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Analysis of Super Mean Labeling Of Graph

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Abstract
Let G be a graph of (p, q) and let f: V(G) {1,2,3, ... ,p + q} be an injection. Let f(u) + f(v))/2 if f(u) + f(v) is equivalent for each edge e = uv and f(e) = (f(u) + f(v)+1)/2 if f(u)+f(v) is odd for each edge e = uv. If f(V){f ∪ (e): e ∈ E(G)} = {1,2,3, ... ,p+q}, then f is regarded as super mean marking. A graph admitting super mean marking is referred to as a super mean graph. S(Pn ⊙ K1),S(P2 x P4),S(Bn, n),'Bn, n: Pm',Cn ⊙ K2,n ≥ 3, generalised antiprism $(\mathcal{A}_n^m)$ and the double triangular snake D(Tn) are super mean graphs in this article.
Keywords: mega m-mean labelling of Smarandachely, mega m-mean graph of Smarandachely, super mean labelling, super mean graph.

Introduction
We mean a finite, simple and undirected one by means of a line. V (G) and E(G) respectively denote the vertex set and the edge set of a graph G. The graph G1 ∪ G2 with V (G1∪ G2) = V (G1) ∪ V (G2) and E(G1 ∪ G2) = E(G1) ∪ E(G2) is the disjoint union of the two graphs G1 and G2. A mG denotes the disjoint union of m copies of graph G. By taking one copy of G1 (with p vertices) and p copies of G2, and then joining the ith vertex of G1 to each vertex in the ith copy of G2, the corona G1 ∪ G2 of graphs G1 and G2 is obtained. The Cn-Pm armed crown is a graph obtained from a cycle Cn by defining at each vertex of the chain the pending vertex of a direction Pm. A graph obtained from a loop Cn by defining the dependent vertices of two disjoint vertex paths of similar length m-1 at each vertex of the loop is the bi-armed crown. By Cn 2Pm, we denote a bi-armed crown where Pm is a direction of length m-1. The graph obtained from direction v1 , v2, v3,. is the double triangular snake D(Tn). By combining vi and vi+1 with two fresh vertices ii and wi for 1 I n-1, vn. A graph obtained from K2 by connecting m pendant edges to one end of K2 and n suspension edges to the other end of K2 is the bistar Bm, n.
The vertex set V = {vji: 1 ≤I ≤ n and 1 ≤ j ≤m} has the simplified prism graph Cn x Pm and the edge set E = $\{v_i^j v_{i+1}^j, v_n^j v_1^j$: 1 ≤ i ≤ n−1 and 1 ≤ j ≤ m}∪$\{v_i^j v_{i-1}^{j+1}, v_1^j v_n^{j+1}$: 2 ≤ i ≤ n and 1 ≤ j ≤ m – 1
By inserting the sides, the generalised antiprism Amn is obtained by completing the generalised prism Cn x Pm.
$v_i^j v_i^{j+1}$ for 1 ≤ i ≤ n and 1 ≤ j ≤ m − 1. In the context of Harary[1], terminology and notations not specified here are used.

Preliminary Results
Let G be a graph and let f be an injection: V (G) → {1 , 2 , 3, • •, |V | + |E(G)|} . The induced Smarandachely edge m-labeling f*S is described by the Smarandachely edge m-labeling f*S for each edge e= uv and an integer m = 2,

$$f_S^*(e) = \left\lceil \frac{f(u) + f(v)}{m} \right\rceil .$$

If f(V(G)))∪ {f (e): e ∈ E(G)} = {1, 2, 3, • • •, + |V | + |E(G)|}. f is therefore named Smarandachely super m-mean marking. The Smarandachely super m-mean graph is considered a graph that admits a Smarandachely super mean m-labeling. In fact, if m = 2, then we know that

$$f^*(e) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even;} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd.} \end{cases}$$

A super-mean labelling of G is considered such a labelling f if f(V(G)) ∪ (f(e): e ∈ E(G)} = {1 , 2, 3,. To p + q}. A graph admitting super mean marking is referred to as a super mean graph. In [7] and further addressed in [2-6], the principle of super mean labelling was added. In the corresponding theorems, we use the following results.

Theorem 2.1([7]) A super mean graph for m = n or n + 1. is the bistar Bm, n.

Theorem 2.2([2]) A super-mean graph is the graph(Bn, n: w) obtained by subdividing the central edge of Bn, n with a vertex w.

Theorem 2.3([2]) For odd n ≥ 3 and m ≥ 2, the bi-armed crown Cn 2Pm is a super mean curve.

Theorem 2.4([7]) Let G1 = (p1, q1) and G2 = (p2, q2) be two super-mean graphs with super-mean f and g names, respectively. Let f(u) = p1+q1 and g(v) = 1. Let f(u) = 1. So a super mean graph is indeed the graph (G1)f * (G2)g acquired from G1 and G2 by defining the vertices u and v.

Super Mean Graphs

If G is a graph, then S(G) is a graph that is obtained by subdividing a vertex between each edge of G.

Theorem 3.1 The S(Pn ⊙K1) graph is a super-mean graph.

Proof Let V(Pn⊙ K1)={ui, vi:1 ≤ I ≤ n}. Let V(Pn≤ K1). Let xi(1 ≤ I ≤ n) be the vertex separating the edge uivi(1 ≤ I ≤ n) and yi(1 ≤ I ≤ n − 1) be the vertex separating the edge uiui+1(1 ≤ I ≤ n − 1). Then V (S(Pn ⊙ K1)) = {ui, vi, xi, yj: 1 ≤ I ≤ n, 1 ≤ j ≤ n − 1}. Then V(S(Pn ⊙ K1))

define f: V (S(Pn ⊙ K1)) {1, 2, 3,. Through, p + q = 8n − 3} by

$$f(v_1) = 1; f(v_2) = 14; f(v_{2+i}) = 14 + 8i \text{ for } 1 \leq i \leq n-4;$$

$$f(v_{n-1}) = 8n - 11; f(v_n) = 8n - 10; f(x_1) = 3;$$

$$f(x_{1+i}) = 3 + 8i \text{ for } 1 \leq i \leq n-2; f(x_n) = 8n - 7;$$

$$f(u_1) = 5; f(u_2) = 9; f(u_{2+i}) = 9 + 8i \text{ for } 1 \leq i \leq n-3;$$

$$f(u_n) = 8n - 5; f(y_i) = 8i - 1 \text{ for } 1 \leq i \leq n-2; f(y_{n-1}) = 8n - 3.$$

It can be confirmed that f is a super-mean S(Pn ⊙ K1) label. S(Pn ⊙ K1) is thus a super-mean graph.

Example 3.2 In Fig.1, the super average S(P5 ⊙K1) labelling is given.

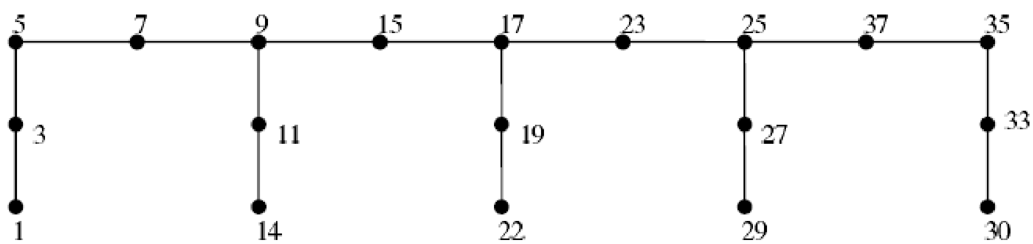

Fig.1

Theorem 3.2 A super-mean graph is the graph S(P2 x Pn).

Proof Let V(P2 xPn)={ui, vi:1 I ≤ n}. Let V(P2 x Pn)={ui, vi:1 I ≤ n}. Let u1i, v1 I (1 ≤ I ≤ n−1) be the vertices that separate the uiui+1, vivi+1(1 ≤ I ≤ n−1) sides. Let the vertex that separates the edge uivi be wi(1 ≤ I ≤n). V (S(P2-Pn)) = {ui, vi, wi: 1≤ I ≤n} {u1i, v1 I 1 ≤I ≤ n-1}.

Define f : V (S(P2 × Pn)) → {1, 2, 3, . . . , p + q = 11n − 6} by

$f(u_1) = 1; f(u_2) = 9; f(u_3) = 27;$

$f(u_i) = f(u_{i-1}) + 5$ for $4 \leq i \leq n$ and i is even

$f(u_i) = f(u_{i-1}) + 17$ for $4 \leq i \leq n$ and i is odd

$f(v_1) = 7; f(v_2) = 16;$

$f(v_i) = f(v_{i-1}) + 5$ for $3 \leq i \leq n$ and i is odd

$f(v_i) = f(v_{i-1}) + 17$ for $3 \leq i \leq n$ and i is even

$f(w_1) = 3; f(w_2) = 12;$

$f(w_{2+i}) = 12 + 11i$ for $1 \leq i \leq n - 2;$

$f(u_1^1) = 6; f(u_2^1) = 24;$

$f(u_i^1) = f(u_{i-1}^1) + 6$ for $3 \leq i \leq n - 1$ and i isodd

$f(u_i^1) = f(u_{i-1}^1) + 16$ for $3 \leq i \leq n - 1$ and i iseven

$f(v_1^1) = 13; f(v_i^1) = f(v_{i-1}^1) + 6$ for $2 \leq i \leq n - 1$ and i iseven

$f(v_i^1) = f(v_{i-1}^1) + 16$ for $2 \leq i \leq n - 1$ and i isodd.

Testing that f is a super-mean S(P2x Pn) mark is simple. S(P2 x Pn) is thus a super-mean graph.

Example 3.4 In Fig.2 the super median labelling of S(P2 xP6) is given.

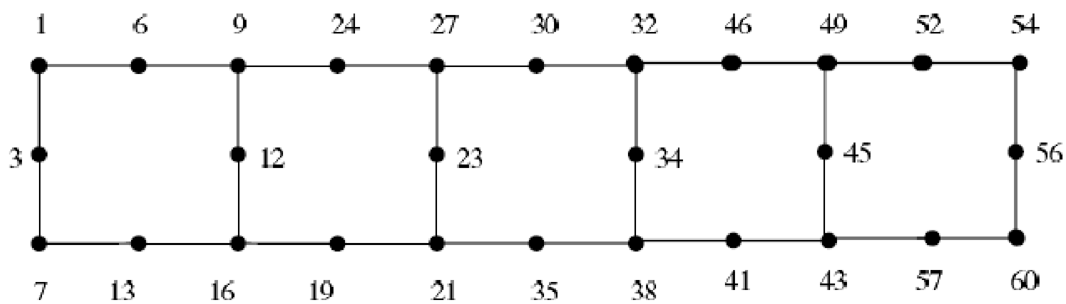

Fig.2

Theorem 3.5 A super mean graph is the graph S(Bn, n).

Proof Let V (Bn, n) = {u, ui, v, vi: 1 I ≤ n} and E(Bn, n) = {uui, vvi, uv: 1 I ≤ n}. Let V (Bn, n) = {uui, vvi, uv: 1 ≤ I ≤ n}. Let w, xi, yi, (1 ≤ I ≤ n) be the vertices that break the uv, uui, vvi(1 ≤ I ≤ n) edges respectively. Then V (S(Bn, n)) = {u, ui, v, vi, xi, yi, w: 1 = I ≤ n} and E(S(Bn, n)) = {uxi, xiui, uw, wv, vyi, yivi: 1 = I ≤ n}.

Define F: V (S(Bn, n)) for {1 , 2, 3,. , p + q = 8n + 5} by 8n + 5}

f(u) = 1; f(xi) = 8i − 5 for 1 ≤ i ≤ n; f(ui) = 8i − 3 for 1 ≤ i ≤ n; f(w) = 8n + 3; f(v) = 8n + 5; f(yi) = 8i − 1 for 1 ≤ i ≤ n; f(vi) = 8i + 1 for 1 ≤ i ≤ n. It can be checked that f is a S(Bn, n) super-mean label. S(Bn, n) is thus a super-mean graph.

Example 3.6 In Fig.3, the super median labelling of S(Bn, n) is given.

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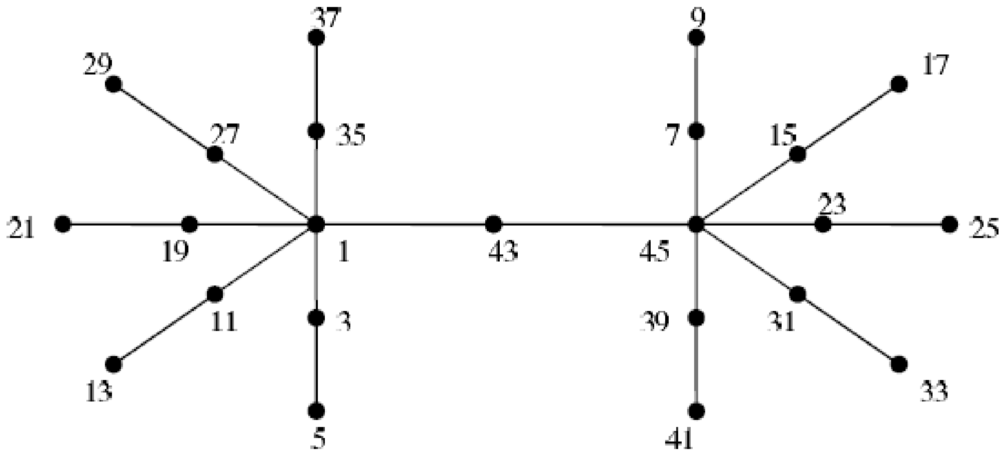

Fig.3

First, we demonstrate that the hBn, n: Pmi graph is a super-mean graph. HBm, n: Pki is a graph obtained by a direction Pk of length k-1 connecting the central vertices of the stars K1,m and K1,n.

Theorem 3.7 For both n ≥ 1 and m > 1., the graph (Bn, n: Pm) is a super mean graph.

Proof Let V (hBn,n : Pmi) = {ui, vi, u, v,wj : 1 ≤ i ≤ n, 1 ≤ j ≤ m with u = w1, v = wm} and E(hBn,n : Pmi) = {uui, vvi,wjwj+1 : 1 ≤ i ≤ n, 1 ≤ j ≤ m− 1}.

Case 1 n equals even.

Subcase 1 m is bizarre.

By Theorem 2.2, a super mean graph is (Bn, n: P3). In m > 3,define f : V (Bn,n : Pm) →{1, 2, 3, . . . , p + q = 4n + 2m− 1} by

$$f(u) = 1; f(u_i) = 4i - 1 \text{ for } 1 \leq i \leq n \text{ and for } i \neq \frac{n}{2} + 1;$$

$$f(u_{\frac{n}{2}+1}) = 2n + 2; f(v_i) = 4i + 1 \text{ for } 1 \leq i \leq n; f(v) = 4n + 3;$$

$$f(w_2) = 4n + 4; f(w_3) = 4n + 9;$$

$$f(w_{3+i}) = 4n + 9 + 4i \text{ for } 1 \leq i \leq \frac{m-5}{2}; f(w_{\frac{m+3}{2}}) = 4n + 2m - 4$$

$$f(w_{\frac{m+3}{2}+i}) = 4n + 2m - 4 - 4i \text{ for } 1 \leq i \leq \frac{m-5}{2}.$$

It can be checked that f is a super-mean (Bn, n: Pm) label.

Subcase number 2 m is even.

By Theorem 2.1, a super mean graph is (Bn, n: P2). For m > 2, define f : V (Bn,n : Pm) → {1, 2, 3, . . . , p + q = 4n + 2m− 1} by

$$f(u) = 1; f(u_i) = 4i - 1 \text{ for } 1 \leq i \leq n \text{ and for } i \neq \frac{n}{2} + 1;$$

$$f(u_{\frac{n}{2}+1}) = 2n + 2; f(v_i) = 4i + 1 \text{ for } 1 \leq i \leq n; f(v) = 4n + 3;$$

$$f(w_2) = 4n + 4; f(w_{2+i}) = 4n + 4 + 2i \text{ for } 1 \leq i \leq \frac{m-4}{2};$$

$$f(w_{\frac{m+2}{2}}) = 4n + m + 3;$$

$$f(w_{\frac{m+2}{2}+i}) = 4n + m + 3 + 2i \text{ for } 1 \leq i \leq \frac{m-4}{2}.$$

It can be checked that f is a super-mean (Bn, n: Pm) label.

Case 2 n is unique.

Subcase 1 m is bizarre.

By Theorem 2.1, a super mean graph is (Bn, n: P2). For m > 2, the define f : V (Bn,n : Pm) →{1, 2, 3, . . . , p + q = 4n + 2m− 1} by

$f(u) = 1; f(v) = 4n + 3; f(u_i) = 4i - 1$ for $1 \leq i \leq n$;

$f(v_i) = 4i + 1$ for $1 \leq i \leq n$ and for $i \neq \frac{n+1}{2}$;

$f(v_{\frac{n+1}{2}}) = 2n + 2; f(w_2) = 4n + 4$;

$f(w_{2+i}) = 4n + 4 + 2i$ for $1 \leq i \leq \frac{m-4}{2}$; $f(w_{\frac{m+2}{2}}) = 4n + m + 3$;

$f(w_{\frac{m+2}{2}+i}) = 4n + m + 3 + 2i$ for $1 \leq i \leq \frac{m-4}{2}$.

It can be confirmed that f is a (Bn, n: Pm) super-mean marking.

Subcase number 2 m is even.

By Theorem 2.2, a super mean graph is (Bn, n: P3). In m > 3, define f : V (Bn,n : Pm) →{1, 2, 3, . . . , p + q = 4n + 2m− 1} by

$f(u) = 1; f(v) = 4n + 3; f(u_i) = 4i - 1$ for $1 \leq i \leq n$;

$f(v_i) = 4i + 1$ for $1 \leq i \leq n$ and for $i \neq \frac{n+1}{2}$; $f(v_{\frac{n+1}{2}}) = 2n + 2$;

$f(w_2) = 4n + 4; f(w_3) = 4n + 9$;

$f(w_{3+i}) = 4n + 9 + 4i$ for $1 \leq i \leq \frac{m-5}{2}$; $f(w_{\frac{m+3}{2}}) = 4n + 2m - 4$;

$f(w_{\frac{m+3}{2}+i}) = 4n + 2m - 4 - 2i$ for $1 \leq i \leq \frac{m-5}{2}$.

It can be checked that f is a super-mean (Bn, n: Pm) label. (Bn, n: pm) is thus a super mean graph for all n ≥ 1 and m > 1.

Example 3.8 In Fig.4, the super mean labelling of (B4,4: P5) is given.

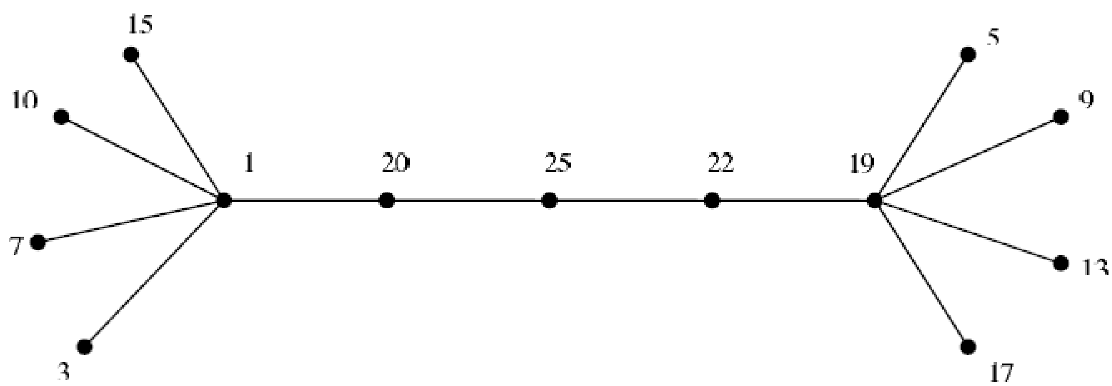

Fig.4

Theorem 3.9 For all n≥ 3, the corona graph Cn ⊙ K2 is a super mean graph.

Proof Let V (Cn) = {u1, u2, . . . , un} and V (Cn ⊙K2) = {ui, vi,wi : 1 ≤ i ≤ n}. Then E(Cn ⊙ K2) = {uiui+1, unu1, ujvj , ujwj : 1 ≤ i ≤ n − 1 and 1 ≤ j ≤ n}.

Case 1 n is unique.

From Theorem 2.3, the evidence proceeds by taking m = 2.

Case 2 n will be even.

For a certain k, take n = 2k. Defines f : V (Cn ⊙ K2) → {1, 2, 3, . . . , p + q = 6n} by

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$f(u_i) = 6i - 3$ for $1 \leq i \leq k-1$; $f(u_k) = 6k - 2$;

$f(u_{k+i}) = 6k - 2 + 6i$ for $1 \leq i \leq k-2$; $f(u_{2k-1}) = 12k - 2$;

$f(u_{2k}) = 12k - 9$; $f(v_i) = 6i - 5$ for $1 \leq i \leq k-1$; $f(v_k) = 6k - 6$;

$f(v_{k+1}) = 6k + 2$; $f(v_{k+1+i}) = 6k + 2 + 6i$ for $1 \leq i \leq k-3$; $f(v_{2k-1}) = 12k$;

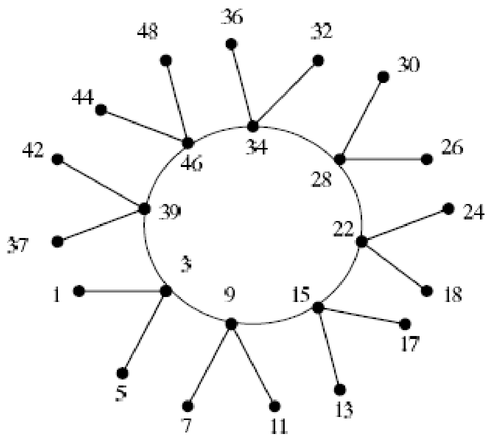
$f(v_{2k}) = 12k - 6$; $f(w_i) = 6i - 1$ for $1 \leq i \leq k-1$; $f(w_k) = 6k$;

$f(w_{k+i}) = 6k + 6i$ for $1 \leq i \leq k-2$; $f(w_{2k-1}) = 12k - 4$;

$f(w_{2k}) = 12k - 11$.

It can be verified that f(V) ∪ (f_(e) : e ∈ E} = {1, 2, 3, . . . , 6n}. Cn ⊙ K2 is also a super-mean graph.

Example 3.10 In Fig.5 the super mean marking of Cn ⊙ K2 is given.



Theorem 3.11 A super-mean graph is the double triangular snake D(Tn).

Proof Induction on n. We prove this outcome. In Fig.6, the super mean labelling of G1 = D(T2) is given.

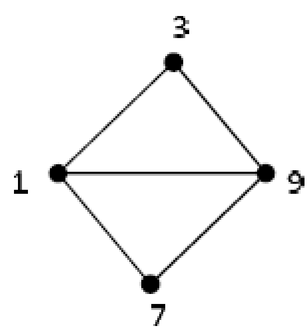


Fig.6

The result is, however, valid for n = 2. Let f be the super-mean G1 mark as seen in the figure above. Now D(T3) = (G1)f * (G1)f, D(T3) is a super mean graph according to Theorem 2.4. The result is, however, valid for n = 3. Assume that D(Tn−1) with the super mean marking g is a super mean graph. Today, D(Tn−1))g * (G1)f = D(Tn) is a super mean graph in Theorem 2.4. The finding is also valid for the n. Thus the outcome is valid for all n by inference theory. D(Tn) is therefore a super-mean graph.

Example 3.12 In Fig.7, the super-mean D(T6) labelling is given.

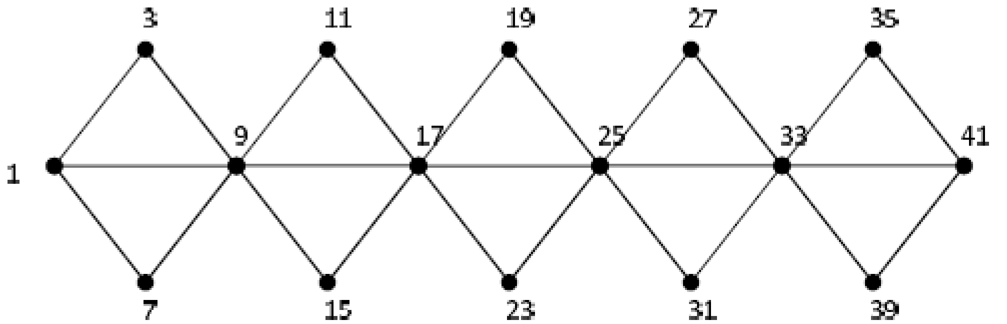


Fig.7

Theorem 3.13 A super mean graph for all m ≥2, n≥ 3 with the exception of n = 4 is the generalised antiprism $(\mathcal{A}_n^m)$.

Proof Let V $(\mathcal{A}_n^m)$ = {vj i : 1 ≤ i ≤ n, 1 ≤ j ≤ m} and E$(\mathcal{A}_n^m)$ = {vj i vj i+1, vjn vj 1 : 1 ≤ i ≤ n − 1, 1 ≤ j ≤ m} ∪ {vj i vj+1 i−1 , vj 1vj+1 n : 2 ≤ i ≤ n, 1 ≤ j ≤ m − 1} ∪ {vj i vj+1i : 1 ≤ i ≤ n and 1 ≤ j ≤ m− 1}.

Case 1 n is unique.

Define f : V $(\mathcal{A}_n^m)$ → {1, 2, 3, . . . , p + q = 4mn − 2n} by

$$f(v_i^j) = 4(j-1)n + 2i - 1 \text{ for } 1 \leq i \leq \frac{n+1}{2} \text{ and } 1 \leq j \leq m;$$

$$f(v^j_{\frac{n+3}{2}}) = 4(j-1)n + n + 3 \text{ for } 1 \leq j \leq m;$$

$$f(v^j_{\frac{n+3}{2}+i}) = 4(j-1)n + n + 3 + 2i \text{ for } 1 \leq i \leq \frac{n-3}{2} \text{ and } 1 \leq j \leq m.$$

F is therefore a super-mean marking of$(\mathcal{A}_n^m)$ f. This is a super-mean line, thus$(\mathcal{A}_n^m)$.

Case 2 n is even and n 6≠4.

Define f : V $(\mathcal{A}_n^m)$ → {1, 2, 3, . . . , p + q = 4mn − 2n} by

$$f(v_1^j) = 4(j-1)n + 1 \text{ for } 1 \leq j \leq m; f(v_2^j) = 4(j-1)n + 3 \text{ for } 1 \leq j \leq m;$$

$$f(v_3^j) = 4(j-1)n + 7 \text{ for } 1 \leq j \leq m; f(v_4^j) = 4(j-1)n + 12 \text{ for } 1 \leq j \leq m;$$

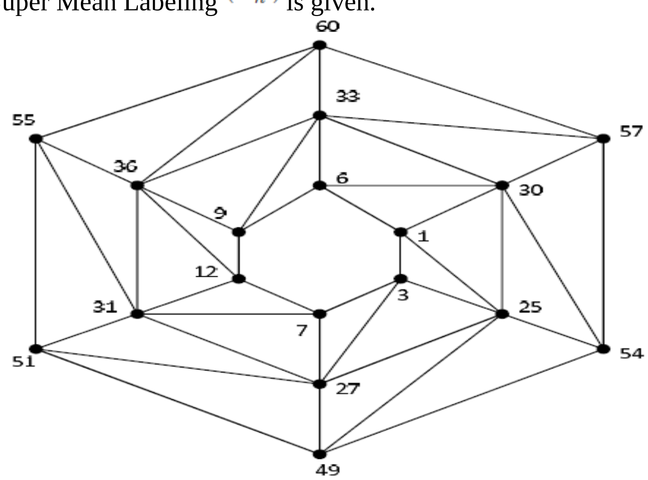
$$f(v_{4+i}^j) = 4(j-1)n + 12 + 4i \text{ for } 1 \leq j \leq m \text{ and } 1 \leq i \leq \frac{n-6}{2};$$

$$f(v^j_{\frac{n+2}{2}+i}) = 4(j-1)n + 2n + 1 - 4i \text{ for } 1 \leq j \leq m \text{ and } 1 \leq i \leq \frac{n-6}{2};$$

$$f(v_{n-1}^j) = 4(j-1)n + 9 \text{ for } 1 \leq j \leq m; f(v_n^j) = 4(j-1)n + 6 \text{ for } 1 \leq j \leq m.$$

F is therefore a super-mean marking of$(\mathcal{A}_n^m)$ f. This is a super-mean line, thus$(\mathcal{A}_n^m)$.

Example 3.14 In Fig.8, the Super Mean Labeling $(\mathcal{A}_n^m)$ is given.



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