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SHEAR WAVE PROPAGATION IN A NON-HOMOGENEOUS ANISOTROPIC MEDIUM UNDER THE INFLUENCE OF GRAVITY FIELD WITH COUPLE STRESS

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ABSTRACT: The effect of gravity field with couple stress on the propagation of shear waves in a non-homogeneous, anisotropic, incompressible and initially stressed medium is discussed. The frequency equation for the problem that determines the phase velocity of the shear waves using linear inhomogeneties has been obtained. Several particular cases are also examined. The numerical calculations are carried out for distinct values of rigidity parameter, density parameter, anisotropic factor, initial stress parameter, gravity field parameter, couple stress parameter as well as wave number and the results are shown graphically using MATLAB.

KEYWORDS: incompressible, anisotropic, shear wave, couple stress, gravity field.

I. INTRODUCTION

Wave motion in an anisotropic solid is fundamentally different from motion in an isotropic solid. Any homogeneous uniform material whose properties vary with direction is anisotropic, and its elastic behaviour with respect to appropriate seismic wavelengths can be described by effective elastic constants in one of a range of anisotropic symmetry systems. Seismic waves penetrating such anisotropic material display a number of characteristic and diagnostic effects, which are different from those of waves propagating in isotropic solids.

In 1965, Biot [1] observed that the initial stresses present in the medium have notable effect on the propagation of elastic waves and an approximate representation of a laminated medium by a continuous structure with anisotropic properties. Das., et.al. [4] have investigated the effect of initial stresses in the propagation of edge waves in homogeneous isotropic plates. In his study, composite structures of thinly laminated materials were discussed and are used in the study of buckling and vibrations. The propagation characteristics of elastic waves in a layered laminated medium under initial and couple stress were discussed by Pal Roy [5]. These studies also consider the dynamics of a laminated medium of Maxwell type solids and surface instability of a laminated material. Pijush Pal Roy., et. al. [6] discussed about the propagation of edge waves in a thinly layered laminated medium with stress couples under initial stresses. Mainly his work is based on an approximate representation of a laminated medium by an equivalent anisotropic continuum with average initial and couple stresses. In his paper he concluded that for a specific compression, the presence of couple stresses increases the velocity of wave propagation with the increase of wave numbers, whereas the reverse is the case when there is no couple stress. Abd-Alla., et.al. [7] discussed the propagation of Rayleigh waves in magneto-elastic half space of orthotropic material under gravity field and initial stress. The propagation of "Edge Waves" in an initially stressed anisotropic plate of finite thickness and infinite length was investigated by Dey, S., et.al.[8]. In his paper, the velocity of an edge wave has been computed for various initial stress parameter and different anisotropy ratio. Abd-Alla, A.M., et.al. [9] discussed the effect of gravity field on the propagation of S-wave in a non-homogeneous anisotropic incompressible and initially stressed medium. Selim [10] inspected the thermal effects on propagation of transverse waves and attenuation of seismic body waves in dissipative medium under initial and couple stresses. Shear waves can be used to analyze the directionally dependent mechanical properties of anisotropic media. Anitha, L., et.al.[11] have investigated the efficiency of gravity field, initial stress, and dry sand with couple stress on shear wave propagation in an inhomogeneous, anisotropic, thinly layered laminated medium.

In this work, an attempt has been made to find the velocities of propagation, damping of shear waves in a non-homogeneous, anisotropic, incompressible and initially stressed medium under the effect of gravity field with couple stress. Also the frequency equation for the problem has been obtained to determine the phase velocity of the shear waves using linear inhomogeneties and analyze the results graphically using MATLAB.

II. Formulation of the problem

We consider a laminated medium of n thin layers and of total thickness H under the average initial compressive stress P along the x- axis. Suppose that the y- axis is normal to the plane of the laminations, the stress-strain relations of the composite medium are given by [1]

$$s_{11} = 2Ne_{xx} + s \qquad (2.1a)$$

$$s_{22} = 2Ne_{yy} + s \qquad (2.1b)$$

$$s_{12} = 2Qe_{xy} \qquad (2.1c)$$

where s_{11} *and* s_{22} are principal stress components along x and y directions respectively, s_{12} is the shear stress component in the xy-plane and

$$s_{33} = s_{13} = s_{23} = 0 \ and \ s = \frac{1}{2}(s_{11} + s_{22}) \qquad (2.1d)$$

The bending rigidity of the medium taken into accounts by introducing couple stresses [2]. The total bending moment of the medium is

$$D = r\frac{\partial^2 v}{\partial x^2} \qquad (2.1e)$$

where r is the couple stress factor.

According to Biot [1], in the absence of external forces, the basic dynamical equations of motion for an infinite, initially stressed medium in plane strain, when the influence of couple stress is taken into account are given by [3]

$$\frac{\partial s_{11}}{\partial x} + \frac{\partial s_{12}}{\partial y} - P\frac{\partial \omega}{\partial y} - \rho g\frac{\partial v}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2} \qquad (2.2a)$$

$$\frac{\partial s_{12}}{\partial x} + \frac{\partial s_{22}}{\partial y} - P\frac{\partial \omega}{\partial x} + \rho g\frac{\partial u}{\partial x} = \rho\frac{\partial^2 v}{\partial t^2} + r\frac{\partial^4 v}{\partial x^4} \qquad (2.2b)$$

where ω is the rotational component in the z direction given by

$$\omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad (2.3)$$

The incremental strain components given by [1]

$$e_{xx} = \frac{\partial u}{\partial x},\ e_{yy} = \frac{\partial v}{\partial y},\ e_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \qquad (2.4)$$

The condition of incompressibility $e_{xx} + e_{yy} = 0$ is satisfied by

$$u = -\frac{\partial \varphi}{\partial y} \ and \ v = \frac{\partial \varphi}{\partial x} \qquad (2.5)$$

where $\varphi = \varphi(x, y, t)$.

III. Solution of the problem

Substituting from equations (2.1a) to (2.1d) and (2.3) to (2.5) in (2.2a) and (2.2b), we obtain

$$\frac{\partial s}{\partial x}-2N\frac{\partial^3\phi}{\partial x^2\partial y}+\frac{\partial}{\partial y}\left[Q\left(\frac{\partial^2\phi}{\partial x^2}-\frac{\partial^2\phi}{\partial y^2}\right)\right]-\frac{P}{2}\left(\frac{\partial^3\phi}{\partial x^2\partial y}+\frac{\partial^3\phi}{\partial y^3}\right)=\rho\left(g\frac{\partial^2\phi}{\partial x^2}-\frac{\partial^3\phi}{\partial t^2\partial y}\right) \tag{3.1}$$

$$\frac{\partial s}{\partial y}+Q\left(\frac{\partial^3\phi}{\partial x^3}-\frac{\partial^3\phi}{\partial x\partial y^2}\right)+\frac{\partial}{\partial y}\left(2N\frac{\partial^2\phi}{\partial x\partial y}\right)-\frac{P}{2}\left(\frac{\partial^3\phi}{\partial x^3}+\frac{\partial^3\phi}{\partial x\partial y^2}\right)=\rho\left(g\frac{\partial^2\phi}{\partial x\partial y}+\frac{\partial^3\phi}{\partial t^2\partial x}\right)+r\frac{\partial^5\varphi}{\partial x^5} \tag{3.2}$$

Assuming non-homogeneities as

$$N=N_1(1+by)\ ,\ Q=Q_1(1+ay)\ ,\ \rho=\rho_1(1+cy) \tag{3.3}$$

where N_1 and Q_1 are rigidities and ρ_1 is the density in homogeneous isotropic medium.

Substituting from (3.3) into equations (3.1) and (3.2), we get

$$\left\{\begin{array}{l}\left[Q_1(1+ay)+\frac{P}{2}\right]\frac{\partial^4\varphi}{\partial y^4}-\left[Q_1(1+ay)-\frac{P}{2}\right]\frac{\partial^4\varphi}{\partial x^4}+\\ \left[4N_1(1+by)-2Q_1(1+ay)\right]\frac{\partial^4\varphi}{\partial x^2\partial y^2}\\ +\left[2bN_1-aQ_1\right]\frac{\partial^3\varphi}{\partial x^2\partial y}+aQ_1\frac{\partial^3\varphi}{\partial y^3}\end{array}\right\}=\left\{\begin{array}{l}\rho_1(1+cy)\left[\frac{\partial^4\varphi}{\partial t^2\partial x^2}+\frac{\partial^4\varphi}{\partial t^2\partial y^2}\right]\\ -\rho_1 c\left(g\frac{\partial^2\phi}{\partial x^2}-\frac{\partial^3\varphi}{\partial t^2\partial y}\right)+r\frac{\partial^6\varphi}{\partial x^6}\end{array}\right\} \tag{3.4}$$

For propagation of sinusoidal waves in any direction, we take the solution of equation (3.6) as

$$\varphi(x,y,t)=\mathrm{M}e^{ik(x\cos\theta+y\sin\theta-c_1t)} \tag{3.5}$$

where θ is the angle made by the direction of propagation with the x-axis, c_1 and k are the velocity of propagation and wave number respectively.

Using equation (3.5) in (3.4) and equating real and imaginary parts separately, we get

$$\left(\frac{c_1}{\beta}\right)^2=\frac{1}{1+cy}\left\{\begin{array}{l}\left[(1+ay)+\frac{P}{2Q_1}\right]\sin^4\theta+\left[(1+ay)-\frac{P}{2Q_1}\right]\cos^4\theta\\ +2\left[\frac{2N_1}{Q_1}(1+by)-(1+ay)\right]\sin^2\theta\cos^2\theta-\frac{gc}{k^2\beta^2}\cos^2\theta+\frac{rk^2}{Q_1}\cos^6\theta\end{array}\right\} \tag{3.6}$$

$$\left(\frac{c_1}{\beta}\right)^2=\left[\frac{2bN_1}{cQ_1}-\frac{a}{c}\right]\cos^2\theta+\frac{a}{c}\sin^2\theta \tag{3.7}$$

where $\beta=\left(\frac{Q_1}{\rho_1}\right)^{\frac{1}{2}}$, the velocity of shear wave in homogeneous isotropic medium.

IV. Analysis of problem in homogeneous medium

(i)Analysis of equation (3.6):

Case I: In this case Q is homogeneous ($a\to 0$)

i.e., rigidity along vertical direction is constant

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$$\left(\frac{c_1}{\beta}\right)^2 = \frac{1}{1+cy}\left\{\begin{array}{l}\left[1+\frac{P}{2Q_1}\right]\sin^4\theta + \left[1-\frac{P}{2Q_1}\right]\cos^4\theta \\ +2\left[\frac{2N_1}{Q_1}(1+by)-1\right]\sin^2\theta\cos^2\theta - \frac{gc}{k^2\beta^2}\cos^2\theta + \frac{rk^2}{Q_1}\cos^6\theta\end{array}\right\} \tag{4.1}$$

The velocity along $x-direction\ (\cos\theta=1, \sin\theta=0, c_1=c_{11})\ is$

$$c_{11}^2 = \frac{\beta^2}{1+cy}\left[1-\frac{P}{2Q_1}-\frac{gc}{k^2\beta^2}+\frac{rk^2}{Q_1}\right] \tag{4.2}$$

Equation (4.2) depends on the initial stress and average couple stress coefficient.

The velocity of propagation along $y-direction\ (\cos\theta=0, \sin\theta=1, c_1=c_{22})\ is$

$$c_{22}^2 = \frac{\beta^2}{1+cy}\left[1+\frac{P}{2Q_1}\right] \tag{4.3}$$

Case II: In this case N is homogeneous ($b\to 0$)

i.e., rigidity along horizontal direction is constant.

$$\left(\frac{c_1}{\beta}\right)^2 = \frac{1}{1+cy}\left\{\begin{array}{l}\left[(1+ay)+\frac{P}{2Q_1}\right]\sin^4\theta + \left[(1+ay)-\frac{P}{2Q_1}\right]\cos^4\theta \\ +2\left[\frac{2N_1}{Q_1}-(1+ay)\right]\sin^2\theta\cos^2\theta - \frac{gc}{k^2\beta^2}\cos^2\theta + \frac{rk^2}{Q_1}\cos^6\theta\end{array}\right\} \tag{4.4}$$

The velocity along $x-direction\ (\cos\theta=1, \sin\theta=0, c_1=c_{11})\ is$

$$c_{11}^2 = \frac{\beta^2}{1+cy}\left[(1+ay)-\frac{P}{2Q_1}-\frac{gc}{k^2\beta^2}+\frac{rk^2}{Q_1}\right] \tag{4.5}$$

Equation (4.5) depends on depth, wave number and average couple stress coefficient.

The velocity of propagation along $y-direction\ (\cos\theta=0, \sin\theta=1, c_1=c_{22})\ is$

$$c_{22}^2 = \frac{\beta^2}{1+cy}\left[(1+ay)+\frac{P}{2Q_1}\right] \tag{4.6}$$

For $P>0$, the velocity along y-direction may increase considerably and the wave becomes dispersive.

Case III: In this case N, Q and ρ are homogeneous ($a\to 0, b\to 0, c\to 0$)

$$\left(\frac{c_1}{\beta}\right)^2 = \left\{\begin{array}{l}\left[1+\frac{P}{2Q_1}\right]\sin^4\theta + \left[1-\frac{P}{2Q_1}\right]\cos^4\theta \\ +2\left[\frac{2N_1}{Q_1}-1\right]\sin^2\theta\cos^2\theta + \frac{rk^2}{Q_1}\cos^6\theta\end{array}\right\} \tag{4.7}$$

In the absence of initial stress, the velocity equation becomes

$$\left(\frac{c_1}{\beta}\right)^2 = \sin^4\theta + \cos^4\theta + 2\left[\frac{2N_1}{Q_1}-1\right]\sin^2\theta\cos^2\theta + \frac{rk^2}{Q_1}\cos^6\theta \tag{4.8}$$

The velocity along $x-direction\ (\cos\theta=1, \sin\theta=0, c_1=c_{11})\ is$

$$c_{11}^2 = \beta^2\left[1+\frac{rk^2}{Q_1}\right] \tag{4.9}$$

The velocity along $y-direction\ (\cos\theta=0, \sin\theta=1, c_1=c_{22})\ is$

$$c_{22}^2 = \beta^2$$

It does not depend on the anisotropy. However, in other directions the anisotropy affects the velocity.

Case IV: In the absence of initial stress $P\to 0$, the velocity is obtained by

$$\left(\frac{c_1}{\beta}\right)^2 = \frac{1}{1+cy}\left\{\begin{matrix}(1+ay)\sin^4\theta+(1+ay)\cos^4\theta \\ +2\left[\frac{2N_1}{Q_1}(1+by)-(1+ay)\right]\sin^2\theta\cos^2\theta-\frac{gc}{k^2\beta^2}\cos^2\theta+\frac{rk^2}{Q_1}\cos^6\theta\end{matrix}\right\} \tag{4.10}$$

The velocity along $x-direction\ (\cos\theta=1, \sin\theta=0, c_1=c_{11})\ is$

$$\left(\frac{c_{11}}{\beta}\right)^2 = \frac{1}{1+cy}\left\{(1+ay)-\frac{gc}{k^2\beta^2}+\frac{rk^2}{Q_1}\right\} \tag{4.11}$$

The velocity along $y-direction\ (\cos\theta=0, \sin\theta=1, c_1=c_{22})\ is$

$$\left(\frac{c_{22}}{\beta}\right)^2 = \frac{1+ay}{1+cy} \tag{4.12}$$

(ii) Analysis of Equation (3.7):

In the absence of the initial stress P in equation (3.7), following three cases have been analyzed.

Case I: In this case Q is homogeneous ($a\to 0$)

i.e., rigidity along vertical direction is constant.

$$\left(\frac{c_1}{\beta}\right)^2 = \frac{2bN_1}{cQ_1}\cos^2\theta \tag{4.13}$$

This shows that velocity of shear wave is always damped.

The velocity along $x-direction\ (\cos\theta=1, \sin\theta=0, c_1=c_{11})\ is$

$$\left(\frac{c_{11}}{\beta}\right)^2 = \frac{2bN_1}{cQ_1} \tag{4.14}$$

i.e., the velocity in x-direction is damped by $\frac{2bN_1}{cQ_1}$.

The velocity along $y-direction\ (\cos\theta=0, \sin\theta=1, c_1=c_{22})\ is$

$$\left(\frac{c_{22}}{\beta}\right)^2 = 0 \tag{4.15}$$

i.e., no damping takes place.

Case II: In this case N is homogeneous ($b\to 0$) i.e., rigidity along horizontal direction is constant.

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$$\left(\frac{c_1}{\beta}\right)^2 = -\frac{a}{c}\cos^2\theta + \frac{2a}{c}\sin^2\theta \tag{4.16}$$

The velocity along $x-direction\ (\cos\theta = 1, \sin\theta = 0, c_1 = c_{11})\ is$

$$\left(\frac{c_{11}}{\beta}\right)^2 = -\frac{a}{c} \tag{4.17}$$

i.e., the velocity in x-direction is damped by $-\frac{a}{c}$.

The velocity along $y-direction\ (\cos\theta = 0, \sin\theta = 1, c_1 = c_{22})\ is$

$$\left(\frac{c_{22}}{\beta}\right)^2 = \frac{2a}{c} \tag{4.18}$$

This shows that actual velocity in $y-direction$ is damped by $\frac{2a}{c}$.

Case III: In this case N and Q are homogeneous($a \to 0, b \to 0$),

$$\left(\frac{c_1}{\beta}\right)^2 = 0 \tag{4.19}$$

i.e., no damping takes place.

V. Numerical Analysis and Discussion

To get numerical information on the velocity of shear wave in the non-homogeneous initially stressed medium we introduce the following non-dimensional parameters:

$A = \frac{a}{b}$ (rigidity parameter), $B = by,\ C = \frac{c}{b},\ C_1 = \frac{c_1}{\beta}$, $\overline{N} = \frac{N_1}{Q_1}$ (anisotropy factor),

$\overline{P} = \frac{P}{2Q_1}$ (initial stress factor), $G = \frac{gb}{k^2\beta^2}$ (gravity parameter), $R = \frac{r}{Q_1}$ (couple stress parameter)

Using these parameters in the equation (3.8), we obtain

$$C_1^2 = \frac{1}{1+BC}\left\{\begin{aligned}&\left[(1+AB)+\overline{P}\right]\sin^4\theta + \left[(1+AB)-\overline{P}\right]\cos^4\theta \\ &+ 2\left[2\overline{N}(1+B)-(1+AB)\right]\cos^2\theta\sin^2\theta - CG\cos^2\theta + Rk^2\cos^6\theta\end{aligned}\right\}$$

Various graphs are plotted with the help of MATLAB by taking the parameters as A=4; B=0 to 6; C=0.7; $\overline{P}$ =0.5; $\overline{N}$ =2.5; R=0.0876765; k=1.

The effect of a non-homogeneity, anisotropy, initial stress, couple stress, gravity field and wave number on shear wave velocity C_1 with respect to depth B is shown in Figures [1-11]. It is obvious that shear wave velocity increases with the increasing of the depth B.

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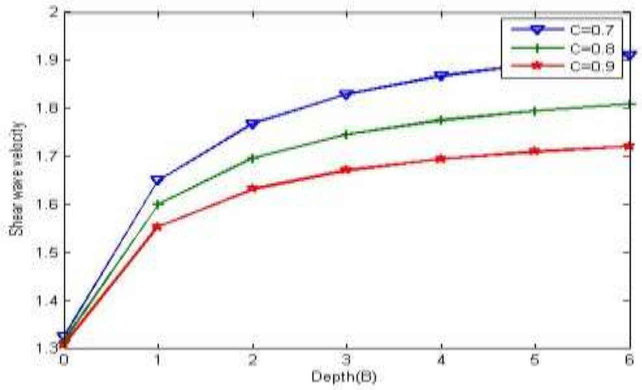

Fig-1 Variation in velocities of shear wave in the direction of 30° with $x-axis$ at different depth and different values of density parameter C: A=4; k=1; B=0 to 6; $\overline{P}$ =0.5; G=0.3; $\overline{N}$ =2.5; R=0.0876765; C=0.7,0.8,0.9.

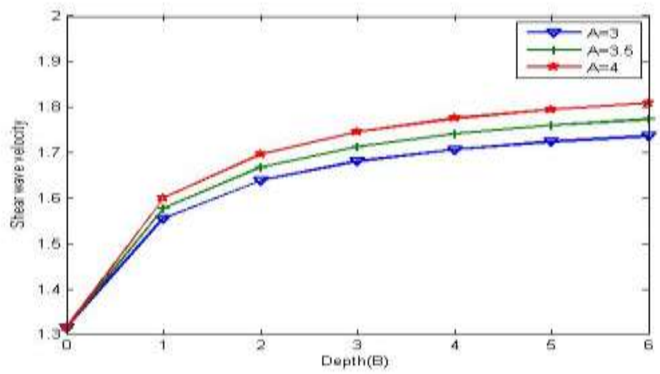

Fig-2 Variation in velocities of shear wave in the direction of 30° with $x-axis$ at different depth and different values of rigidity parameter A: k=1; $\overline{P}$ =0.5; B=0 to 6; $\overline{N}$ =2.5; C=0.8; R=0.0876765; G=0.3; A=3, 3.5, 4.

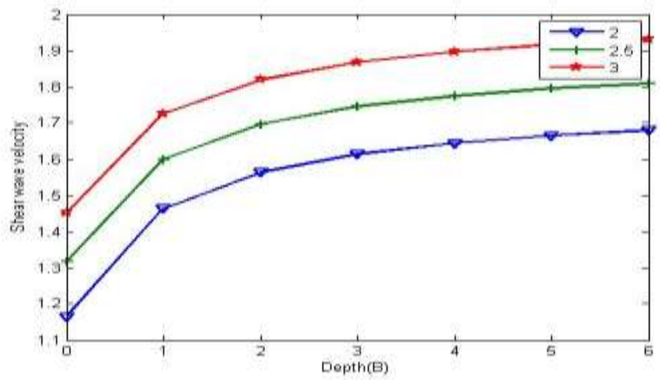

Fig-3 Variation in velocities of shear wave in the direction of 30° with $x-axis$ at different depth and different values of $\overline{N}$ (anisotropy): k=1; $\overline{P}$ =0.5; B=0 to 6; C=0.8; R=0.0876765; G=0.3; A=4; $\overline{N}$ =2, 2.5, 3.

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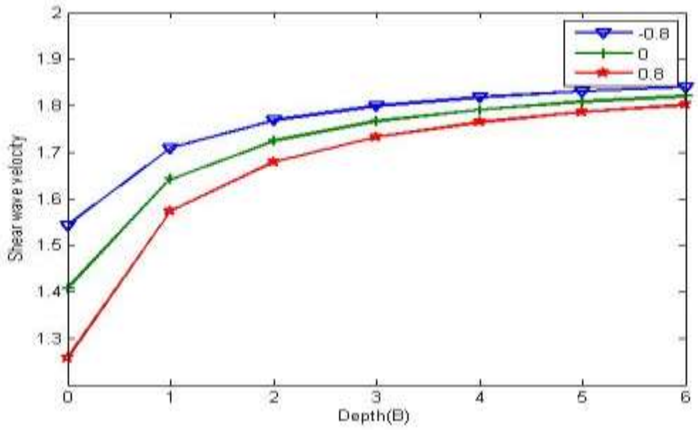

Fig-4 Variation in velocities of shear wave in the direction of 30° with $x-axis$ at different depth and different values of initial stress parameter $\overline{P}$: k=1; $\overline{P}$ =-0.8, 0, 0.8; B=0 to 6; C=0.8; R=0.0876765; G=0.3; A=4; $\overline{N}$ =2.5.

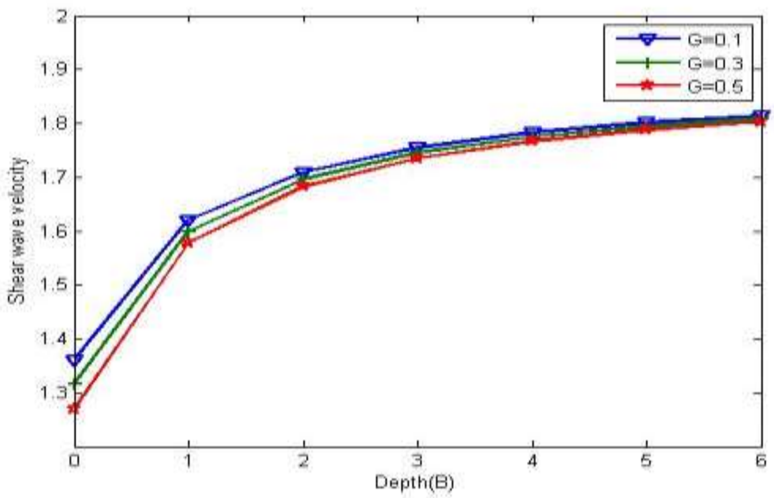

Fig-5 Variation in velocities of shear wave in the direction of 30° with $x-axis$ at different depth and different values of gravity parameter G: k=1; $\overline{P}$ =0.5; B=0 to 6; C=0.8; R=0.0876765; G=0.1, 0.3, 0.5; A=4; $\overline{N}$ =2.5.

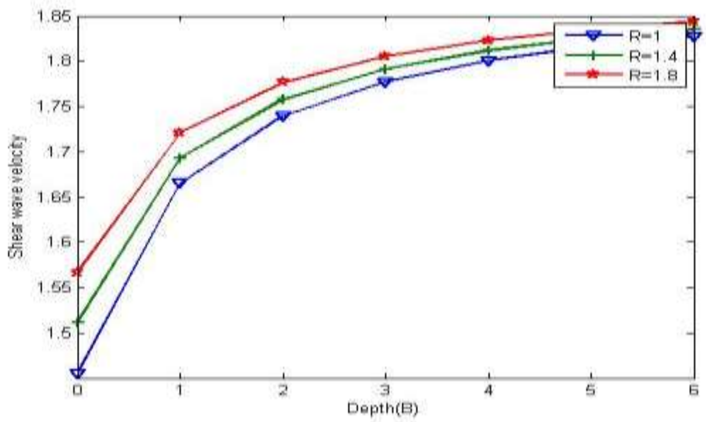

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Fig-6 Variation in velocities of shear wave in the direction of 30° with $x-axis$ at different depth and different values of couple stress parameter R: k=1; $\overline{P}$ =0.5; B=0 to 6; C=0.8; R=1, 1.4, 1.8; G=0.3; A=4; $\overline{N}$ =2.5.

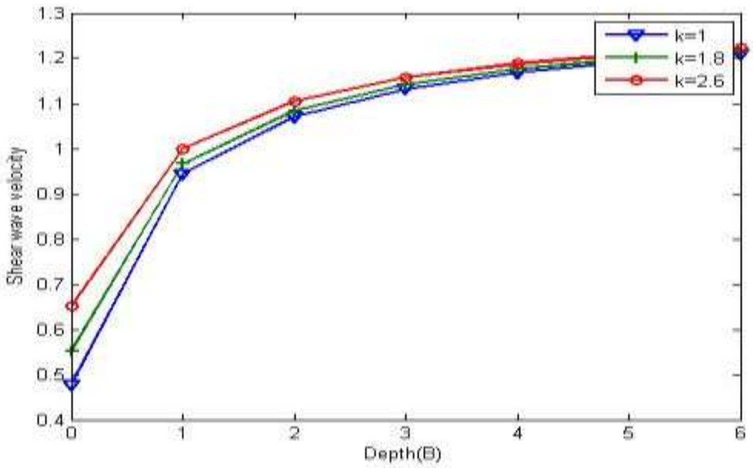

Fig-7 Variation in velocities of shear wave in the direction of 30° with $x-axis$ at different depth and different values of wave numbers k: $\overline{P}$ =0.5; B=0 to 6; C=0.8; R=0.08; G=0.3; A=4; $\overline{N}$ =2.5; k=1, 1.8, 2.6.

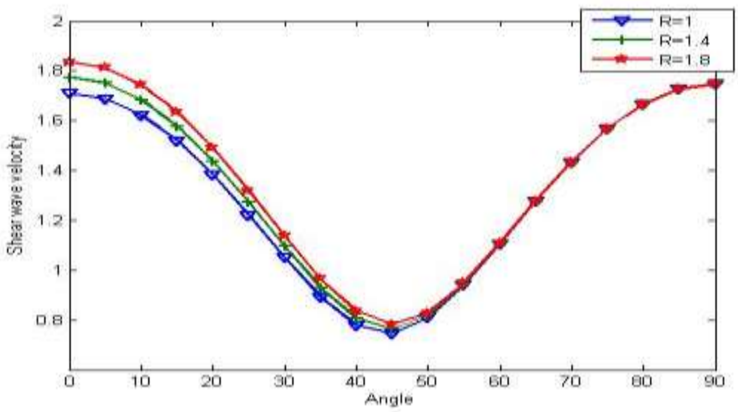

Fig-8 Variation in velocities of shear wave with various angle ϕ and different values of couple stress parameter R=1, 1.4, 1.8.

VI. Conclusion

The anisotropy, non-homogeneity of the medium, the initial stress, gravity field, couple stress, the direction of wave propagation and the depth have significant effect on the phase velocity of shear waves. The velocity of the shear wave propagation is proportional to the rigidity parameter, couple stress parameter, wave number as well as anisotropy parameter and also inversely proportional to the density parameter, gravity parameter as well as the initial stress parameter. Finally, it is observed that the increasing values of couple stress increase the velocity of shear wave propagation within the range $(0^{\circ}, 76^{\circ})$ but the velocity is equal within the range $(77^{\circ}, 90^{\circ})$.

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VII. REFERENCES

[1] Biot, M.A. (1965). Mechanics of Incremental Deformations, John Wiley and Sons, New York.

[2] Biot, M.A. (1972). A new approach to the mechanics of orthotropic multilayered plates, Int. J. Solids Struct. 8: 475-490.

[3] Biot, M.A. (1972). Simplified dynamics of multilayered orthotropic viscoelastic plates, Int. J. Solids Struct. 8: 490-509.

[4] Das, S.C. and Dey, S. (1970). Edge waves under initial stress, Appl. Sci. Res. 22:382-389.

[5] Pal Roy, P. (1984). Wave Propagation in a Thinly Two-layered Laminated Medium with Stress Couples Under Initial Stresses, Acta Mech, 54: 1-21.

[6] Pijush Pal Roy., Lokenath Debnath. (1988). Propagation of Edge Waves in a Thinly Layered Laminated Medium With Stress Couples Under Initial Stresses, Journal of Applied Mathematics and Simulation, 1(4): 271-286.

[7] Abd-Alla, A.M., Hammad, A.H., Abo-Dahab. (2004). Rayleigh waves in magneto elastic half-space of orthotropic material under influence of initial stress and gravity field, Applied Mathematics and Computations, 154: 583-597.

[8] Dey, S., De, P.K. (2009). Edge wave propagation in an incompressible anisotropic initially stressed plate of finite thickness, International Journal of Computational Cognition, 7(3): 55-60.

[9] Abd-Alla, A.M., Mahmoud, S.R., Abo-Dahab, S.M., Helmy, M.I. (2011). Propagation of S-wave in a non-homogeneous anisotropic incompressible and initially stressed medium under influence of gravity field, Applied Mathematics and Computation, 217: 4321-4332.

[10] Selim, M.M. (2016). Thermal Effects on Propagation of Transverse Waves in Anisotropic Incompressible Dissipative Pre-Stressed Plate, Appl. Math. Inf. Sci., 10(3): 1091-1095.

[11] Anitha, L., Selvi, S. (2020). Propagation of S-waves in an Elastic Half-Space of Anisotropic Incompressible Sandy Medium under Gravity Field and Couple Stress, Test Engineering and Management, 83(March-April 2020): 14629 – 14636.