0763, 1.0893] | 4.576 ,6.0451] |

## 5.CONCLUSIONS:

Interval mathematics approach can be easily applied to deal with uncertain input data for power flow problems in an efficient manner. On extensive study an idea of the Interval Newton's method is being proposed and tested on standard IEEE –11, 13, 30 ill and well-conditioned bus systems. The proposed method has been validated against Monte Carlo simulation.Interval methods have proven computationally superior to Monte Carlo simulations. Moreover, in the initial stages of planning load flow studies, the proposedmethod is found to be useful tool save on time, effort and resources required.

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