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Review Article

SOME FIXED POINT RESULTS OF C∗ - ALGEBRA VALUED FUZZY SOFT METRIC SPACES WITH APPLICATIONS

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Abstract

In this paper, we establish unique common coupled fixed-point results for two pairs of ω-compatible mappings satisfying different contractive conditions in C ∗-algebra-valued fuzzy soft metric spaces. We also furnish an example to support our obtained results. Moreover, the paper provides an application to prove the existence and uniqueness of a solution for a non-linear integral equation.

Keywords—ω-compatible, C*-algebra-valued fuzzy soft metric, coincidence point, coupled fixed point.

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INTRODUCTION

In recent times the study of fixed point theory has been gained an important role in nonlinear analysis because of its wide applications in integral, integro-differential and impulsive differential equations.The fundamental work in fixed point theory is due to Banach Contraction Principle [1]. Thangaraj Beaula et. al [2] initiated the notion of fuzzy soft metric space in terms of fuzzy soft points and proved some results. On the other hand Thangaraj Beaula, R. Raja proved few results on complete fuzzy soft metric spaces ([3]).

The notion of C - algebra valued metric space introduced by Ma et.al [4] and they proved some fixed and coupled fixed point results. This line of research was continued in (see [5]-[8]). Recently, we introduced the concept of C∗- algebra valued fuzzy soft metric space and proved the convergence properties, completeness and some fixed point results in a C∗-algebra valued fuzzy soft metric space, see ([9],[10]).

The aim of this paper is to present some common coupled fixed point results for two pair of ω-compatible mappings satisfying generalized contractive conditions in C∗- algebra valued fuzzy soft metric spaces.

PRELIMINARIES

Lets recall some basic definitions and notations in this section. In the entire analysis, U refers to an initial universe, κ the set of all parameters for $\mathcal{U}$, R(C)∗ is set of non-negative fuzzy soft real numbers and A˜ refer to C∗-algebra. The particulars on C*-algebras are available in [11]. An element $\tilde{x}\epsilon\tilde{A}$is called a positive element if $\tilde{x} \in \tilde{x}^*$, we write it as $\tilde{x} \geq \tilde{0}_{\tilde{A}}$. one can define partial ordering on $\tilde{A}$ as follows; $\tilde{x} \preccurlyeq \tilde{\mathcal{Y}}$ if and only if $\tilde{0}_{\tilde{A}} \preccurlyeq \tilde{\mathcal{Y}} - \tilde{x}\preccurlyeq$. From now on,$\tilde{A}_+ = \{\tilde{x} \in \tilde{A} : \tilde{0}_{\tilde{A}} \preccurlyeq \tilde{x}$ and $\tilde{A}^{|} = \{\tilde{x} \in \tilde{A} : \tilde{x}\tilde{y} \preccurlyeq \tilde{y}\tilde{x}$.

Definition 1.1: ([9]) Let C ⊆κ and $\tilde{\kappa}$ = F_κ(κ) = $\tilde{1}$ for all κ ∈ K be the absolute fuzzy soft set. Let $\tilde{A}$ denote theC∗-algebra. Suppose that $\tilde{d}_{c^*}: \tilde{\kappa} \times \tilde{\kappa} \to \tilde{A}$ be a mapping satisfying the following conditions.

(S1) $\tilde{0}_{\tilde{A}}\preccurlyeq\tilde{d}_{c^*}(G_{\kappa_1}, G_{\kappa_2})$ for all $G_{\kappa_1}, G_{\kappa_2} \in \tilde{\kappa}$.

(S2) $\tilde{d}_{c^*}(G_{\kappa_1}, G_{\kappa_2}) = \tilde{0}_{\tilde{A}} \Leftrightarrow G_{\kappa_1} = G_{\kappa_2}$

(S3) $\tilde{d}_{c^*}(G_{\kappa_1}, G_{\kappa_2}) = \tilde{d}_{c^*}(G_{\kappa_2}, G_{\kappa_1})$

(S4) $\tilde{d}_{c^*}(G_{\kappa_1}, G_{\kappa_3}) \preccurlyeq \tilde{d}_{c^*}(G_{\kappa_1}, G_{\kappa_2}) + \tilde{d}_{c^*}(G_{\kappa_1}, G_{\kappa_2})$ $\forall G_{\kappa_1}, G_{\kappa_2}, G_{\kappa_3} \in \tilde{\kappa}$.

Then the mapping $\tilde{d}_{c^*}$ is called a C∗-algebra valued fuzzy soft metric on $\tilde{\kappa}$ and the triple $(\tilde{\kappa}, \tilde{A}, \tilde{d}_{c^*})$ is called the C∗- algebra valued fuzzy soft metric space.

Definition 1.2: ([9]) Let $(\tilde{\kappa}, \tilde{A}, \tilde{d}_{c^*})$ be a C∗-algebra valued fuzzy soft metric space. A sequence $\{G_{\kappa_n}\}$ is said to converges to $G_{\kappa^|}$ in $\tilde{\kappa}$with respect to $\tilde{A}$. If $\left\|\tilde{d}_{c^*}(G_{\kappa_n}, G_{\kappa^|})\right\|_{\tilde{A}} \to \tilde{0}_{\tilde{A}}$ as n → ∞ that is for every $\tilde{0}_{\tilde{A}} \prec \tilde{\epsilon}$, there exists $\tilde{0}_{\tilde{A}} \prec \tilde{\delta}$ and a positive integer N = N($\tilde{\epsilon}$) such that $\left\|\tilde{d}_{c^*}(G_{\kappa_n}, G_{\kappa^|})\right\| \to \tilde{\delta}$ implies $\left\|\mu^t_{G_{\kappa_n}}(r) - \mu^t_{G_{\kappa^|}}(r)\right\| < \tilde{\epsilon}$

whenever $n \geq N$. It is denoted as $\lim_{n\to\infty} G_{\kappa_n} = G_{\kappa^|}$.

Definition 1.3: ([9]) Let $(\tilde{\kappa}, \tilde{A}, \tilde{d}_{c^*})$ be a C∗-algebra valued fuzzy soft metric space. A sequence $\{G_{\kappa_n}\}$ is said to Cuachy sequence if to every $\tilde{0}_{\tilde{A}} \prec \tilde{\epsilon}$, there exists $\tilde{0}_{\tilde{A}} \prec \tilde{\delta}$ and a positive integer N = N($\tilde{\epsilon}$) such that $\left\|\tilde{d}_{c^*}(G_{\kappa_n}, G_{\kappa^|})\right\| \to \tilde{\delta}$ implies $\left\|\mu^t_{G_{\kappa_n}}(r) - \mu^t_{G_{\kappa_m}}(r)\right\| < \tilde{\epsilon}$ whenever $n \geq N$.

That is $\left\|\tilde{d}_{c^*}(G_{\kappa_n}, G_{\kappa_m})\right\|_{\tilde{A}} \to \tilde{0}_{\tilde{A}}$ as n, m → ∞

Definition 1.4: ([9]) A C∗-algebra valued fuzzy soft metric space $(\tilde{\kappa}, \tilde{A}, \tilde{d}_{c^*})$ is said to be complete, if every Cauchysequence in $\tilde{\kappa}$ converges to some fuzzy soft point of $\tilde{\kappa}$.

Lemma 1.5: ([9]) Let $\tilde{A}$ be a C*-algebra with the identity element $\tilde{I}_{\tilde{A}}$ and $\tilde{\theta}$ be a positive element of $\tilde{A}$. If $\tilde{x} \in \tilde{A}$ is such that $\|\tilde{x}\| < 1$ then for m < n, we have

$$\lim_{n\to\infty} \sum_{k=m}^{n} (\tilde{x}^*)^k \tilde{\theta} (\tilde{x})^k = \tilde{I}_{\tilde{A}} \left\| \widetilde{(\theta)}^{\frac{1}{2}} \right\|^2 \left(\frac{\|\tilde{x}\|^m}{1-\|\tilde{x}\|} \right) \quad (1)$$

$$\sum_{k=m}^{n} (\tilde{x}^*)^k \tilde{\theta} (\tilde{x})^k \to \tilde{0}_{\tilde{A}} \text{ as } m \to \infty \quad (2)$$

Lemma 1.6: ([9]) Suppose that $\tilde{A}$ be a C*-algebra with unit $\tilde{1}$.

(i) If $\tilde{x} \in \tilde{A}_+$ with $\|\tilde{x}\| < \frac{1}{2}$ then $\tilde{I} - \tilde{x}$ is invertible and $\|\tilde{x}(\tilde{I} - \tilde{x})^{-1}\| < 1$

(ii) Suppose that, $\tilde{x}, \tilde{y} \in \tilde{A}$ with $\tilde{x}, \tilde{y} \succcurlyeq \tilde{0}_{\tilde{A}}$ and $\tilde{x}\tilde{y} = \tilde{y}\tilde{x} then \tilde{x}, \tilde{y} \succcurlyeq \tilde{0}_{\tilde{A}}$

(iii) $\tilde{A}^{|}$ we denote the set $\{\tilde{x} \in \tilde{A} / \tilde{x}\tilde{y} = \tilde{y}\tilde{x}\ \forall\ \tilde{y} \in \tilde{A}\}$. Let $\tilde{x} \in \tilde{A}^{|}$, if $\tilde{y}, \tilde{z} \in \tilde{A}$ with $\tilde{y} \succcurlyeq \tilde{z} \succcurlyeq \tilde{0}_{\tilde{A}}$ and $\tilde{I} - \tilde{x} \in \tilde{A}_+$ is an invertible operator then $(\tilde{I} - \tilde{x})^{-1}\tilde{y} \succcurlyeq (\tilde{I} - \tilde{x})^{-1}\tilde{z}$.

Notices that in C∗-algebra , if $\tilde{0}_{\tilde{A}} \preccurlyeq \tilde{x}, \tilde{y}$, one can't conclude that$\tilde{0}_{\tilde{A}} \preccurlyeq \tilde{x}\tilde{y}$. Indeed, consider the C∗-algebra M2(R(C)∗) and

$$\tilde{x} = \begin{bmatrix} G_{\kappa_1}(p) & G_{\kappa_2}(p) \\ G_{\kappa_2}(r) & G_{\kappa_1}(q) \end{bmatrix} = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$$

and $\tilde{y} = \begin{bmatrix} G_{\kappa_1}(r) & G_{\kappa_2}(r) \\ G_{\kappa_2}(c) & G_{\kappa_1}(s) \end{bmatrix} = \begin{bmatrix} 0.4 & 0.5 \\ 0.5 & 0.6 \end{bmatrix}$

then clearly $\tilde{x} \succcurlyeq \tilde{0}$ and $\tilde{y} \succcurlyeq \tilde{0}$ but $\tilde{x}, \tilde{y} \in$ M2(R(C)∗)+ while $\tilde{x}\tilde{y}$ is not.

MAIN RESULTS

In this section we shall prove some coupled fixed point theorems in C∗-algebra valued fuzzy soft metric spaces byusing different contractive conditions.

Definition 2.1:$(\tilde{\kappa}, \tilde{A}, \tilde{d}_{c^*})$ be a C*-algebra valued fuzzy soft metric space. Let S : $\tilde{\kappa} \times \tilde{\kappa} \to \tilde{\kappa}$ be a mapping, an element $(F_{\kappa_1}, G_{\kappa_1}) \in \tilde{\kappa} \times \tilde{\kappa}$ is called coupled fixed point of S if S $(F_{\kappa_1}, G_{\kappa_1}) = F_{\kappa_1}$ and S$(F_{\kappa_1}, G_{\kappa_1}) = G_{\kappa_1}$

Definition 2.2:$\tilde{\kappa}$ be absolute fuzzy soft set. An element $(F_{\kappa_1}, G_{\kappa_1}) \in \tilde{\kappa} \times \tilde{\kappa}$ is called

(i) a coupled coincidence point of mappings S : $\tilde{\kappa} \times \tilde{\kappa} \to \tilde{\kappa}$ and f:$\tilde{\kappa} \to \tilde{\kappa}$ if
$fF_{\kappa_1} = S\ (F_{\kappa_1}, G_{\kappa_1})$ and $fG_{\kappa_1} = S\ (G_{\kappa_1}, F_{\kappa_1})$

(ii) a common coupled fixed point of mappings S : $\tilde{\kappa} \times \tilde{\kappa} \to \tilde{\kappa}$ and f:$\tilde{\kappa} \to \tilde{\kappa}$ if
$F_{\kappa_1} = fF_{\kappa_1} = S\ (F_{\kappa_1}, G_{\kappa_1})$
and $G_{\kappa_1} = fG_{\kappa_1} = S\ (G_{\kappa_1}, F_{\kappa_1})$

Definition 2.3: Let $\tilde{\kappa}$ be absolute fuzzy soft set and S : $\tilde{\kappa} \times \tilde{\kappa} \to \tilde{\kappa}$ and f:$\tilde{\kappa} \to \tilde{\kappa}$. Then {S, f} is said to be ω-compatible pairs if $f\left(S(F_{\kappa_1}, G_{\kappa_1})\right) = S(fF_{\kappa_1}, fG_{\kappa_1})$ and $f\left(S(G_{\kappa_1}, F_{\kappa_1})\right) = S(fG_{\kappa_1}, fF_{\kappa_1})$

Definition 2.4: Let $(\tilde{\kappa}, \tilde{A}, \tilde{d}_{c^*})$ be a C*-algebra valued fuzzy soft metric space. Suppose that S,T : $\tilde{\kappa} \times \tilde{\kappa} \to \tilde{\kappa}$ and f,g:$\tilde{\kappa} \to \tilde{\kappa}$ be satisfying

(2.4.1) $S(\tilde{\kappa} \times \tilde{\kappa}) \subseteq g(\tilde{\kappa})$ and $T(\tilde{\kappa} \times \tilde{\kappa}) \subseteq f(\tilde{\kappa})$;

(2.4.2) {S, f} and {T, g} are ω-compatible pairs;

(2.4.3) one of $f(\tilde{\kappa})$ or $g(\tilde{\kappa})$ is complete C*-algebra valued fuzzy soft metrics of $\tilde{\kappa}$;

(2.4.4) $\tilde{d}_{c^*}\left(S(F_{\kappa_1}, G_{\kappa_1}), T(F_{\kappa_2}, G_{\kappa_2})\right) \preccurlyeq$

$$\tilde{a}^* \max \left\{ \begin{array}{c} \tilde{d}_{c^*}(fF_{\kappa_1}, gF_{\kappa_2}), \tilde{d}_{c^*}(fG_{\kappa_1}, gG_{\kappa_2}), \\ \tilde{d}_{c^*}\left(fF_{\kappa_1}, S(F_{\kappa_1}, G_{\kappa_1})\right), \tilde{d}_{c^*}\left(gF_{\kappa_2}, T(F_{\kappa_2}, G_{\kappa_2})\right), \\ \tilde{d}_{c^*}\left(fG_{\kappa_1}, S(G_{\kappa_1}, F_{\kappa_1})\right), \tilde{d}_{c^*}\left(gG_{\kappa_2}, T(G_{\kappa_2}, F_{\kappa_2})\right) \end{array} \right\} \tilde{a}$$

$\forall\ F_{\kappa_1}, G_{\kappa_1}, F_{\kappa_2}, G_{\kappa_2} \in \tilde{\kappa}$ where $\tilde{a} \to \tilde{A}$ with $\|\sqrt{2}\tilde{a}\| < 1$. Then S, T, f and g have unique common coupled fixed point in $\tilde{\kappa} \times \tilde{\kappa}$.

Proof: Let $F_{\kappa_0}, G_{\kappa_0} \in \tilde{\kappa}$. From (2.4.1) we can construct the sequences $\{F_{\kappa_{2n}}\}_{2n=1}^{\infty}, \{G_{\kappa_{2n}}\}_{2n=1}^{\infty}, \{I_{\kappa_{2n}}\}_{2n=1}^{\infty} and \{J_{\kappa_{2n}}\}_{2n=1}^{\infty}$such that

$$S(F_{\kappa_{2n}}, G_{\kappa_{2n}}) = gF_{\kappa_{2n+1}} = I_{\kappa_{2n}}$$

$$T(F_{\kappa_{2n+1}}, G_{\kappa_{2n+1}}) = fF_{\kappa_{2n+2}} = I_{\kappa_{2n+1}}$$

$$S(G_{\kappa_{2n}}, F_{\kappa_{2n}}) = gG_{\kappa_{2n+1}} = J_{\kappa_{2n}}$$

$$T(G_{\kappa_{2n+1}}, F_{\kappa_{2n+1}}) = fG_{\kappa_{2n+2}} = J_{\kappa_{2n+1}}$$

for n = 0, 1, 2, · · ·

Notices that in C∗-algebra, if $\tilde{a}, \tilde{b} \in \tilde{A} and \tilde{a} \preccurlyeq \tilde{b}$, then for any $\tilde{x} \in \tilde{A}_+$ both $\tilde{x}^*\tilde{a}\tilde{x} and \tilde{x}^*\tilde{b}\tilde{x}$ are positive elements and $\tilde{x}^*\tilde{a}\tilde{x} \preccurlyeq \tilde{x}^*\tilde{b}\tilde{x}$. From (2.4.4), we get

$$\tilde{d}_{c^*}(I_{\kappa_{2n+1}}, I_{\kappa_{2n+2}}) = \tilde{d}_{c^*}\left(S(F_{\kappa_{2n+1}}, G_{\kappa_{2n+1}}), T(F_{\kappa_{2n+1}}, G_{\kappa_{2n+2}})\right)$$

$$\preccurlyeq \tilde{a}^* \max \left\{ \begin{array}{c} \tilde{d}_{c^*}(fF_{\kappa_{2n+1}}, gF_{\kappa_{2n+2}}), \\ \tilde{d}_{c^*}(fG_{\kappa_{2n+1}}, gG_{\kappa_{2n+2}}), \\ \tilde{d}_{c^*}\left(fF_{\kappa_{2n+1}}, S(F_{\kappa_{2n+1}}, G_{\kappa_{2n+1}})\right), \\ \tilde{d}_{c^*}\left(gF_{\kappa_{2n+2}}, T(F_{\kappa_{2n+2}}, G_{\kappa_{2n+2}})\right), \\ \tilde{d}_{c^*}\left(fG_{\kappa_{2n+1}}, S(G_{\kappa_{2n+1}}, F_{\kappa_{2n+1}})\right), \\ \tilde{d}_{c^*}\left(gG_{\kappa_{2n+2}}, T(G_{\kappa_{2n+2}}, F_{\kappa_{2n+2}})\right) \end{array} \right\} \tilde{a}$$

$$\preccurlyeq \tilde{a}^* \max \left\{ \begin{array}{c} \tilde{d}_{c^*}\left(I_{\kappa_{2n}}, I_{\kappa_{2n+1}}\right), \tilde{d}_{c^*}\left(J_{\kappa_{2n}}, J_{\kappa_{2n+1}}\right), \\ \tilde{d}_{c^*}(I_{\kappa_{2n}}, I_{\kappa_{2n+1}}), \tilde{d}_{c^*}(I_{\kappa_{2n+1}}, I_{\kappa_{2n+2}}), \\ \tilde{d}_{c^*}(J_{\kappa_{2n}}, J_{\kappa_{2n+1}}), \tilde{d}_{c^*}(J_{\kappa_{2n+1}}, J_{\kappa_{2n+2}}) \end{array} \right\} \tilde{a}$$

$$\preccurlyeq \tilde{a}^* \max \left\{ \begin{array}{c} \tilde{d}_{c^*}\left(I_{\kappa_{2n}}, I_{\kappa_{2n+1}}\right), \tilde{d}_{c^*}\left(J_{\kappa_{2n}}, J_{\kappa_{2n+1}}\right), \\ \tilde{d}_{c^*}(I_{\kappa_{2n+1}}, I_{\kappa_{2n+2}}), \\ \tilde{d}_{c^*}(J_{\kappa_{2n+1}}, J_{\kappa_{2n+2}}) \end{array} \right\} \tilde{a} \quad (3)$$

Similarly,

$$\tilde{d}_{c^*}(J_{\kappa_{2n+1}}, J_{\kappa_{2n+2}}) \preccurlyeq$$

$$\tilde{a}^* \max \left\{ \begin{array}{c} \tilde{d}_{c^*}\left(I_{\kappa_{2n}}, I_{\kappa_{2n+1}}\right), \tilde{d}_{c^*}\left(J_{\kappa_{2n}}, J_{\kappa_{2n+1}}\right), \\ \tilde{d}_{c^*}(I_{\kappa_{2n+1}}, I_{\kappa_{2n+2}}), \\ \tilde{d}_{c^*}(J_{\kappa_{2n+1}}, J_{\kappa_{2n+2}}) \end{array} \right\} \tilde{a} (4)$$

Combining (3) and (4), we get

$$\max\{\tilde{d}_{c^*}(I_{\kappa_{2n+1}}, I_{\kappa_{2n+2}}), \tilde{d}_{c^*}(J_{\kappa_{2n+1}}, J_{\kappa_{2n+2}})\}$$

$$\preccurlyeq \tilde{a}^* \max \left\{ \begin{array}{c} \tilde{d}_{c^*}\left(I_{\kappa_{2n}}, I_{\kappa_{2n+1}}\right), \tilde{d}_{c^*}\left(J_{\kappa_{2n}}, J_{\kappa_{2n+1}}\right), \\ \tilde{d}_{c^*}(I_{\kappa_{2n+1}}, I_{\kappa_{2n+2}}), \\ \tilde{d}_{c^*}(J_{\kappa_{2n+1}}, J_{\kappa_{2n+2}}) \end{array} \right\} \tilde{a}$$

If $\tilde{d}_{c^*}(I_{\kappa_{2n+1}}, I_{\kappa_{2n+2}}) > \tilde{d}_{c^*}\left(I_{\kappa_{2n}}, I_{\kappa_{2n+1}}\right)$

$and \tilde{d}_{c^*}(J_{\kappa_{2n+1}}, J_{\kappa_{2n+2}}) > \tilde{d}_{c^*}\left(J_{\kappa_{2n}}, J_{\kappa_{2n+1}}\right)$ for some n, then from the above inequality, we have

$$\max\{\tilde{d}_{c^*}(I_{\kappa_{2n+1}}, I_{\kappa_{2n+2}}), \tilde{d}_{c^*}(J_{\kappa_{2n+1}}, J_{\kappa_{2n+2}})\} \preccurlyeq \tilde{a}^* \max\left\{\begin{matrix}\tilde{d}_{c^*}(I_{\kappa_{2n+1}}, I_{\kappa_{2n+2}}),\\ \tilde{d}_{c^*}(J_{\kappa_{2n+1}}, J_{\kappa_{2n+2}})\end{matrix}\right\}\tilde{a}$$

a contradiction.

Hence, $\tilde{d}_{c^*}(I_{\kappa_{2n+1}}, I_{\kappa_{2n+2}}) \le \tilde{d}_{c^*}\left(I_{\kappa_{2n}}, I_{\kappa_{2n+1}}\right) and \tilde{d}_{c^*}(J_{\kappa_{2n+1}}, J_{\kappa_{2n+2}}) \le \tilde{d}_{c^*}\left(J_{\kappa_{2n}}, J_{\kappa_{2n+1}}\right)$ for all $n \in N$. Also, by above inequality, we obtain

$$\max\{\tilde{d}_{c^*}(I_{\kappa_{2n+1}}, I_{\kappa_{2n+2}}), \tilde{d}_{c^*}(J_{\kappa_{2n+1}}, J_{\kappa_{2n+2}})\} \preccurlyeq \tilde{a}^* \max\left\{\begin{matrix}\tilde{d}_{c^*}\left(I_{\kappa_{2n}}, I_{\kappa_{2n+1}}\right),\\ \tilde{d}_{c^*}\left(J_{\kappa_{2n}}, J_{\kappa_{2n+1}}\right)\end{matrix}\right\}\tilde{a}$$

Let $\alpha_{2n+1} = \max\{\tilde{d}_{c^*}(I_{\kappa_{2n+1}}, I_{\kappa_{2n+2}}), \tilde{d}_{c^*}(J_{\kappa_{2n+1}}, J_{\kappa_{2n+2}})\}$

Now by above inequality, we have

$$\alpha_{2n+1} \preccurlyeq \tilde{a}^*\alpha_{2n}\tilde{a}$$
$$\vdots$$
$$\preccurlyeq (\tilde{a}^*)^{2n+1}\alpha_0(\tilde{a})^{2n+1}.$$

Thus,

$$\tilde{d}_{c^*}(I_{\kappa_{2n+1}}, I_{\kappa_{2n+2}}) \preccurlyeq (\tilde{a}^*)^{2n+1}\alpha_0(\tilde{a})^{2n+1}$$

$$\tilde{d}_{c^*}(J_{\kappa_{2n+1}}, J_{\kappa_{2n+2}}) \preccurlyeq (\tilde{a}^*)^{2n+1}\alpha_0(\tilde{a})^{2n+1}.$$

Now, we can obtain for any n ∈ N

$$\alpha_n = \max\left\{\tilde{d}_{c^*}\left(I_{\kappa_{2n}}, I_{\kappa_{2n+1}}\right), \tilde{d}_{c^*}\left(J_{\kappa_{2n}}, J_{\kappa_{2n+1}}\right)\right\}$$
$$\preccurlyeq \tilde{a}^*\alpha_{n-1}\tilde{a}$$
$$\vdots$$
$$\preccurlyeq (\tilde{a}^*)^n\alpha_0(\tilde{a})^n.$$

Thus,

$$\tilde{d}_{c^*}\left(I_{\kappa_{2n}}, I_{\kappa_{2n+1}}\right) \preccurlyeq (\tilde{a}^*)^n\alpha_0(\tilde{a})^n$$

$$\tilde{d}_{c^*}\left(J_{\kappa_{2n}}, J_{\kappa_{2n+1}}\right) \preccurlyeq (\tilde{a}^*)^n\alpha_0(\tilde{a})^n$$

If $\alpha_0 = \tilde{0}_{\tilde{A}}$ then from (S2) of Definition-1.1, we know $(I_{\alpha_0}, J_{\alpha_0})$ is a coupled fixed point of S, T.f and g.

Now letting $\tilde{0}_{\tilde{A}} \preccurlyeq \alpha_0$, we get for any n ∈ N, for any p ∈ N and using triangle inequality

$$\tilde{d}_{c^*}\left(I_{\kappa_{2n+p}}, I_{\kappa_{2n}}\right) + \tilde{d}_{c^*}\left(J_{\kappa_{2n+p}}, J_{\kappa_{2n}}\right)$$
$$\preccurlyeq \left(\tilde{d}_{c^*}\left(I_{\kappa_{2n+p}}, I_{\kappa_{2n+p-1}}\right), \tilde{d}_{c^*}\left(J_{\kappa_{2n+p}}, J_{\kappa_{2n+P-1}}\right)\right)$$
$$+\left(\tilde{d}_{c^*}\left(I_{\kappa_{2n+p+1}}, I_{\kappa_{2n+p-2}}\right), \tilde{d}_{c^*}\left(J_{\kappa_{2n+p+1}}, J_{\kappa_{2n+P-2}}\right)\right)$$
$$+\ldots+\left(\tilde{d}_{c^*}\left(I_{\kappa_{2n+1}}, I_{\kappa_{2n}}\right), \tilde{d}_{c^*}\left(J_{\kappa_{2n+1}}, J_{\kappa_{2n}}\right)\right)$$
$$\preccurlyeq \sum_{m=2n}^{2n+p-1}\left[\left(\sqrt{2}\tilde{a}\right)^*\right]^m \alpha_0(\sqrt{2}\tilde{a})^m$$

and then,

$$\left\|\tilde{d}_{c^*}\left(I_{\kappa_{2n+p}}, I_{\kappa_{2n}}\right) + \tilde{d}_{c^*}\left(J_{\kappa_{2n+p}}, J_{\kappa_{2n}}\right)\right\|$$
$$\le \sum_{m=2n}^{2n+p-1}\left\|\sqrt{2}\tilde{a}\right\|^{2m}\alpha_0$$
$$\le \sum_{m=n}^{\infty}\left\|\sqrt{2}\tilde{a}\right\|^{2m}\alpha_0$$
$$= \frac{\left\|\sqrt{2}\tilde{a}\right\|^{2n}}{1-\left\|\sqrt{2}\tilde{a}\right\|^2}\alpha_0 \to 0 \text{ as } n \to \infty$$

Which together with

$$\tilde{d}_{c^*}\left(I_{\kappa_{2n+p}}, I_{\kappa_{2n}}\right) \preccurlyeq max\left\{\tilde{d}_{c^*}\left(I_{\kappa_{2n+p}}, I_{\kappa_{2n}}\right) + \tilde{d}_{c^*}\left(J_{\kappa_{2n+p}}, J_{\kappa_{2n}}\right)\right\}$$

and

$$\tilde{d}_{c^*}\left(J_{\kappa_{2n+p}}, J_{\kappa_{2n}}\right) \preccurlyeq \max\left\{\tilde{d}_{c^*}\left(I_{\kappa_{2n+p}}, I_{\kappa_{2n}}\right) + \tilde{d}_{c^*}\left(J_{\kappa_{2n+p}}, J_{\kappa_{2n}}\right)\right\}$$

implies $\{I_{\kappa_{2n}}\}$ *and* $\{J_{\kappa_{2n}}\}$ are Cauchy sequences in $\tilde{\kappa}$ with respect to $\tilde{A}$. It follows that $\{I_{\kappa_{2n+1}}\}$ *and* $\{J_{\kappa_{2n+1}}\}$are also Cauchy sequences in $\tilde{\kappa}$ with respect to $\tilde{A}$. Thus $\{I_{\kappa_{2n}}\}$ *and* $\{J_{\kappa_{2n}}\}$ are Cauchy sequences in $(\tilde{\kappa}, \tilde{A}, \tilde{d}_{c^*})$.

Suppose $f(\tilde{\kappa})$ is complete subspace of $(\tilde{\kappa}, \tilde{A}, \tilde{d}_{c^*})$. Then the sequences $\{I_{\kappa_n}\}$ *and* $\{J_{\kappa_n}\}$ are converge to $I_{k^|}$, $J_{k^|}$respectively in $f(\tilde{\kappa})$. Thus there exist $F_{k^|}$, $G_{k^|} in f(\tilde{\kappa})$ such that

$$\lim_{n\to\infty} I_{\kappa_n} = I_{k^|} = fF_{k^|} \text{ and } \lim_{n\to\infty} J_{\kappa_n} = J_{k^|} = fG_{k^|} \qquad (5)$$

We now claim that $S(F_{k^|}, G_{k^|}) = I_{k^|}$ and $S(G_{k^|}, F_{k^|}) = J_{k^|}$

From (2.4.4) and using the triangular inequality

$$\tilde{0}_{\tilde{A}} \preccurlyeq \tilde{d}_{c^*}(S(F_{k^|}, G_{k^|}), I_{k^|})$$
$$\preccurlyeq \tilde{d}_{c^*}(S(F_{k^|}, G_{k^|}), I_{\kappa_{2n+1}}) + \tilde{d}_{c^*}\left(I_{\kappa_{2n+1}}, I_{k^|}\right)$$
$$\preccurlyeq \tilde{d}_{c^*}(S(F_{k^|}, G_{k^|}), T(F_{\kappa_{2n+1}}, G_{\kappa_{2n+1}}) + \tilde{d}_{c^*}\left(I_{\kappa_{2n+1}}, I_{k^|}\right)$$
$$\preccurlyeq \tilde{a}^*\max\left\{\begin{matrix}\tilde{d}_{c^*}(fF_{k^|}, gF_{\kappa_{2n+1}}),\\ \tilde{d}_{c^*}(fG_{k^|}, gG_{\kappa_{2n+1}}),\\ \tilde{d}_{c^*}\left(fF_{k^|}, S(F_{k^|}, G_{k^|})\right),\\ \tilde{d}_{c^*}(gF_{\kappa_{2n+1}}, \mathrm{T}(F_{\kappa_{2n+1}}, G_{\kappa_{2n+1}})),\\ \tilde{d}_{c^*}\left(fG_{k^|}, S(G_{k^|}, F_{k^|})\right),\\ \tilde{d}_{c^*}(gG_{\kappa_{2n+1}}, \mathrm{T}(G_{\kappa_{2n+1}}, F_{\kappa_{2n+1}}))\end{matrix}\right\}\tilde{a} + \tilde{d}_{c^*}\left(I_{\kappa_{2n+1}}, I_{k^|}\right)$$
$$\preccurlyeq \tilde{a}^*\max\left\{\begin{matrix}\tilde{d}_{c^*}(fF_{k^|}, I_{\kappa_{2n}}), \tilde{d}_{c^*}(fG_{k^|}, J_{\kappa_{2n}}),\\ \tilde{d}_{c^*}\left(fF_{k^|}, S(F_{k^|}, G_{k^|})\right), \tilde{d}_{c^*}(I_{\kappa_{2n}}, I_{\kappa_{2n+1}}),\\ \tilde{d}_{c^*}\left(fG_{k^|}, S(G_{k^|}, F_{k^|})\right), \tilde{d}_{c^*}(J_{\kappa_{2n}}, J_{\kappa_{2n+1}})\end{matrix}\right\}\tilde{a}$$
$$+ \tilde{d}_{c^*}\left(I_{\kappa_{2n+1}}, I_{k^|}\right)$$

Taking the limit as n $\to\infty$ in the above relation, we obtain

$$\tilde{d}_{c^*}(S(F_{k^|}, G_{k^|}), I_{k^|}) \preccurlyeq \tilde{a}^*\max\left\{\begin{matrix}\tilde{d}_{c^*}\left(I_{k^|}, S(F_{k^|}, G_{k^|})\right),\\ \tilde{d}_{c^*}\left(J_{k^|}, S(G_{k^|}, F_{k^|})\right)\end{matrix}\right\}\tilde{a}$$

Similarly, we get

$$\tilde{d}_{c^*}\left(S\left(G_{k^|},F_{k^|}\right),J_{k^|}\right) \preccurlyeq \tilde{\alpha}^* \max\begin{Bmatrix} \tilde{d}_{c^*}\left(j_{k^|},S\left(G_{k^|},F_{k^|}\right)\right), \\ \tilde{d}_{c^*}\left(I_{k^|},S\left(F_{k^|},G_{k^|}\right)\right) \end{Bmatrix}\tilde{\alpha}$$

Therefore,

$$\max\{\tilde{d}_{c^*}\left(S\left(F_{k^|},G_{k^|}\right),I_{k^|}\right),\tilde{d}_{c^*}\left(S\left(G_{k^|},F_{k^|}\right),J_{k^|}\right) \preccurlyeq \tilde{\alpha}^* \max\begin{Bmatrix} \tilde{d}_{c^*}\left(j_{k^|},S\left(G_{k^|},F_{k^|}\right)\right), \\ \tilde{d}_{c^*}\left(I_{k^|},S\left(F_{k^|},G_{k^|}\right)\right) \end{Bmatrix}\tilde{\alpha}$$

for each n ∈ N and $\|\tilde{\alpha}\| < \frac{1}{\sqrt{2}} < 1$ Then $\tilde{d}_{c^*}\left(S\left(F_{k^|},G_{k^|}\right),I_{k^|}\right) = \tilde{0}_{\tilde{A}}$ and $\tilde{d}_{c^*}\left(S\left(G_{k^|},F_{k^|}\right),J_{k^|}\right) = \tilde{0}_{\tilde{A}}$.

Hence, $S\left(F_{k^|},G_{k^|}\right) = I_{k^|}$ and $S\left(G_{k^|},F_{k^|}\right),J_{k^|}$.

Therefore, it follows $S\left(F_{k^|},G_{k^|}\right) = I_{k^|} = fI_{k^|}$ and $S\left(G_{k^|},F_{k^|}\right),J_{k^|} = fJ_{k^|}$.

Since, $\{S,f\}$ is ω-compatible pair, we have $S\left(I_{k^|},J_{k^|}\right) = fI_{k^|}$ and $S\left(J_{k^|},I_{k^|}\right) = fJ_{k^|}$

Now to prove that $fI_{k'} = I_{k'}$ and $fJ_{k'} = J_{k'}$

$$\tilde{0}_{\tilde{A}} \preccurlyeq \tilde{d}_{C^*}\left(fI_{k'},I_{k_{2n+1}}\right)$$

$$= \tilde{d}_{C^*}\left(s(I_{k'},J_{k'}),T\left(F_{k_{2n+1}},G_{k_{2n+1}}\right)\right)$$

$$\preccurlyeq \tilde{\alpha}^* max\begin{Bmatrix} \tilde{d}_{C^*}\left(fI_{k'},gF_{k_{2n+1}}\right),\tilde{d}_{C^*}\left(fJ_{k'},gG_{k_{2n+1}}\right), \\ \tilde{d}_{C^*}\left(fI_{k'},S(I_{k'},J_{k'})\right), \\ \tilde{d}_{C^*}\left(gF_{k_{2n+1}},T\left(F_{k_{2n+1}},G_{k_{2n+1}}\right)\right. \\ \tilde{d}_{C^*}\left(fJ_{k'},S(J_{k'},I_{k'})\right), \\ \tilde{d}_{C^*}\left(gG_{k_{2n+1}},T\left(G_{k_{2n+1}},F_{k_{2n+1}}\right)\right. \end{Bmatrix}\tilde{\alpha}$$

$$\preccurlyeq \tilde{\alpha}^* max\begin{Bmatrix} \tilde{d}_{C^*}\left(fI_{k'},I_{k_{2n}}\right),\tilde{d}_{C^*}\left(fJ_{k'},J_{k_{2n}}\right), \\ \tilde{d}_{C^*}\left(fI_{k'},S(I_{k'},J_{k'})\right),\tilde{d}_{C^*}\left(I_{k_{2n}},I_{k_{2n+1}}\right) \\ \tilde{d}_{C^*}\left(fJ_{k'},S(J_{k'},I_{k'})\right),\tilde{d}_{C^*}\left(J_{k_{2n}},J_{k_{2n+1}}\right) \end{Bmatrix}\tilde{\alpha}$$

Taking the limit as $n \to \infty$ in the above relation, we obtain

$$\tilde{d}_{C^*}(fI_{k'},I_{k'}) \preccurlyeq \tilde{\alpha}^* max\{\tilde{d}_{C^*}(fI_{k'},I_{k'}),\tilde{d}_{C^*}(fJ_{k'},J_{k'})\}\tilde{\alpha}$$

Similarly we can prove that

$$\tilde{d}_{C^*}(fJ_{k'},J_{k'}) \preccurlyeq \tilde{\alpha}^* max\{\tilde{d}_{C^*}(fJ_{k'},J_{k'}),\tilde{d}_{C^*}(fI_{k'},I_{k'})\}\tilde{\alpha}$$

Thus, we have

$$max\{\tilde{d}_{C^*}(fI_{k'},I_{k'}),\tilde{d}_{C^*}(fJ_{k'},J_{k'})\} \preccurlyeq \tilde{\alpha}^* max\{\tilde{d}_{C^*}(fI_{k'},I_{k'}),\tilde{d}_{C^*}(fJ_{k'},J_{k'})\}\tilde{\alpha}$$

Since, $\|\tilde{\alpha}\| < \frac{1}{\sqrt{2}} < 1$ then $\tilde{d}_{C^*}(fI_{k'},I_{k'}) = 0_{\tilde{A}}$ and $\tilde{d}_{C^*}(fJ_{k'},J_{k'}) = 0_{\tilde{A}}$ which implies $fI_{k'} = I_{k'}$ and

$fJ_{k'} = J_{k'}$. therefore $S(I_{k'},J_{k'}) = fI_{k'} = I_{k'}$ and

$S(J_{k'},I_{k'}) = fJ_{k'} = J_{k'}$.

Thus $(I_{k'},J_{k'})$ is common coupled fixed point of S and f. since $S(\tilde{K} \times \tilde{K}) \subseteq g(\tilde{K})$. So there exist $K_{k'},L_{k'} \in \tilde{K}$ such that $S(I_{k'},J_{k'}) = I_{k'} = gK_{k'}$ and $S(J_{k'},I_{k'}) = J_{k'} = gL_{k'}$.

Now from (2.4.4) and using the triangular inequality

$$\tilde{0}_{\tilde{A}} \preccurlyeq \tilde{d}_{C^*}(I_{k'},T(K_{k'},L_{k'}) = \tilde{d}_{C^*}(S(I_{k'},J_{k'}),T(K_{k'},L_{k'})$$

$$\preccurlyeq \tilde{\alpha}^* max\begin{Bmatrix} \tilde{d}_{C^*}(fI_{k'},gK_{k'}),\tilde{d}_{C^*}(fJ_{k'},gL_{k'}), \\ \tilde{d}_{C^*}\left(fI_{k'},S(I_{k'},J_{k'})\right),\tilde{d}_{C^*}\left(gK_{k'},T(K_{k'},L_{k'})\right) \\ \tilde{d}_{C^*}\left(fJ_{k'},S(J_{k'},I_{k'})\right),\tilde{d}_{C^*}\left(gL_{k'},T(L_{k'},K_{k'})\right) \end{Bmatrix}\tilde{\alpha}$$

$$\preccurlyeq \tilde{\alpha}^* max\{\tilde{d}_{C^*}(I_{k'},T(K_{k'},L_{k'})),\tilde{d}_{C^*}(J_{k'},T(L_{k'},K_{k'})\}\tilde{\alpha}$$

Similarly we get

$$\tilde{0}_{\tilde{A}} \preccurlyeq \tilde{d}_{C^*}(J_{k'},T(L_{k'},K_{k'}))$$

$$\preccurlyeq \tilde{\alpha}^* max\{\tilde{d}_{C^*}(J_{k'},T(L_{k'},K_{k'})),\tilde{d}_{C^*}(I_{k'},T(K_{k'},L_{k'})\}\tilde{\alpha}$$

Thus we have $max\{\tilde{d}_{C^*}(I_{k'},T(K_{k'},L_{k'})),\tilde{d}_{C^*}(J_{k'},T(L_{k'},K_{k'})\} \preccurlyeq \tilde{\alpha}^* max\{\tilde{d}_{C^*}(I_{k'},T(K_{k'},L_{k'})),\tilde{d}_{C^*}(J_{k'},T(L_{k'},K_{k'})\}\tilde{\alpha}$

Since, $\|\tilde{\alpha}\| < \frac{1}{\sqrt{2}} < 1$.then, we have $\tilde{d}_{C^*}(I_{k'},T(K_{k'},L_{k'})) = 0$ and $\tilde{d}_{C^*}(J_{k'},T(L_{k'},K_{k'}) = 0$ which means

$I_{k'} = T(K_{k'},L_{k'})$ and $T(L_{k'},K_{k'}) = J_{k'}$ since $\{T,g\}$ is ω-compatible pair, we have $T(I_{k'},J_{k'}) = gI_{k'}$ and $T(J_{k'},I_{k'}) = gJ_{k'}$ now we prove that $gI_{k'} = I_{k'}$ and $gJ_{k'} = J_{k'}$

$$\tilde{0}_{\tilde{A}} \preccurlyeq \tilde{d}_{C^*}(I_{k'},T(L_{k'},gI_{k'}))$$

$$= \tilde{d}_{C^*}(S(I_{k'},J_{k'}),T(I_{k'},J_{k'})$$

$$\preccurlyeq \tilde{\alpha}^* max\begin{Bmatrix} \tilde{d}_{C^*}(fI_{k'},gI_{k'}),\tilde{d}_{C^*}(fJ_{k'},gJ_{k'}), \\ \tilde{d}_{C^*}\left(fI_{k'},S(I_{k'},J_{k'})\right),\tilde{d}_{C^*}\left(gI_{k'},T(I_{k'},J_{k'})\right) \\ \tilde{d}_{C^*}\left(fJ_{k'},S(J_{k'},I_{k'})\right),\tilde{d}_{C^*}\left(gJ_{k'},T(J_{k'},J_{k'})\right) \end{Bmatrix}\tilde{\alpha}$$

$$\preccurlyeq \tilde{\alpha}^* max\{\tilde{d}_{C^*}(I_{k'},gI_{k'}),\tilde{d}_{C^*}(J_{k'},gJ_{k'})\} \quad (6)$$

And similarly

$$\tilde{0}_{\tilde{A}} \preccurlyeq \tilde{d}_{C^*}(J_{k'},gJ_{k'})$$

$$\preccurlyeq \tilde{\alpha}^* max\{\tilde{d}_{C^*}(J_{k'},gJ_{k'}),\tilde{d}_{C^*}(I_{k'},gI_{k'})\} \quad (7)$$

From (6) and (7)

$$\tilde{0}_{\tilde{A}} \preccurlyeq max\{\tilde{d}_{C^*}(I_{k'},gI_{k'}),\tilde{d}_{C^*}(J_{k'},gJ_{k'})\}$$

$$\preccurlyeq \tilde{\alpha}^* max\{\tilde{d}_{C^*}(I_{k'},gI_{k'}),\tilde{d}_{C^*}(J_{k'},gJ_{k'})\}\tilde{\alpha}$$

Therefore,

$$\tilde{0} \leq \|max\{\tilde{d}_{C^*}(I_{k'},gI_{k'}),\tilde{d}_{C^*}(J_{k'},gJ_{k'})\}\|$$

$$\leq \|\tilde{\alpha}\|^2\|max\{\tilde{d}_{C^*}(I_{k'},gI_{k'}),\tilde{d}_{C^*}(J_{k'},gJ_{k'})\}\|$$

Since $\|\tilde{\alpha}\| < \frac{1}{\sqrt{2}} < 1$ then

$$\|max\{\tilde{d}_{C^*}(I_{k'},gI_{k'}),\tilde{d}_{C^*}(J_{k'},gJ_{k'})\}\| = 0$$

Hence $gI_{k'} = I_{k'}$ and $gJ_{k'} = J_{k'}$

Therefore, we have $T(I_{k'},J_{k'}) = gI_{k'} = I_{k'}$ and $T(J_{k'},I_{k'}) = gJ_{k'} = J_{k'}$. Thus $(I_{k'},J_{k'})$ is common coupled fixed point of S,T,f and g. in the following we will show the uniqueness of common coupled fixed point in $(I_{k''},J_{k''})$ of S,T,f and g. then

$$\tilde{d}_{C^*}\left(I_{k'},I_{k''}\right) \preccurlyeq \tilde{d}_{C^*}(S(I_{k'},J_{k'}),T\left(I_{k''},J_{k''}\right))$$

$$\preccurlyeq \tilde{\alpha}^* max\begin{Bmatrix} \tilde{d}_{C^*}\left(fI_{k'},gI_{k''}\right),\tilde{d}_{C^*}\left(fJ_{k'},gJ_{k''}\right), \\ \tilde{d}_{C^*}(fI_{k'},S(I_{k'},J_{k'})),\tilde{d}_{C^*}\left(gI_{k''},T(I_{k''},J_{k''})\right), \\ \tilde{d}_{C^*}(fJ_{k'},S(J_{k'},I_{k'})),\tilde{d}_{C^*}\left(gJ_{k''},T(J_{k''},J_{k''})\right) \end{Bmatrix}\tilde{\alpha}$$

$$\preccurlyeq \tilde{\alpha}^* max\{\tilde{d}_{C^*}\left(I_{k'},I_{k''}\right),\tilde{d}_{C^*}\left(J_{k'},J_{k''}\right)\}\tilde{\alpha} \quad (8)$$

And similarly

$$\tilde{d}_{C^*}(J_{k'},J_{k''}) \preccurlyeq \tilde{\alpha}^* max\{\tilde{d}_{C^*}\left(J_{k'},J_{k''}\right),\tilde{d}_{C^*}\left(I_{k'},I_{k''}\right)\}\tilde{\alpha} \quad (9)$$

From (8) and (9) we get

$$max\{\tilde{d}_{C^*}(I_{k'},I_{k''}),\tilde{d}_{C^*}(J_{k'},J_{k''})\} \preccurlyeq \tilde{a}^* max\{\tilde{d}_{C^*}(I_{k'},I_{k''}),\tilde{d}_{C^*}(J_{k'},J_{k''})\}\tilde{a}$$

Which further induces that

$$\|max\{\tilde{d}_{C^*}(I_{k'},I_{k''}),\tilde{d}_{C^*}(J_{k'},J_{k''})\}\| \leq \|\tilde{a}\|^2 \|max\{\tilde{d}_{C^*}(I_{k'},I_{k''}),\tilde{d}_{C^*}(J_{k'},J_{k''})\}\|$$

Since $\|\tilde{a}\| < 1$ then

$$\|max\{\tilde{d}_{C^*}(I_{k'},I_{k''}),\tilde{d}_{C^*}(J_{k'},J_{k''})\}\| = 0$$

Hence we get $(I_{k'},J_{k'}) = (I_{k''},,J_{k''})$ which means the coupled fixed point is unique.

In order to prove that S,T,f and g have a unique fixed point, we only have to prove $I_{k'} = J_{k'}$. We have

$$\tilde{d}_{C^*}(I_{k'},J_{k'}) = \tilde{d}_{C^*}(S(I_{k'},J_{k'}),T(J_{k'},I_{k'})$$

$$\preccurlyeq \tilde{a}^* max\left\{\begin{matrix} \tilde{d}_{C^*}(fI_{k'},gJ_{k'}),\tilde{d}_{C^*}(fJ_{k'},gI_{k'}), \\ \tilde{d}_{C^*}(fI_{k'},S(I_{k'},J_{k'})),\tilde{d}_{C^*}(gJ_{k'},T(J_{k'},I_{k'})), \\ \tilde{d}_{C^*}(fJ_{k'},S(J_{k'},I_{k'})),\tilde{d}_{C^*}(gI_{k'},T(I_{k'},J_{k'})) \end{matrix}\right\}\tilde{a}$$

$$\preccurlyeq \tilde{a}^* \max\{\tilde{d}_{C^*}(I_{k'},J_{k'}),\tilde{d}_{C^*}(J_{k'},I_{k'})\}\tilde{a}$$

$$\preccurlyeq \tilde{a}^* \tilde{d}_{C^*}(I_{k'},J_{k'})\tilde{a}$$

$\|\tilde{d}_{C^*}(I_{k'},J_{k'})\| \leq \|\tilde{a}\|^2 \|\tilde{d}_{C^*}(I_{k'},J_{k'})\|$.

It follows from the fact $\|\tilde{a}\| < \frac{1}{\sqrt{2}}$ that $\|\tilde{d}_{C^*}(I_{k'},J_{k'})\| = 0$, thus $I_{k'} = J_{k'}$. Which means that S,T,f and g have a unique fixed point in $\tilde{K}$.

Corollary 2.5: let $(\tilde{K},\tilde{A},\tilde{d}_{C^*})$ be C^* algebra valued fuzzy soft metric space. Suppose $S: \tilde{K} \times \tilde{K} \to \tilde{K}$ and

$f,g: \tilde{K} \to \tilde{K}$ be satisfying

(2.5.1) $S(\tilde{K} \times \tilde{K}) \subseteq f(\tilde{K}) and S(\tilde{K} \times \tilde{K}) \subseteq g(\tilde{K})$

(2.5.2) $\{S,f\} and \ \{S,g\}$ are ω- compatible pairs.

(2.5.3) one of $f(\tilde{K})$ or $g(\tilde{K})$ is complete C^* algebra valued fuzzy soft metric space of $\tilde{K}$

(2.5.4) $\tilde{d}_{C^*}(S(F_{K_1},G_{K_1}),S(F_{K_2},G_{K_2})$

$$\leq \tilde{a}^* max\left\{\begin{matrix} \tilde{d}_{C^*}(fF_{K_1},gF_{K_2}),\tilde{d}_{C^*}(fG_{K_1},gG_{K_2}), \\ \tilde{d}_{C^*}\left(fF_{K_1},S(F_{K_1},G_{K_1})\right), \\ \tilde{d}_{C^*}\left(gF_{K_2},S(F_{K_2},G_{K_2})\right) \\ \tilde{d}_{C^*}\left(fG_{K_1},S(G_{K_1},F_{K_1})\right), \\ \tilde{d}_{C^*}\left(gG_{K_2},S(G_{K_2},F_{K_2})\right) \end{matrix}\right\}\tilde{a}$$

For all $F_{K_1},F_{K_2},G_{K_1},G_{K_2} \in \tilde{K}$ where $\tilde{a} \in \tilde{A}$ with $\|\sqrt{2}\tilde{a}\| < 1$ then S and f,g have a unique fixed point in $\tilde{K}$

Corollary 2.6: let $(\tilde{K},\tilde{A},\tilde{d}_{C^*})$ be C^* algebra valued fuzzy soft metric space. Suppose $S: \tilde{K} \times \tilde{K} \to \tilde{K}$ and

$f: \tilde{K} \to \tilde{K}$ be satisfying

(2.6.1) $S(\tilde{K} \times \tilde{K}) \subseteq f(\tilde{K})$

(2.6.2) $\{S,f\}$is ω- compatible pairs.

(2.6.3) $f(\tilde{K})$ is complete C^* algebra valued fuzzy soft metric space of $\tilde{K}$

(2.6.4) $\tilde{d}_{C^*}(S(F_{K_1},G_{K_1}),S(F_{K_2},G_{K_2}) \preccurlyeq$

$$\leq \tilde{a}^* max\left\{\begin{matrix} \tilde{d}_{C^*}(fF_{K_1},fF_{K_2}),\tilde{d}_{C^*}(fG_{K_1},fG_{K_2}), \\ \tilde{d}_{C^*}\left(fF_{K_1},S(F_{K_1},G_{K_1})\right), \\ \tilde{d}_{C^*}\left(fF_{K_2},S(F_{K_2},G_{K_2})\right) \\ \tilde{d}_{C^*}\left(fG_{K_1},S(G_{K_1},F_{K_1})\right), \\ \tilde{d}_{C^*}\left(fG_{K_2},S(G_{K_2},F_{K_2})\right) \end{matrix}\right\}\tilde{a}$$

For all $F_{K_1},F_{K_2},G_{K_1},G_{K_2} \in \tilde{K}$ where $\tilde{a} \in \tilde{A}$ with $\|\sqrt{2}\tilde{a}\| < 1$ then S and f have a unique fixed point in $\tilde{K}$

Corollary 2.7: let $(\tilde{K},\tilde{A},\tilde{d}_{C^*})$ be C^* algebra valued fuzzy soft metric space. Suppose $S,T: \tilde{K} \times \tilde{K} \to \tilde{K}$ satisfies

(2.7.1)) $\tilde{d}_{C^*}(S(F_{K_1},G_{K_1}),S(F_{K_2},G_{K_2})$

$$\preccurlyeq \tilde{a}^* max\left\{\begin{matrix} \tilde{d}_{C^*}(F_{K_1},F_{K_2}),\tilde{d}_{C^*}(G_{K_1},G_{K_2}), \\ \tilde{d}_{C^*}\left(F_{K_1},S(F_{K_1},G_{K_1})\right),\tilde{d}_{C^*}\left(F_{K_2},T(F_{K_2},G_{K_2})\right) \\ \tilde{d}_{C^*}\left(G_{K_1},S(G_{K_1},F_{K_1})\right),\tilde{d}_{C^*}\left(G_{K_2},T(G_{K_2},F_{K_2})\right) \end{matrix}\right\}\tilde{a}$$

For all $F_{K_1},F_{K_2},G_{K_1},G_{K_2} \in \tilde{K}$ where $\tilde{a} \in \tilde{A}$ with $\|\sqrt{2}\tilde{a}\| < 1$ then S and f have a unique fixed point in $\tilde{K}$

Corollary 2.8: let $(\tilde{K},\tilde{A},\tilde{d}_{C^*})$ be complete C^* algebra valued fuzzy soft metric space. Suppose $S: \tilde{K} \times \tilde{K} \to \tilde{K}$ satisfies

(2.8.1)) $\tilde{d}_{C^*}(S(F_{K_1},G_{K_1}),S(F_{K_2},G_{K_2})$

$$\leq \tilde{a}^* max\left\{\begin{matrix} \tilde{d}_{C^*}(F_{K_1},F_{K_2}),\tilde{d}_{C^*}(G_{K_1},G_{K_2}), \\ \tilde{d}_{C^*}\left(F_{K_1},S(F_{K_1},G_{K_1})\right), \\ \tilde{d}_{C^*}\left(F_{K_2},S(F_{K_2},G_{K_2})\right) \\ \tilde{d}_{C^*}\left(G_{K_1},S(G_{K_1},F_{K_1})\right),\tilde{d}_{C^*}\left(G_{K_2},S(G_{K_2},F_{K_2})\right) \end{matrix}\right\}\tilde{a}$$

For all $F_{K_1},F_{K_2},G_{K_1},G_{K_2} \in \tilde{K}$ where $\tilde{a} \in \tilde{A}$ with $\|\sqrt{2}\tilde{a}\| < 1$ then S and f have a unique fixed point in $\tilde{K}$

Example 2.9: let $K = \{k_1,k_2,k_3\}, u = \{p,q,r,s\}$ and C and D are two subsets K where $C = \{k_1,k_2,k_3\}, D = \{k_1,k_2\}$. Define fuzzy soft as,

$$(F_K,C) = \{k_1 = \{p_{0.5},q_{0.6},r_{0.7},s_{0.4}\}, k_2 = \{p_{0.7},q_{0.8},r_{0.5},s_{0.6}\}, k_3 = \{p_{0.8},q_{0.9},r_{0.6},s_{0.7}\}\}$$

$$(G_K,D) = \{k_1 = \{p_{0.6},q_{0.7},r_{0.2},s_{0.5}\}, k_2 = \{p_{0.9},q_{0.6},r_{0.3},s_{0.8}\}\}$$

$$F_{K_1} = \mu F_{K_1} = \{p_{0.5},q_{0.6},r_{0.7},s_{0.4}\}$$

$$F_{K_2} = \mu F_{K_2} = \{p_{0.7},q_{0.8},r_{0.5},s_{0.6}\}$$

$$F_{K_3} = \mu F_{K_3} = \{p_{0.8},q_{0.9},r_{0.6},s_{0.7}\}$$

$$G_{K_1} = \mu G_{K_1} = \{p_{0.6},q_{0.7},r_{0.2},s_{0.5}\}$$

$$G_{K_2} = \mu G_{K_2} = \{p_{0.9},q_{0.6},r_{0.3},s_{0.8}\}$$

And $FSC(F_K) = \{F_{K_1},F_{K_2},F_{K_3},G_{K_1},G_{K_2}\}$, Let for all $k \in K, K(k) = 1$ be absolute fuzzy soft set and $\tilde{A} = M_2(R(C)^*)$ be the C^* algebra.

Define $\tilde{d}_{C^*}: \tilde{K} \times \tilde{K} \to \tilde{A}$ by $\tilde{d}_{C^*}(G_{K_1},G_{K_2}) = (\inf\{|G_{K_1}(p) - G_{K_2}(p)| \ /p \in C\},0)$ then obviously $(\tilde{K},\tilde{A},\tilde{d}_{C^*})$ is a complete C^* algebra valued fuzzy soft metric space.

We define $S: \tilde{K} \times \tilde{K} \to \tilde{K}$ by $(F_{K_1},G_{K_1})(p) = \left(\frac{F_{K_1}+G_{K_1}}{2}\right)^5$, $T: \tilde{K} \times \tilde{K} \to \tilde{K}$ by

$T(F_{K_1}, G_{K_1})(p) = \left(\frac{F_{K_1}+G_{K_1}}{2}\right)^4$,

$f: \widetilde{K} \times \widetilde{K} \to \widetilde{K}$ by $fF_{K_1} = \left(\frac{F_{K_1}}{2}\right)^2$ and $g: \widetilde{K} \times \widetilde{K} \to \widetilde{K}$ by

$gF_{K_1} = \left(\frac{F_{K_1}}{2}\right)^4$ for all $p \in u and F_{K_1}, G_{K_1} \in \widetilde{K}$

Then obviously $S(\widetilde{K} \times \widetilde{K}) \subseteq g(\widetilde{K})$ and $T(\widetilde{K} \times \widetilde{K}) \subseteq f(\widetilde{K})$. Furthermore, the pairs {S,f} and {T,g} are ω-compatible

Notice that,

$$fF_{K_1} = \left(\frac{F_{K_1}}{2}\right)^2 = \{0.0625, 0.09, 0.1225, 0.04\}$$

$gF_{K_2} = \left(\frac{F_{K_2}}{2}\right)^4 = \{0.0150, 0.0256, 0.0039, 0.0081\}$. thus

$\inf\{\left|\mu^p_{fF_{K_1}}(t) - \mu^p_{gF_{K_2}}(t)\right| / t \in C\} = \{0.0475, 0.0644, 0.1186, 0.0319\} = 0.319.$

hence

$$\tilde{d}_{C^*}(fF_{K_1}, gF_{K_2}) = \begin{bmatrix} 0.0319 & 0 \\ 0 & 0.0319 \end{bmatrix}$$

Also

$$fG_{K_1} = \left(\frac{G_{K_1}}{2}\right)^2 = \{0.09, 0.1225, 0.010, 0.0625\}$$

$gG_{K_2} = \left(\frac{G_{K_2}}{2}\right)^4 = \{0.04100.00810.000500.0256\}$ thus

$\inf\{\left|\mu^p_{fG_{K_1}}(t) - \mu^p_{gG_{K_2}}(t)\right| / t \in C\} = \{0.049, 0.0114, 0.0095, 0.0369\} = 0.0095.$

$\tilde{d}_{C^*}(fG_{K_1}, gG_{K_2}) = \begin{bmatrix} 0.0095 & 0 \\ 0 & 0.0095 \end{bmatrix}$ Moreover

$S(F_{K_1}, G_{K_1})(p) = \left(\frac{F_{K_1}+G_{K_1}}{2}\right)^5 = \{0.0503, 0.0116, 0.0184, 0.0184\}$ and

$T(F_{K_2}, G_{K_2})(p) = \left(\frac{F_{K_2}+G_{K_2}}{2}\right)^4 = \{0.4096, 0.2401, 0.0256, 0.2401\}$ then

$$\tilde{d}_{C^*}\left(S(F_{K_1}, G_{K_1}), T(F_{K_2}, G_{K_2})\right) = \begin{bmatrix} 0.0072 & 0 \\ 0 & 0.0072 \end{bmatrix}$$

$$\preccurlyeq \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 0.0319 & 0 \\ 0 & 0.0319 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \end{bmatrix} + \begin{bmatrix} \frac{\sqrt{3}}{3} & 0 \\ 0 & \frac{\sqrt{3}}{3} \end{bmatrix} \begin{bmatrix} 0.0095 & 0 \\ 0 & 0.0095 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{3} & 0 \\ 0 & \frac{\sqrt{3}}{3} \end{bmatrix}$$

$$\preccurlyeq \tilde{c}^* \max\{\left(\tilde{d}_{C^*}(fF_{K_1}, gF_{K_2}), \tilde{d}_{C^*}(fG_{K_1}, gG_{K_2})\right)\}\tilde{c}$$

$$\preccurlyeq \tilde{c}^* \max\left\{\begin{matrix} \tilde{d}_{C^*}(fF_{K_1}, gF_{K_2}), \tilde{d}_{C^*}(fG_{K_1}, gG_{K_2}), \\ \tilde{d}_{C^*}\left(fF_{K_1}, S(F_{K_1}, G_{K_1})\right), \tilde{d}_{C^*}\left(gF_{K_2}, T(F_{K_2}, G_{K_2})\right) \\ \tilde{d}_{C^*}\left(fG_{K_1}, S(G_{K_1}, F_{K_1})\right), \tilde{d}_{C^*}\left(gG_{K_2}, T(G_{K_2}, F_{K_2})\right) \end{matrix}\right\}\tilde{c}$$

Hence $\tilde{c} = \begin{bmatrix} \frac{\sqrt{5}}{5} & 0 \\ 0 & \frac{\sqrt{5}}{5} \end{bmatrix} with \|\tilde{c}\| = \frac{1}{\sqrt{5}} < \frac{1}{\sqrt{2}}$ therefore all the conditions of theorem 2.4 satisfied. Hence S,T,f and g have a unique coupled fixed point.

APPLICATIONS TO INTEGRAL EQUATIONS

THEOREM 3.1: Let us consider the integral equation

$$F_{K_1}(x) = \int_c \left(T_1(x, y, F_{K_1}(y) + T_1\left(x, y, F_{K_1}(y)\right)\right) dy, x \in C$$

$$F_{K_1}(x) = \int_c \left(I_1(x, y, F_{K_1}(y) + I_2(x, y, F_{K_1}(y)\right) dy, x \in C$$

Where C is a lebesgue measurable set. Suppose that

$T_1, T_2: C \times C \times R(C)^* \to R(C)^*$

$and I_1, I_2: C \times C \times R(C)^* \to R(C)^*$

There exist two continuous functions $\emptyset, \varphi: C \times C \times R(C)^* and r \in (0,1)$ such that for $v, u \in c and F_{K_1}(v), F_{K_2}(v) \in R(C)^*$

$$\inf\left\{\left|T_1\left(u, v, F_{K_1}(v)\right) - I_1\left(u, v, F_{K_2}(v)\right)\right|\right\}$$
$$\leq r inf\{|\emptyset(u,v)|\}. \inf\{\left|\left(F_{K_1}(v) - F_{K_2}(v)\right)\right|\}$$

and

$$\inf\left\{\left|T_2\left(u, v, F_{K_1}(v)\right) - I_2\left(u, v, F_{K_2}(v)\right)\right|\right\}$$
$$\leq t inf\{|\{\varphi(u,v)|\}. \inf\{\left|\left(F_{K_1}(v) - F_{K_2}(v)\right)\right|\}$$

$$\sup_{x \in C} \int_C \inf\{|\emptyset(u,v)|\} dv \leq 1$$

and $\sup_{x \in C} \int_C \inf\{|\varphi(u,v)|\} dv \leq 1$

Then the integral equation has a unique solution in $L^\infty(C)$.

Proof: let $K = C = [0,1] and \widetilde{K} = L^\infty(C)$ be the set of essential bounded measurable function on C and $H = L^2(C)$. The set of bounded linear operators on Hilbert space H denoted by L(H). Consider $\tilde{d}_{C^*}: \widetilde{K} \times \widetilde{K} \to L(H)$ by $\tilde{d}_{C^*}(F_{K_1}, F_{K_2}) = M_{\inf\{\left|\mu^p_{F_{K_1}}(y) - \mu^p_{F_{K_2}}(y)\right| / y \in C\}}$ for all $F_{K_1}, F_{K_2} \in \widetilde{K}$ where $M_h: H \to H$ is the multiplication operator defined by $M_h(\emptyset) = h.\emptyset$ for $\emptyset \in H$. Then $\tilde{d}_{C^*}$ is a C^* algebra valued fuzzy soft metric and $(\widetilde{K}, L(h), \tilde{d}_{C^*})$ is a complete a C^* algebra valued fuzzy soft metric space.

Define two self mappings $S, T: \widetilde{K} \times \widetilde{K} \to \widetilde{K}$ by

$$S(F_{K_1}, G_{K_1})(x) = \int_c \left(T_1(x, y, F_{K_1}(y) + T_1\left(x, y, G_{K_1}(y)\right)\right) dy, x \in C$$

$$T(F_{K_2}, G_{K_2})(x) = \int_c \left(I_1(x, y, F_{K_2}(y) + I_2\left(x, y, G_{K_2}(y)\right)\right) dy, x \in C$$

Notice that

$$\tilde{d}_{C^*}\left(S(F_{K_1}, G_{K_1}), T(F_{K_2}, G_{K_2})\right) = M_{\inf\{\left|\mu^p_{S(F_{K_1}, G_{K_1})}(y) - \mu^p_{T(F_{K_2}, G_{K_2})}(y)\right| / y \in C\}}$$

Then we have

$$\left\|\tilde{d}_{C^*}\left(S(F_{K_1}, G_{K_1}), T(F_{K_2}, G_{K_2})\right)\right\|$$

$$= \sup_{\|h\|=1} \left(M_{\inf\{\left|\mu^p_{S(F_{K_1}, G_{K_1})}(y) - \mu^p_{T(F_{K_2}, G_{K_2})}(y)\right| / y \in C\}} h, h\right)$$

$$= \sup_{\|h\|=1} \int_C \left[\inf\left\{\left| \mu^p_{S(F_{K_1},G_{K_1})}(y) - \mu^p_{T(F_{K_2},G_{K_2})}(y) \right| \Big/ y \in C\right\}\right] h(x)\overline{h(x)}dx$$

$$\leq \sup_{\|h\|=1} \int_C \left[\int_C \inf\left\{\left| T_1(x,y,F_{K_1}(y)) - I_1(x,y,F_{K_2}(y)) \right|\right\} dy\right] |h(x)|^2 dx$$

$$+ \sup_{\|h\|=1} \int_C \left[\int_C \inf\left\{\left| T_2(x,y,G_{K_1}(y)) - I_2(x,y,G_{K_2}(y)) \right|\right\} dy\right] |h(x)|^2 dx$$

$$\leq \sup_{\|h\|=1} \int_C \left[\int_C \inf\left\{\left| \emptyset(x,y)(F_{K_1}(y) - F_{K_2}(y)) \right|\right\} dy\right] |h(x)|^2 dx$$

$$+ \sup_{\|h\|=1} \int_C \left[\int_C \inf\left\{\left| \varphi(x,y)(G_{K_1}(y) - G_{K_2}(y) \right|\right\} dy\right] |h(x)|^2 dx$$

$$\leq r \sup_{\|h\|=1} \int_C \left[\int_C inf|\emptyset(x,y)|\inf\{\left| F_{K_1}(y) - F_{K_2}(y) \right|\} dy\right] |h(x)|^2 dx$$

$$+t \sup_{\|h\|=1} \int_C \left[\int_C inf|\emptyset(x,y)|\inf\{\left| G_{K_1}(y) - G_{K_2}(y) \right|\} dy\right] |h(x)|^2 dx$$

$$\leq r \sup_{\|h\|=1} \int_C \left[\int_C \inf\{|\emptyset(x,y)|\} dy\right] |h(x)|^2 dx \; . \left\|\inf\{\left|F_{K_1}(y) - F_{K_2}(y)\right|\}\right\|_\infty$$

$$+ t \sup_{\|h\|=1} \int_C \left[\int_C \inf\{|\varphi(x,y)|\} dy\right] |h(x)|^2 dx \; . \left\|\inf\{\left|G_{K_1}(y) - G_{K_2}(y)\right|\}\right\|_\infty$$

$$\leq r \sup_{\|h\|=1} \int_C \inf\{|\emptyset(x,y)|\} dy. \sup_{\|h\|=1} \int_C |h(x)|^2 dx \; . \left\|\inf\{\left|F_{K_1}(y) - F_{K_2}(y)\right|\}\right\|_\infty +$$

$$\leq t \sup_{\|h\|=1} \int_C \inf\{|\varphi(x,y)|\} dy. \sup_{\|h\|=1} \int_C |h(x)|^2 dx \; . \left\|\inf\{\left|G_{K_1}(y) - G_{K_2}(y)\right|\}\right\|_\infty$$

$$\leq r\left\|\inf\{\left|F_{K_1}(y) - F_{K_2}(y)\right|\}\right\|_\infty + t\left\|\inf\{\left|G_{K_1}(y) - G_{K_2}(y)\right|\}\right\|_\infty$$

$$\leq \zeta max\left\{ \begin{array}{l} \left\|\inf\{\left|F_{K_1}(y) - F_{K_2}(y)\right|\}\right\|_\infty, \\ \left\|\inf\{\left|G_{K_1}(y) - G_{K_2}(y)\right|\}\right\|_\infty \end{array} \right\}.$$

Set $\tilde{a} = \sqrt{\zeta}1_{L(H)}$, then $\tilde{a} \in L(H)$ and $\|\tilde{a}\| = \sqrt{\zeta} < \frac{1}{\sqrt{2}}$. Hence, applying our corollary 2.7, we get the desired result.

CONCLUSIONS

In this paper we conclude some applications to integral equations on fuzzy soft set theory by using fixed point theorems in C^* algebra valued fuzzy soft metric space.

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