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Lexicographic Goal Programming Technique in the Production Sector to Resolve Different Conflicting Objectives

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Abstract:
In the production sector, various problems are arising while dealing with products production, especially in planning. In the production environment, it is apparent that every industry or firm tries to get maximum profit for their efforts while producing different varieties of products but there are more that may want to attain several, sometimes opposing or conflicting goals. In reality, we find it very difficult to gratify all such laid objectives at their highest degree simultaneously. To survive in the present competitive market, industrialists have to choose the best decision model. Thus, in the present research paper we aim to develop the lexicographic goal programming technique in the real-life problem of bakery production system. The results obtained with LINDO 6.1 computer software reveals that all the desired conflicting goals are attained and achieved optimal solution successfully.
Key Words : Lexicographic goal programming, Bakery production, Decision making, Conflicting goals.
MSC2010 : 90C29, 97D30.

1. Introduction:

The economic development of India is much dependent on the production sector such as the agriculture sector, industrial sector, agricultural and livestock production etc. Thus, in the current competitive scenario, it becomes important for industrialists to choose the best decision model to support their production planning so that economy of India would be boosted. In the production system, planning is a very chief part and it is the one most challenging problem for decision-makers in the present world. The planning in the production system governs what, when and how many products should be produced to meet the demands of the customers, compete with other factories in its area and at the same time deliver maximum profit to the factory with all available requirements. In earlier days, profit maximization or cost minimization was only objective or goal in the production atmosphere or the small-scale industrial organizations. The procedure which is available that applies to only a single objective i.e., minimize cost or maximize profit, frequently used is linear programming. In practical life, the decision-maker of any factory wants to fulfil various objectives, not only maximize sales or minimize delivery cost, maximize profit, avoid employee layoffs, get rid of a labour strike, labour and industrial relations, faster delivery but also higher quality, employment stability, labour and industrial relations, etc. though the core objective is an expansion of the profit. Most of these goals are conflicting, so in many important real-world decision-making circumstances, it may not be easy to reduce all such goals into a single objective. With the help of decent and appropriate production planning, a production system accomplishes success. Many tactics deal with multiple objectives in decision-making problems. Goal programming is a prevalent method which deals with these conflicting goals which are desired to be achieved by the decision-maker in his production atmosphere. In other words, we can say that goal programming is the stretched case of linear programming. We see that multi-objective is a very important approach in operation research or management science because of the multi-criteria nature of most real-life problems. The importance of goal programming is in those situations, where a satisfactory solution is sought rather than an optimal one.

Goal programming was first discussed by Charnes and Cooper(1961) and further developed by Ijiri (1965), Lee(1972) and Ignizio(1976; 1985). The goal programming model attempts to obtain a satisfactory level of goal attainment that would be the best feasible solution in the view of priorities (relative importance of the goals). The objective of goal programming is to minimize deviation from each of the goals that have been set from the desired target (instead of maximizing profit or minimizing the cost of linear programming). The basic concept of goal programming is that whether goals are achievable or not, an objective will be stated in which optimization gives a result that comes as close as possible to the desired goals (satisfied solution). The main difference between linear programming and goal programming is that linear programming has to minimize or maximize(optimise) a single objective while in goal programming we have different objectives (multi-objective). For a single goal problem, the design and solution are similar to linear programming withthe prohibiting that, if complete goal accomplishment is not possible, goal programming will offer a solution and information to the decision-makers (Orumie and Ebong, 2013).

The enormous majority of the initial goal programming formulations used the lexicographic goal programming (LGP) variant and it is also called 'pre-emptive' goal programming. The distinguishing feature of lexicographic goal programming to other goal programming models is the presence of several priority levels. Each priority

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level contains some unwanted deviations which have to be minimised subject to constraints. In lexicographic goal programming, the decision-maker must rank the goals of the problem in order of importance. The model is then optimized by focussing merely on the most important goal as closely as possible, before proceeding to the next higher goal, and so on to the least goal i.e., the objective functions are prioritized such that fulfilment of the first goal is far more important than the fulfilment of the second goal which is far more important than attainment of the third goal, etc., such that the optimum value of a higher priority goal is never dishonoured by a lower priority goal.

Charnes and Cooper (1977) discussed the potentials aimed at several additional classes of goal functional are sightseer and demarcated and they emphasized that the classifications are too provided for substitute depictions and plain results from distinct operational belongings including piecewise linear functions. Ignizio(1985) has developed the multi-objective mathematical displaying view multiplex model. Gauss (1986) applied a goal programming model for shaping priority weights. Lin and O`leary (1993) examined the application of goal programming to multi-objective financial management decision situations. Orgyczak (1994) developed the goal programming model of the reflexive point method. Bryson et al. (1995) have developed the goal programming model for pair wise assessment ratio scales. Extended lexicographic and goal programming a unifying approach was developed by Singh et al. (2000) and Romero (2001). Jafari et al.(2008)havepresented the lexicographic linear goal programming approach aimed at recognizing the optimal composite of agriculture product in the rice farmland of Maydonsar Koshteli village from Babol region, a city in the north of Iran and they found that there are many differences among allocated surfaces in different rice. Hadi-Vencheh and Aghajani (2010) offered a production programming model in the textile industry by using a goal programming technique and reachable production programming by them is a new modelling process that uses a single objective model and the tracker price of limitations and management priorities to cover all different goals of the organization without overlooking some. Taghizadeh et al. (2015) have extended the discussion of optimization production planning using fuzzy goal programming procedures and they follow two goals. These goals which they include are reducing costs and increasing revenue for the production of numerous products. Jyoti and Mannan (2016) discussed the implementation of goal programming in the financial estimation of the well popular public school namely St. Brother’s Public School, India and to minimize the total weights and priorities associated with meeting the necessities for optimal Operating cost allocation of the institution. Satkhed et al. (2018) provided the goal programming model for the finest consumption of water, diminishing the labour cost, lessening the chemical fertilizer by endorsing the organic fertilizer and maximizing the soil fruitfulness in 2018. Noviyarsi et.al (2018) proposed the optimization of production planning to maximize the chocolate factory`s profit by the practice of goal programming technique.

The purpose of this study is to formulate lexicographic goal programming (LGP) technique in one of the small-scale bakery factories in India to determine an optimal solution that would help in the economic development of the country. To use lexicographic goal programming, the present study assumes all the goals are analogous in importance, also one or more goals are more important than others. The main focus is on achieving as carefully as possible these first-priority goals. After finding an optimal solution to first priority goals, other priority goals are also optimized. There are two procedures based on linear programming for solving lexicographic goal programming problems. They are consecutive and streamlined procedures. In this study, a consecutive procedure is used. In the first section of the consecutive procedure; only the first priority goal model was solved and deviation variables were strong-minded. After that, the second priority goal model is solved but the first deviation variables are added to this model. The same procedure is repetitive for any lower priority goals.

2. Mathematical Formulation and Analysis:

The proposed lexicographic goal programming model is given as:

Minimize:

$$Z = \sum_{k=1}^{m} q_k ({h_k}^- + {h_k}^+)$$

Subject to:

Goal constraints:

$$\sum_{j=1}^{n} c_{kj}\, t_j + {h_k}^- - {h_k}^+ = p_k \; for\; k = 1,2,3,\dots,m$$

Hard constraints:

$$\sum_{j=1}^{n} c_{kj}\, t_j \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} p_k \;\; for\; k = m+1, \dots, m+g$$

$$t_j, {h_k}^-, h_k^+ \;\geq 0,$$

$(k = 1,2,3,4,\dots,m)$, (j=1, 2, 3, …, n), where

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q_k is the pre-emptive factor/priority level allocated to each relative goal in rank order (i, e., $q_1 > q_2 > \dots > q_n$).
Z = objective function = Summation of all deviations.
h_k^+ denotes the positive deviation variable from overachieving the kthgoal.
h_k^- denotes the negative deviation variable from underachieving the kthgoal.
p_k represents the goals.
t_j represents the decision variables and
c_{kj} denotes the decision variable coefficients.

3. Lexicographic Goal Programming Problem Description:

The production planning problem is exemplified in the real-life bakery manufacturing sector to show the applicability and mathematical justification of the proposed technique and the small size bakery manufacturing production planning problem is considered in the present study. The decision-maker always faces complications in utilizing labourers, raw material and machine time in producing different varieties of bakery products throughout the year. So, the decision-maker requires the perfect plan with available resources, machine time and existing labour force to maximize the profit, meet the daily demand of customers and compete with other factories in its area.

The model is based on the following assumptions:

1. It should be kept in mind that enough machine power and required raw materials are available to produce each product with decent quality.
2. Manpower should be appropriately disseminated to make all products. Also, labourers may be organized into groups according to their proficiency.
3. It may be a guideline that the waste would be minute and the remaining waste products may be applied appropriately or transported somewhere.

Now, the data is obtained from the bakery factory given in the tabular form. The factory is producing five products. The outline of the data set used in the XYZ factory is given in the following tables.

Table 1. The cost, price and profit of every product.

Products	Variable Notation	Costs (Rs)	Price (Rs)	Profit (Rs)
Muffins	t_1	0.20	1.00	0.80
Cake	t_2	1.40	2.90	1.50
Cream puff	t_3	0.42	1.50	1.08
Egg tart	t_4	0.14	1.50	1.28
Cheese tart	t_5	0.20	1.15	0.95

Table 2. The main ingredients required for each product for one item in grams.

Material	Products				
	t_1	t_2	t_3	t_4	t_5
Flour	8.000	9.100	5.012	9.262	8.523
Margarine	5.025	8.520	4.801	4.573	5.000
Sugar	7.472	10.122	2.607	3.010	2.322
Egg	8.000	9.000	8.300	6.322	2.021
Cream cheese	0.000	0.000	0.000	0.000	4.215
Evaporated milk	0.000	0.000	4.231	6.012	0.000

Table 3. The raw material offered for each day.

Raw material	Total available per day(gram)
Flour	4000
Margarine	2800
Sugar	2600
Eggs	3400

Cream cheese	450
Evaporated milk	1100

Table 4. The time essential for each product (in minutes)

	Products				
	t_1	t_2	t_3	t_4	t_5
Labour	1.45	0.30	1.00	0.60	0.80
Machine	1.30	0.80	1.00	0.55	0.50

The flexible goals taken for this purpose are:

1. **Profit goal:**

The decision-maker of the factory would like to attain a satisfactory profit level of 560 Rs per day according to their available resources.

2. **Manpower goal:**

The decision-maker would like to minimize labour over time and does not want to use more than 430 minutes of labour per day.

3. **Machine goal:**

This goal indicates that the decision-maker does not want to use less than 430 minutes of usage of the baking machine to ensure the products are not undercooked.

The above data in tabular form is converted into the lexicographic goal programming model and then the result would be drawn with the help of LINDO 6.1 software.

Goal (1) is given priority 2, goal (2) and goal (3) are given priority 1. The significance of this model is that the satisfaction of goals placed in a higher priority level is stringently chosen to that of goals placed in lower priority levels.

Step 1: Priority 1 (minimize over goal 2 and under goal 3)

Minimize:

Z $=h_2^+ + h_3^-$

Subject to:

Goal Constraints

$0.80t_1 + 1.50t_2 + 1.08t_3 + 1.28t_4 + 0.95t_5 - h_1^+ + h_1^- = 560$(ProfitGoal)

$1.45t_1 + 0.30t_2 + 1.00t_3 + 0.60t_4 + 0.80t_5 - h_2^+ + h_2^- = 430$(Manpower Goal)

$1.30t_1 + 0.80t_2 + 1.00t_3 + 0.55t_4 + 0.50t_5 - h_3^+ + h_3^- = 430$ (Machine Goal)

Hardconstraints

$$8.000t_1 + 9.100t_2 + 5.012t_3 + 9.262t_4 + 8.523t_5 \leq 4000$$
$$5.025t_1 + 8.520t_2 + 4.801t_3 + 4.573t_4 + 5.000t_5 \leq 2800$$
$$7.472t_1 + 10.122t_2 + 2.607t_3 + 3.010t_4 + 2.322t_5 \leq 2600$$
$$8.000t_1 + 9.000t_2 + 8.300t_3 + 6.322t_4 + 2.021t_5 \leq 3400$$
$$0.000t_1 + 0.000t_2 + 0.000t_3 + 0.000t_4 + 4.215t_5 \leq 450$$

$0.000t_1 + 0.000t_2 + 4.231t_3 + 6.012t_4 + 0.000t_5 \leq 1100$, where

$t_1, t_2, t_3, t_4, t_5, h_1^+, h_1^-, h_2^+, h_2^-, h_3^+, h_3^- \geq 0$ and

$t_1$represents the total units of muffins manufactured each day.

$t_2$represents the total units of cakes manufactured each day.

$t_3$represents the total units of cream puff manufactured each day.

t_4 represents the total units of egg tart produced each day.

t_5 represents the total units of cheese tart produced each day.

h_1^+displays the overachievement of desired profit.

h_1^- displays the underachievement of desired profit.

h_2^+ shows the overtime of labour working hours (overutilization).

h_2^- shows the idle time oflabour working hours (underutilization).

h_3^+ is the overutilization of machine practice.

h_3^- is the underutilization of machine practice.

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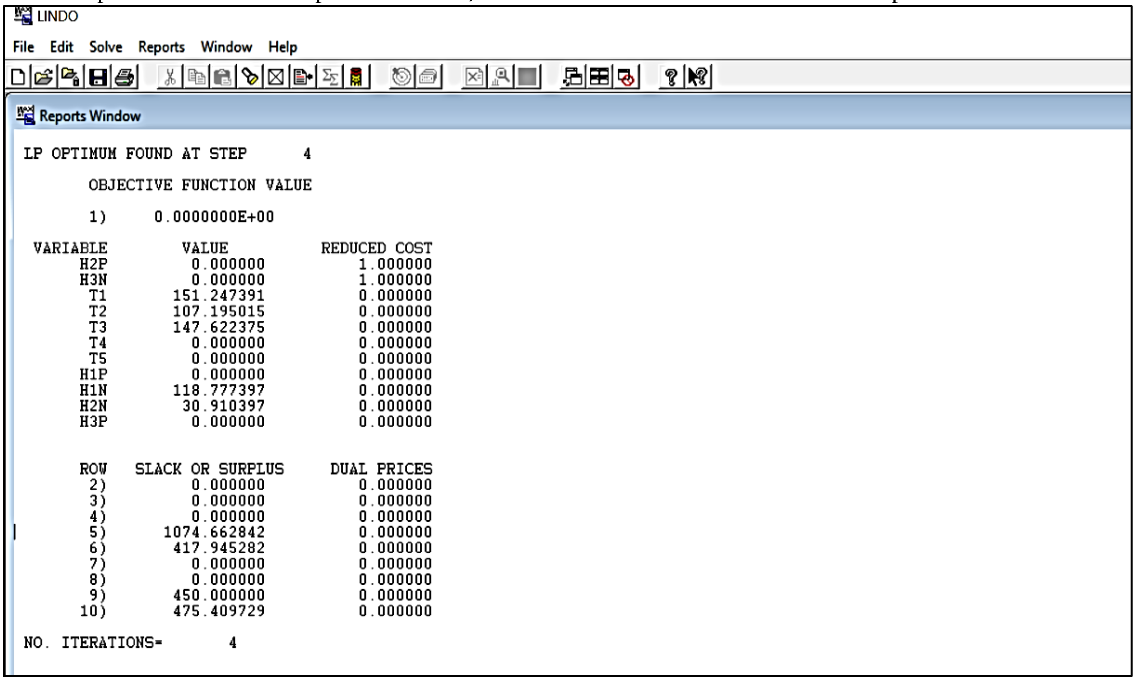
4. Results:

With the help of LINDO 6.1computer software, we draw the result of the above-noted problem as shown below:



```
LINDO
File Edit Solve Reports Window Help
Reports Window

LP OPTIMUM FOUND AT STEP      4

       OBJECTIVE FUNCTION VALUE

       1)      0.0000000E+00

 VARIABLE        VALUE          REDUCED COST
      H2P         0.000000          1.000000
      H3N         0.000000          1.000000
       T1       151.247391          0.000000
       T2       107.195015          0.000000
       T3       147.622375          0.000000
       T4         0.000000          0.000000
       T5         0.000000          0.000000
      H1P         0.000000          0.000000
      H1N       118.777397          0.000000
      H2N        30.910397          0.000000
      H3P         0.000000          0.000000

      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          0.000000
       3)         0.000000          0.000000
       4)         0.000000          0.000000
       5)      1074.662842          0.000000
       6)       417.945282          0.000000
       7)         0.000000          0.000000
       8)         0.000000          0.000000
       9)       450.000000          0.000000
      10)       475.409729          0.000000

NO. ITERATIONS=       4
```

Fig. 1. The LINDO 6.1 output result of step 1

Step 2: Priority 2 {minimize under goal (1)}

and keeping goal (1) and goal (2) fixed as $d_2^+ = 0$ & $d_3^- = 0$

Minimize:

Z = h_1^-

Subject to:

Goal Constraints

$0.80t_1 + 1.50t_2 + 1.08t_3 + 1.28t_4 + 0.95t_5 - h_1^+ + h_1^- = 560$ (Profit Goal)

$1.45t_1 + 0.30t_2 + 1.00t_3 + 0.60t_4 + 0.80t_5 - h_2^+ + h_2^- = 430$ (Manpower Goal)

$1.30t_1 + 0.80t_2 + 1.00t_3 + 0.55t_4 + 0.50t_5 - h_3^+ + h_3^- = 430$ (Machine Goal)

Hard constraints:

$$8.000t_1 + 9.100t_2 + 5.012t_3 + 9.262t_4 + 8.523t_5 \leq 4000$$
$$5.025t_1 + 8.520t_2 + 4.801t_3 + 4.573t_4 + 5.000t_5 \leq 2800$$
$$7.472t_1 + 10.122t_2 + 2.607t_3 + 3.010t_4 + 2.322t_5 \leq 2600$$
$$8.000t_1 + 9.000t_2 + 8.300t_3 + 6.322t_4 + 2.021t_5 \leq 3400$$
$$0.000t_1 + 0.000t_2 + 0.000t_3 + 0.000t_4 + 4.215t_5 \leq 450$$
$$0.000t_1 + 0.000t_2 + 4.231t_3 + 6.012t_4 + 0.000t_5 \leq 1100$$
$$h_2^+ = 0\,,$$

$h_3^- = 0$,

where $t_1, t_2, t_3, t_4, t_5, h_1^+, h_1^-, h_2^+, h_2^-, h_3^+, h_3^- \geq 0$

and $t_1, t_2, t_3, t_4, t_5, h_1^+, h_1^-, h_2^+, h_2^-, h_3^+, h_3^-$ are defined as in ($).

The result of the above problem is drawn with the help of LINDO 6.1 computer software as given below:

```
LINDO
File  Edit  Solve  Reports  Window  Help

Reports Window

LP OPTIMUM FOUND AT STEP      9

       OBJECTIVE FUNCTION VALUE

       1)      8.941554

 VARIABLE        VALUE          REDUCED COST
      H1N         8.941554          0.000000
       T1       100.248390          0.000000
       T2        99.943703          0.000000
       T3       134.373978          0.000000
       T4        58.122505          0.000000
       T5       106.761559          0.000000
      H1P         0.000000          1.000000
      H2P         0.000000          0.000000
      H2N         0.000000          1.007452
      H3P         0.000000          2.336166
      H3N         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000         -1.000000
       3)         0.000000          1.007452
       4)         0.000000         -2.336166
       5)       166.783417          0.000000
       6)         0.000000          0.137244
       7)        66.151825          0.000000
       8)         0.000000          0.210820
       9)         0.000000          0.047411
      10)       182.031235          0.000000
      11)         0.000000          1.007452
      12)         0.000000          2.336166

NO. ITERATIONS=       9
```

Fig. 2. The LINDO 6.1 output result of step 2

5. Discussion:

Table 5. Achieved value for goals and variables in both steps.

Steps						
1. LGP Problem Priority 1	**Variables**	$t_1 =$ 151.247391	t_2 = 107.195015	t_3 =147.622375	$t_4 = 0$	$t_5 = 0$
	Achieved Goal	Goal(1)= 441.2226	Goal(2) = 399.089	Goal(3) = 429.999		
2. LGP Problem Priority 2	**Variables**	t_1 = 100.24839	t_2 = 99.943703	t_3 =134.373978	t_4 = 58.122505	t_5 = 106.761559
	Achieved Goal	Goal (1) = 551.058	Goal (2) = 430	Goal (3) = 430		

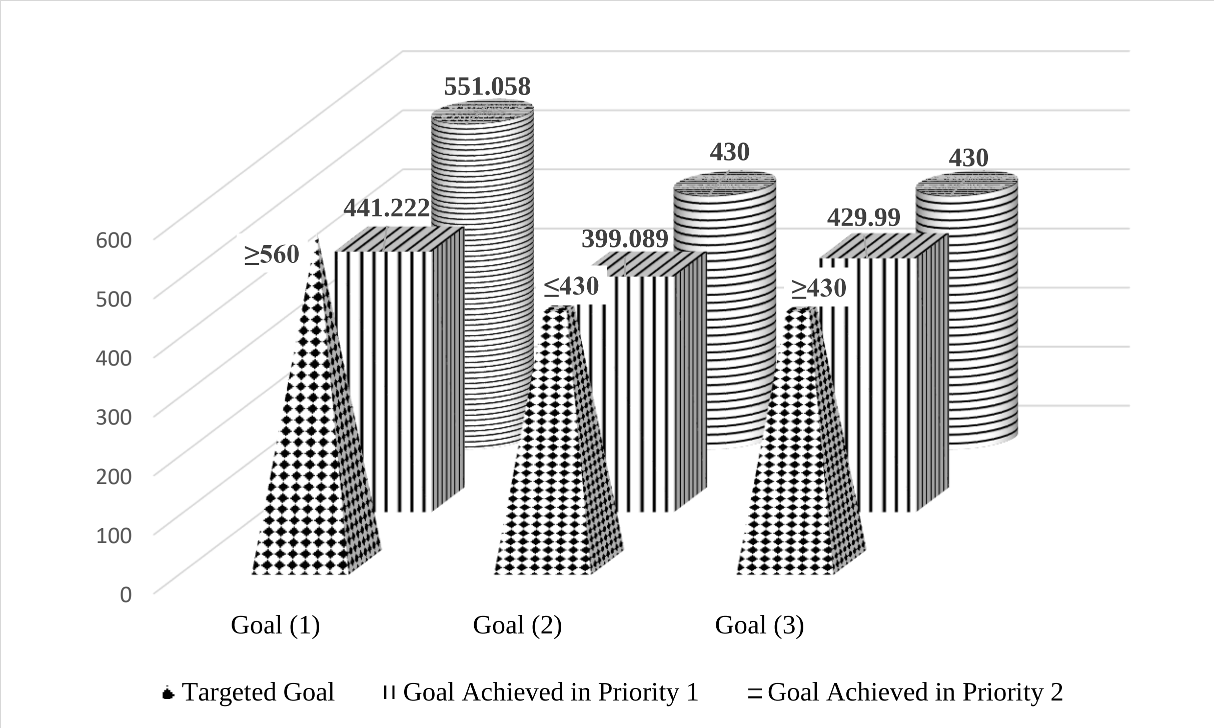


Fig. 3. Chart of goals

It is evident from table 5 and fig. 3, manpower goal and machine goal in priority 1 are satisfied fully and manpower time is reduced by 30.910404 minutes but the profit goal is not satisfied fully with the underachievement value of 118.777397 Rs. In priority 1, the decision-maker would make approximately 151 muffins, 107 cakes and 148 cream puffs to achieve the desired target. In priority 2, manpower goal and machine goal are fully satisfied at their highest degree and the satisfying solution is also obtained for-profit goal because of minimum underachievement deviation of 8.941554.In priority 2, the decision-maker would make approximately 100 muffins, 100 cakes, 135 cream puffs, 58 egg tarts and 107 cheese tarts to achieve the desired target and satisfy the customers' demands. Hence, the decision-maker can accomplish the desired profit subject to the given constraints based on the availability of labour, machine time and required material.

6. Conclusion:

The present study reveals that there is a potential to increase the productivity of industries by applying the lexicographic goal programming technique which would lead to the economic development of the country. In this study, the mathematical technique describing the uncertain life time decides optimum schedules determining maximum production and reducing cost as an indicator of much productivity in the production of bakery products. Thus, the proposed model provides a new approach to the industrialists for supporting their decision planning to obtain several objects simultaneously.

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