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A NEW POPULATION GROWTH MODEL: POPULATION PROJECTION OF INDIA AND SOME INDIAN STATES

Md. Irphan Ahamed[1*], Vikash Kumar[2], Bhuwaneshwar Kumar Gupt[3] and Manoshi Phukon[4]

[1,4]Department of Mathematics, Umshyrpi College, Shillong, Meghalaya-793004, India
[2,3]Department of Statistics, North-Eastern Hill University, Meghalaya-793022, India
[*]Corresponding Author: Md. Irphan Ahamed, Email: ahamed.irphan@gmail.com

Abstract

In this paper, a population growth model is proposed and its basic properties are discussed from the perspective of why the model is a suitable growth model for applying in sparse census data of developing countries like India. Using the proposed model, fitted curves with striking accuracies in the decadal census populations of India and 15 Indian states have been obtained, and hence populations of the country and the states have been estimated from 1901 to 2011 and projected from 2021 to 2091. The estimated and projected populations are found to be reasonable when observed in the light of observed population of census data, and hitherto population estimations and projections available in literature.

Key-words: Census Data, Non-linear Least Square Regression, Population Growth Model, Population Projection, Relative Growth Rate.

1. Introduction

In the history of development of growth curves to model both population dynamics and general biological growth, a large number of different methods have been developed by many population researchers. Verhulst (1938) did the pioneering work by developing logistic equation $\frac{dN(t)}{dt} = rN(t)\left\{1 - \frac{N(t)}{K}\right\}$, where $N(t)$ is population at time t, r is a parameter, K population carrying capacity of a region or a country, which laid the foundation for many successful predictive models gradually developed by later scientists. Verhulst (1838) assumed that a growth model must contain an increasing population of a geographical region must have an upper ceiling called carrying capacity K;the carrying capacity is the maximum population that can be sustained by the environmental resources of the region. However, in this logistic curve, it is observed that as long as population size $N(t)$ is smaller than K, the logistic growth rate curve is sigmoid, but the point of inflection of $N(t)$ is $\frac{K}{2}$which is unreasonable restriction and limiting the generality of the model. Thus,Verhulst's logistic growth curve suffers from this limitation inspite of its epoch-making breakthrough in population growth modeling. Thenceforth a number of researchers have been contributing various extended logistic growth models among which some of the prominent ones are Von Bertalanffy's growth curve (1938), Richards' growth model (1959), Blumberg's equation(1968), generic growth function of Turner et. al., (1969) etc.

Tosoularis-Wallace (2002) proposed a generalized logistic growth function as follows:

$$\frac{dN(t)}{dt} = rN(t)^{\alpha}\left\{1-\left(\frac{N(t)}{K}\right)^{\beta}\right\}^{\gamma},$$

and they obtained a parametric expression for population value at the inflection point as

$$N_{inf} = \left(1+\frac{\beta\gamma}{\alpha}\right)^{1/\beta} K, N_{inf} < N_0.$$

The authors analysed that the above expression of population value at inflection point could contain the inflection points of all the other above-mentioned curves as special cases.

However, all these growth models are found to be not doing well in estimating population of developing countries like India since available population data of developing countries like India are scanty, e.g., sparse decadal census data of population is available for India. Gupta et al.,(2012) explain that in such cases of sparse census data the relative growth rate (RGR) behaves differently unlike the common decreasing trend of logistic curves. The authors propose a population growth model in which they introduce rate modeling instead of size modeling. In rate modeling, the relative change of population size with respect to time is taken and model is made in such a way that RGR is function of time as follows:

$$\frac{1}{N(t)}\frac{dN(t)}{dt} = rt^{a}\left(1-\frac{t}{d}\right)^{c}, 0 \le t \le d \quad (1)$$

$$= 0, t > \mathrm{d}$$

They empirically illustrate RGR modeling (1) and show the advantages of (1) in terms of giving the S-shaped sigmoidal RGR curve while fitting to census data whose RGR behaves typically - increasing, primary increasing and then decreasing with time. Moreover, the growth law of the model can be analytically solved easily for studying the characteristics of the growth curve such as time point at which RGR is maximized, expression of $N(t)$ as function of t, $N(t)$ value at inflection point, etc.

On the other hand, population projection has been worked out by many workers and organisations. Keyfitz (1972), Lutz (2012) and Repetto (1987) dealt with the problem of population projection. Registrar General of India (RGI) conducted population projection (1997, 2006) for India and all her states which does not give any clue of about the nature of growth, saturation point and decay of population in future. Population projection by UNDP, World Bank, Dayson (2004) and Rahul et al., (2007) predicted, to a certain extent, about the nature of population growth trajectory in the future and also provided relatively better estimates of Indian

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population available in census data. Singh et al., (2018) showed precision in estimating Indian male, female and total population available in census data when they used Gaussian curve for population projection. Hence, they predicted Indian population will stabilise by 2050 and thence the population will decline.

In this paper, we have considered the RGR modeling, along the line Gupta et al.,(2012) had considered, and proposed a growth model which is simpler than theirs for solving analytically. But the proposed model has merits in terms of its appropriateness for fitting to the sparse census data and is flexible enough to represent all types of monotonic structure of RGR. By using the proposed model, population of India and 15 Indian states have been estimated and projected.

2. Data and Methods

2.1 ***Data***

The data used in this paper is census data of population of India and her fifteen states of Registrar General of India from the year 1901 to 2011.

2.2. ***Methods***

Considering the rate modeling, as was done by Gupta et al.,(2012) that RGR is a function of time in (1), we have proposed a simpler model as follows:

$$\frac{1}{P(t)}\frac{dP(t)}{dt} = rt^{a}\left\{1-\left(\frac{t}{d}\right)^{c}\right\}, \quad (2)$$

where $P(t)$ is population at time t, d is a parameter - the value of t at which population approaches to upper ceiling/carrying capacity – and *r*, a, and *c* are also parameters of the model.

The proposed model (2) is simpler for studying characteristics of RGR by finding out maximum point of RGR, the points of inflections of RGR and $P(t)$, and the expression of the curve -$P(t)$ in terms of function of *t* along with the parameters r, a, c and d.

The unusual shape of RGR for sparse data set available from census - increasing, primary increasing and then decreasing - were shown by Gupt et al.,(2012). However, the proposed model (2) with its key parameters a and c adapts to the monotonicity of RGR of census data. As RGR is zero at $t = 0$ and $t = d, P(t)$ stablises at $t = d$ when population approaches to the carrying capacity.

By simple integration with respect to time variable t, the solution of the proposed model (2) is obtained as

$$P(t) = P(0)exp\left\{\frac{rt^{a+1}}{a+1} - \frac{r\,t^{a+c+1}}{d^{c}(a+c+1)}\right\}, \quad (3)$$

where$P(0)$ is population at the beginning of time, $t = 0$.

Differentiating (2) with respect to t and equating to zero, we get the maximum of the RGR of (2) at

$$t = d\left(\frac{a}{a+c}\right)^{1/c}. \quad (4)$$

Then, the magnitude of maximum RGR is obtained as:

$$RGR_{max} = r\left(\frac{a}{a+c}\right)^{a/c} d^{a-c}\left\{\frac{d^c(a+c)-a}{a+c}\right\}. \quad (5)$$

By obtaining the second order derivative of (2) with respect to t and equating to zero, we get points of inflection of RGR at

$$t = \left\{\frac{a(a-1)d^c}{(a+c)(a+c-1)}\right\}^{1/(c-3)}. \quad (6)$$

The point of inflections of the curve (3), i.e.,$P(t)_{inf}$ is obtained from the equation:

$$rt^{a+1}\left\{1-\left(\frac{t}{d}\right)^c\right\}^2 - \left(\frac{t}{d}\right)^c(a+c) + a = 0 \;. \quad (7)$$

2.2.1 *Estimation of parameters*

In estimating the parameters $r, a, c, \& \, d$, general method of fitting non-linear regressions (Snedecor and Cochran, 1967) method is applied as follows:

The model (2) is put in the form:

$$log_e\, P(t) = \frac{rt^{a+1}}{a+1} - \frac{rt^{a+c+1}}{d^c(a+c+1)} + log_e P(0). \quad (8)$$

We take $Y = log_e P(t) = f(t, a, r, c, d)$and obtain the following:

$$\frac{\delta f}{\delta r} = \frac{t^{a+1}}{a+1} - \frac{t^{a+c+1}}{d^c(a+c+1)}, \qquad \frac{\delta f}{\delta a} = r\left[\frac{t^{a+1}\{(a+1)log\, t-1\}}{(a+1)^2} - \frac{1}{d^c}\frac{\{(a+c+1)\log t-1\}t^{a+c+1}}{(a+c+1)^2}\right],$$

$$\frac{\delta f}{\delta d} = \frac{r\, c\, t^{a+c+1}}{a+c+1}\frac{1}{d^{c+1}}, \frac{\delta f}{\delta c} = \frac{-rt^{a+c+1}}{d^c(a+c+1)^2}\{(a+c+1)\log t - (a+c+1)\log d - 1\}$$

and we assume $f_a = \frac{\delta f}{\delta a} = X_1,\; f_c = \frac{\delta f}{\delta a} = X_2,\; f_d = \frac{\delta f}{\delta d} = X_3,\; f_r = \frac{\delta f}{\delta r} = X_4$.

By using Taylor's theorem, we obtain

$$Y = f(t, a, c, d, r) = f(a_1, c_1, d_1, r_1) + (a - a_1)f_a + (c - c_1)f_c + (d - d_1)f_d + (r - r_1)f_r$$

f_a, f_c, f_d, and f_r are evaluated at $a = a_1$, $c = c_1$, $d = d_1$ and $r = r_1$, where a_1, c_1, d_1, and r_1 are the initially assumed estimates of the parameters a, c, d and r.

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We write, $Y_{res} = Y - f$ and assume the following linear regression.

$$Y_{res} = (a - a_1)X_1 + (c - c_1)X_2 + (d - d_1)X_3 + (r - r_1)X_4 + \varepsilon \qquad (9)$$

The computation of simple regression of Y_{res} on X_1, X_2, X_3, and X_4 give the regression coefficients $(\hat{a} - a_1)$, $(\hat{c} - c_1)$, $\left(\hat{d} - d_1\right)$, and $(\hat{r} - r_1)$ from which the estimates $\hat{a}$, $\hat{c}$, $\hat{d}$ and $\hat{r}$ are obtained. These estimates are again used in evaluating f_a, f_c, f_d, and f_r and then linear regression (9) is again performed to obtain a set of new estimates of the parameters. Every time, corresponding to each set of estimate of parameters, $P(t)$ is estimated for $t = 0,\ 10,\ 20,\ 30, \ldots, 110$, where the values of t are taken in place of the years 1901, 1911, …, 2011 respectively, until parameters are converged and sum of squares of difference between census $P(t)$ and estimated $P(t)$ is sufficiently minimised. While performing this non-linear least square regression method, we initially choose value of d subject to the following conditions:

(i) $d$is the value of time at which the population $P(t)$ approaches upper ceiling, i.e., carrying capacity.

(ii) While going through the iterations, i.e., successive linear regression of (9), the constraint i.e., carrying capacity of India is 200 crores (Gretchen et al., 1992), is contained.

(iii) While estimating parameters of the model (8) in fitting to census data of Indian states, the proportionate carrying capacity of every state along its population density is taken due care particularly in choosing initial value of d of each state.

(iv) Step factor λ (Box, 1960 & Hartley, 1961) is used in the iterations so as to ensure convergence of d to its most feasible value, and the other parameters so as the convergence of all the parameters minimises sum of squares of difference between predicted population and census population to its lowest possible value.

Thus models for trajectories of population growth for India and 15 Indian states are constructed and shown in Tables 1 to 4 in section 3.

3. Results and Discussion

It is seen that under the proposed model (2), population curve (8) has fitted precisely to the scanty census data of India and her fifteen states, shown in figures1 to 16. The model is helpful in predicting maximum population, below upper ceiling, upto which population of the country or a state may reach, stabilise and then decline. According to this prediction, India may attain its maximum population 1771.29 millions in the year 2048 and then start declining. Assam and Chhattisgarh are relatively lesser population density states which are predicted to rise their population upto 426.15 and 531.93 lakhs respectively in the years 2051 and 2054 and then decline, whereas relatively higher density population state Bihar is predicted to rise its population upto 1647.8 Lakhs in the year 2048 and then decline.

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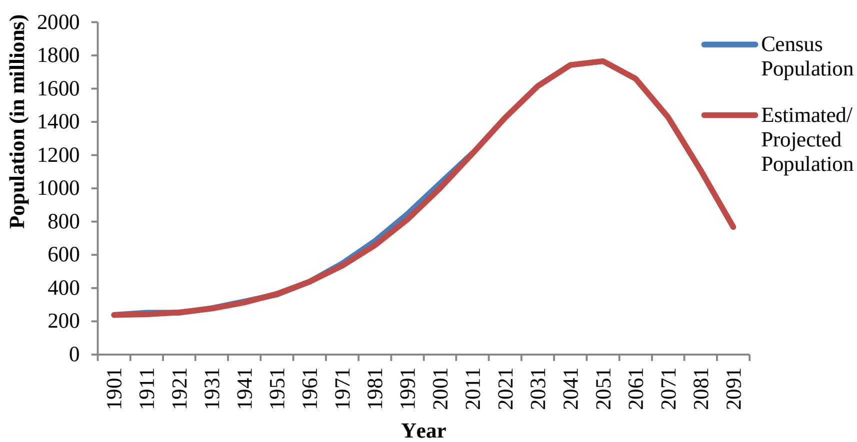
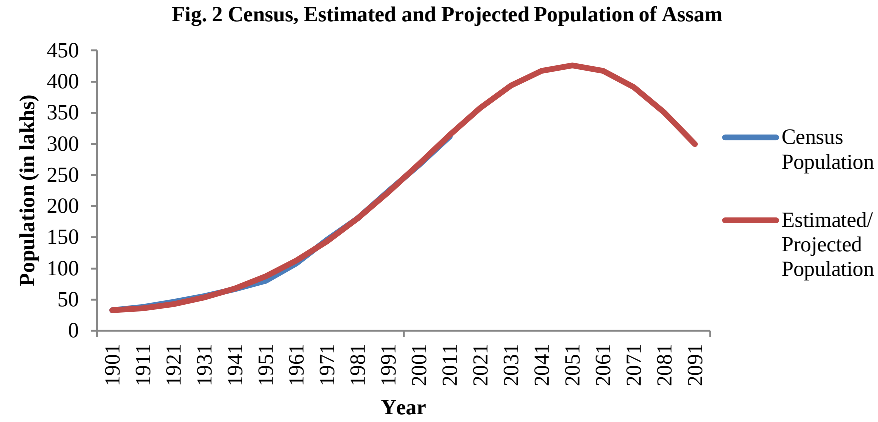
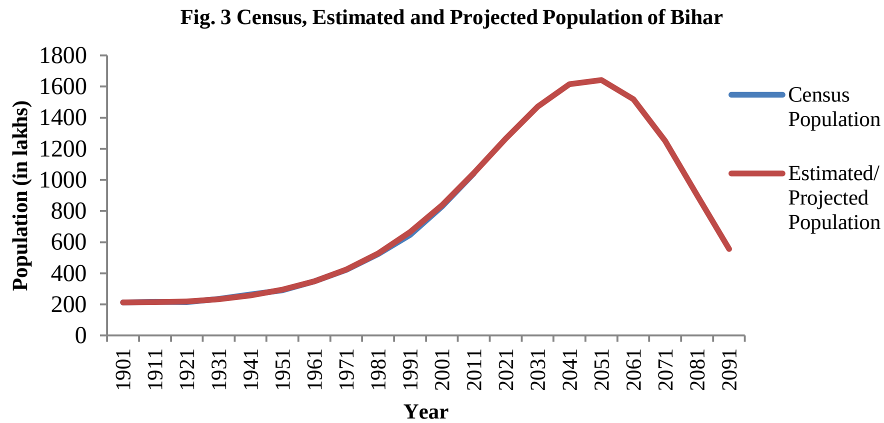
Table 1. Population estimation/projection of India, Assam, Bihar and Chhattisgarh

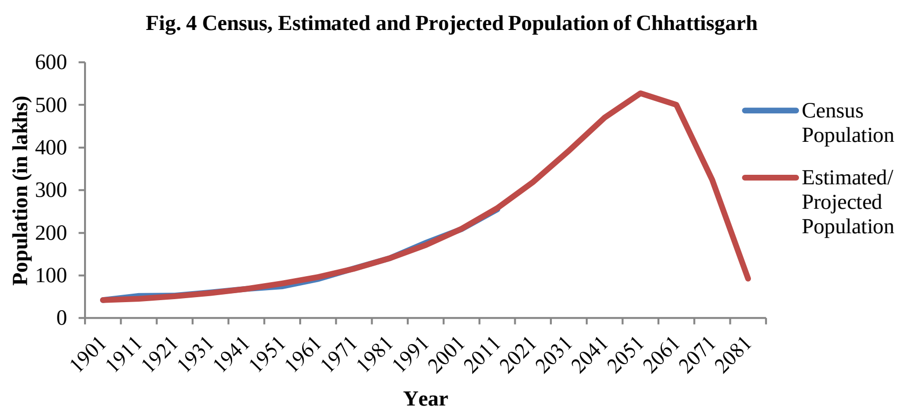
Country /States	India (Population in millions)		Assam (Population in lakhs)		Bihar (Population in lakhs)		Chhattisgarh (Population in lakhs)	
Parameters' estimate	r = 0.0002109 a=1.19 c= 1.3041801 d=147		r = 0.005434616 a=0.80501 c=0.201546713 d=150		r =0.0002549302 a=2.055468736 c=0.018012482 d=147		r = 0.005497314 a=0.2887511 c=13.31722265 d=153	
Years	Census Population	Estimated/ Projected Population	Census Population	Estimated/ Projected Population	Census Population	Estimated/ Projected Population	Census Population	Estimated/ Projected Population
1901	238.4	238.40	32.90	32.90	212.44	212.44	41.82	41.82
1911	252.1	241.92	38.49	36.07	215.67	213.50	51.92	45.43
1921	251.32	254.39	46.37	43.05	213.59	219.40	52.65	51.21
1931	278.98	277.65	55.60	53.60	234.38	232.97	60.29	58.85
1941	318.66	313.87	66.95	68.22	263.03	256.72	68.15	68.60
1951	361.09	366.00	80.29	87.63	290.85	293.64	74.57	80.90
1961	439.23	437.86	108.37	112.55	348.41	347.65	91.54	96.36
1971	548.16	533.86	146.25	143.44	421.26	423.68	116.37	115.76
1981	683.33	658.37	180.41	180.31	523.03	527.47	140.10	140.15
1991	846.42	814.30	224.14	222.43	645.31	664.48	176.15	170.86
2001	1028.74	1000.85	266.56	268.08	829.99	837.54	208.34	209.60
2011	1210.85	1210.38	312.06	314.47	1040.99	1042.87	255.45	258.39
2021		1425.08		357.79		1264.75		319.17
2031		1615.30		393.64		1471.09		392.14
2041		1742.02		417.61		1614.05		470.47
2051		1765.54		426.15		1641.14		527.44
2061		1660.12		417.30		1517.71		500.25
2071		1429.01		391.30		1251.64		324.25
2081		1110.57		350.64		901.69		92.74
2091		768.18		299.72		555.37		4.95

Fig. 1 Census, Estimated and Projected Population of India









Gujarat, Jharkhand, Karnataka and Madhya Pradesh states are predicted to rise their populations upto 877.17, 494.84, 832.54 and 1210.26 lakhs in the years 2050, 2054, 2051 and 2056 respectively and then decline. The estimates of the populations against available census populations are very close which are shown in the data provided in the Table 2 and fitted graphs given in figures 5 to 8. Madhya Pradesh population density is relatively the lowest among these four states and land area is big, that may be the reason why its population may stabilise in the later years compared to that of other states.

Table 2. Population estimation/projection of Gujarat, Jharkhand, Karnataka and Madhya Pradesh

States	Gujarat (Population in lakhs)		Jharkhand (Population in lakhs)		Karnataka (Population in lakhs)		Madhya Pradesh (Population in lakhs)	
Parameters' estimate	r =0.00059665 a=1.159968 c=0.42986 d=149		r =0.00083237 a=1.413817901 c=0.080888135 d=153		r = 0.00045765 a=1.149968 c=0.47986 d=150		r =0.000278375 a=1.170588 c=0.973973 d=155	
Years	Census Population	Estimated/ Projected Population	Census Population	Estimated/ Projected Population	Census Population	Estimated/ Projected Population	Census Population	Estimated/ Projected Population
1901	90.95	90.95	60.68	60.68	130.55	130.55	126.79	126.79
1911	98.04	93.67	67.47	61.91	135.25	133.63	142.49	129.11
1921	101.75	102.10	67.68	66.09	133.78	143.12	139.07	137.01
1931	114.90	116.66	79.09	73.57	146.33	159.22	153.27	151.41
1941	137.02	138.35	88.68	84.83	162.55	182.58	171.76	173.56
1951	162.63	168.68	96.97	100.57	194.02	214.18	186.15	205.23
1961	206.33	209.50	116.06	121.69	235.87	255.18	232.18	248.79
1971	266.97	262.75	142.27	149.13	292.99	306.61	300.17	307.02
1981	340.86	329.96	176.12	183.71	371.36	369.02	381.69	382.84
1991	413.10	411.55	218.44	225.78	449.77	441.99	485.66	478.63
2001	506.71	505.83	269.46	274.82	528.51	523.44	603.48	595.06
2011	604.40	607.96	329.88	328.87	610.95	609.16	726.27	729.54
2021		709.28		384.16		692.41		874.35
2031		797.41		434.94		764.18		1015.35
2041		857.82		473.94		814.14		1132.11
2051		876.92		493.65		832.54		1200.84
2061		846.13		488.15		812.64		1200.36
2071		765.51		455.20		753.00		1119.99
2081		645.17		397.64		658.81		965.98
2091		503.31		323.28		541.37		762.62

Fig. 5 Census, Estimated and Projected Population of Gujarat

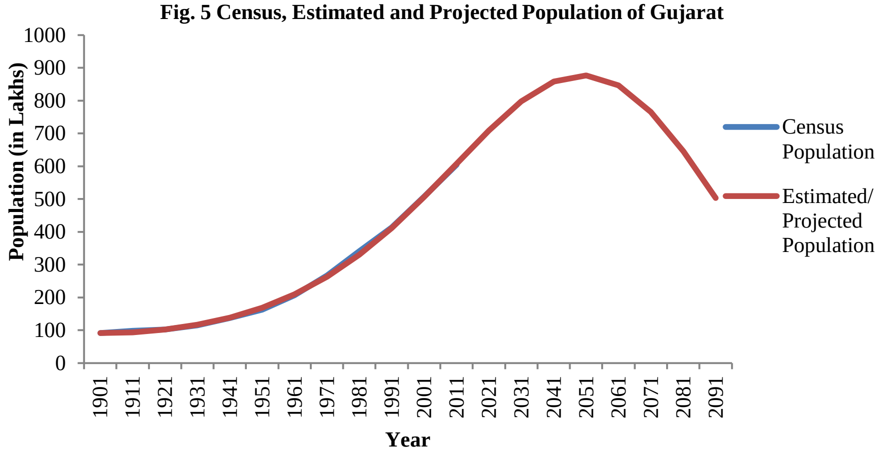


Fig. 6 Census, Estimated and Projected Population of Jharkhand

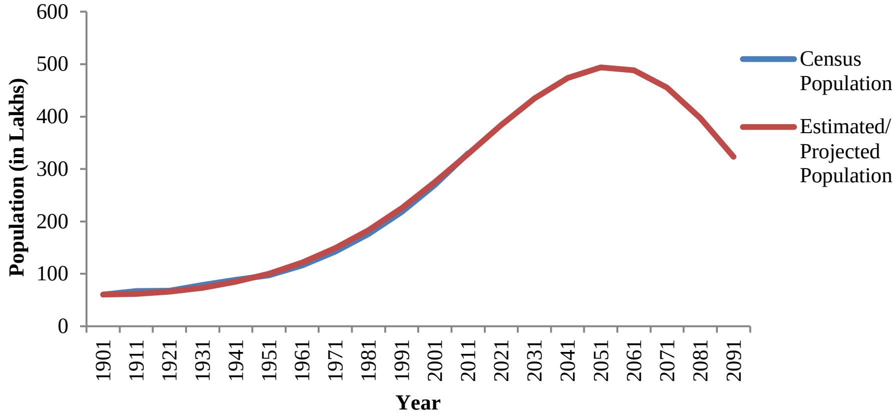


Fig. 7 Census, Estimated and Projected Population of Karnataka

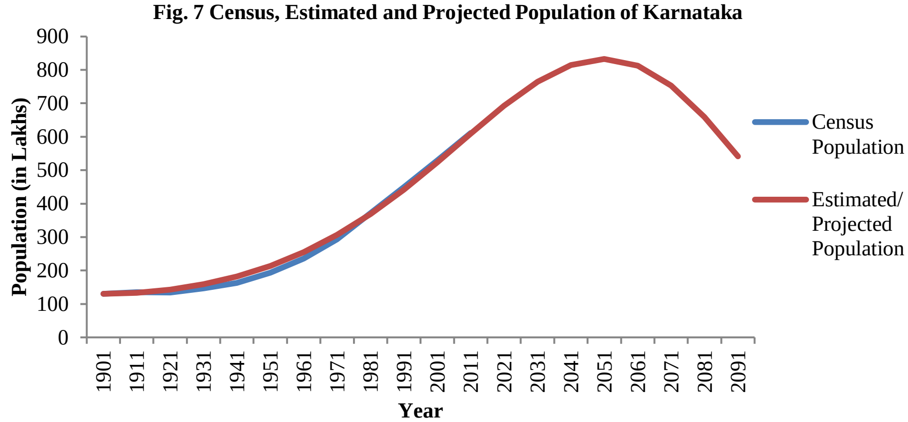
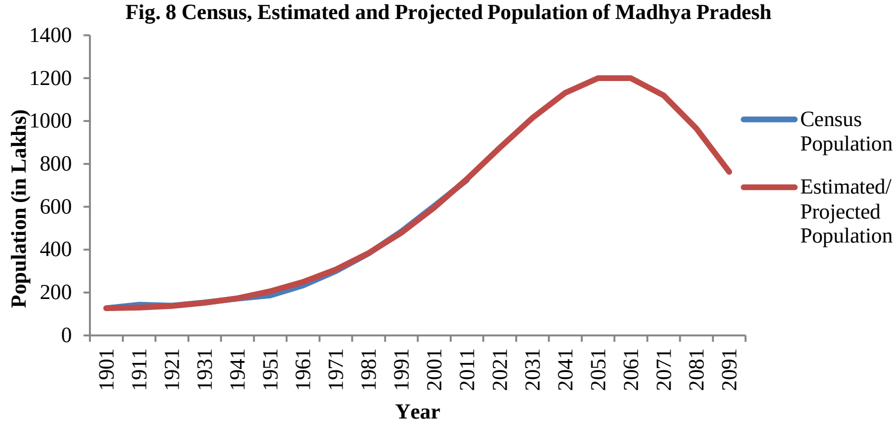


Fig. 8 Census, Estimated and Projected Population of Madhya Pradesh



The proposed model fits to the decadal census data of Maharashtra, Meghalaya, Mizoram and Odisha well as shown in figures 9 to 12. In the very less population density states like Mizoram and Meghalaya, saturation points of population growth come in much later years than that of other states. The highest possible population of states of Maharashtra, Meghalaya, Mizoram and Odisha predicted by the proposed model are 1582.78 lakhs, 5411.25 thousands, 6045.22 thousands and 589.28 lakhs which may be attained in the years 2050, 2058, 2058 and 2053 respectively. It is to be noted that Mizoram is the least density state of all states taken into consideration in this paper.

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Table 3. Population estimation/projection of Maharashtra, Meghalaya, Mizoram and Odisha

States	Maharashtra (Population in lakhs)		Meghalaya (Population in thousands)		Mizoram (Population in thousands)		Odisha (Population in lakhs)	
Parameters' estimate	r =0.0005667 a=1.127999 c=0.48996 d=149		r =0.001548688 a=1.406188286 c=0.05502757 d=157		r =0.005295 a=0.368200729 c=4.900572257 d=157		r =0.000655343 a=1.472605021 c=0.067326765 d=152	
Years	Census Population	Estimated/ Projected Population	Census Population	Estimated/ Projected Population	Census Population	Estimated/ Projected Population	Census Population	Estimated/ Projected Population
1901	193.92	193.92	340.52	340.52	284.47	284.47	103.03	103.03
1911	214.75	199.43	394.01	349.57	346.22	311.36	113.79	104.58
1921	208.50	216.21	422.40	380.32	384.02	359.19	111.59	110.04
1931	239.59	244.83	480.84	436.28	445.61	426.99	124.91	119.88
1941	268.33	286.89	555.82	523.23	512.07	519.33	137.68	134.58
1951	320.03	344.80	605.67	650.02	577.64	643.55	146.46	154.80
1961	395.54	421.47	769.38	828.72	780.04	810.05	175.49	181.33
1971	504.12	519.74	1011.70	1074.23	1072.75	1032.72	219.45	214.93
1981	627.83	641.61	1335.82	1402.82	1420.95	1329.31	263.70	256.13
1991	789.37	786.98	1774.78	1828.92	1837.15	1720.87	316.60	304.88
2001	968.79	952.17	2318.82	2359.41	2293.90	2229.06	368.05	360.14
2011	1123.74	1128.43	2966.89	2985.61	2855.79	2869.40	419.74	419.43
2021		1300.98		3674.09		3637.71		478.51
2031		1449.49		4359.97		4488.35		531.32
2041		1550.53		4947.95		5307.80		570.54
2051		1582.36		5326.28		5899.95		588.72
2061		1531.04		5394.90		6015.49		580.09
2071		1395.96		5101.07		5458.11		542.41
2081		1192.24		4467.49		4247.12		478.30
2091		948.22		3596.22		2710.20		395.30

Fig. 9 Census, Estimated and Projected Population of Maharashtra

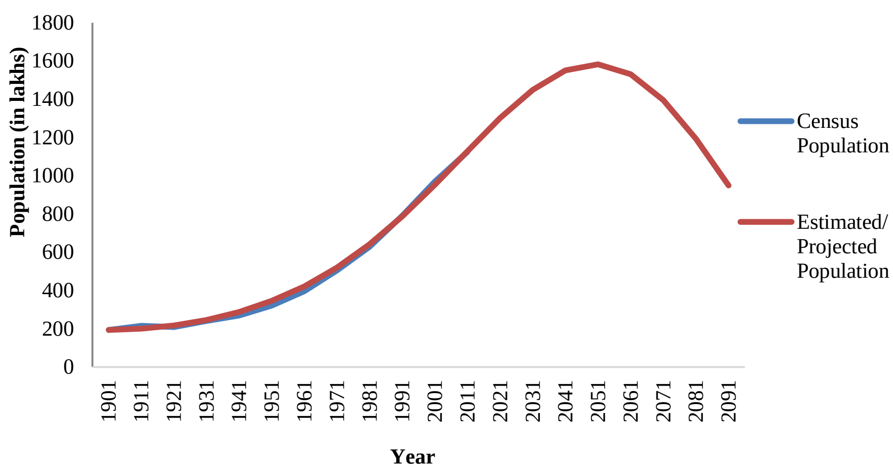

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Fig. 10 Census, Estimated and Projected Population of Meghalaya

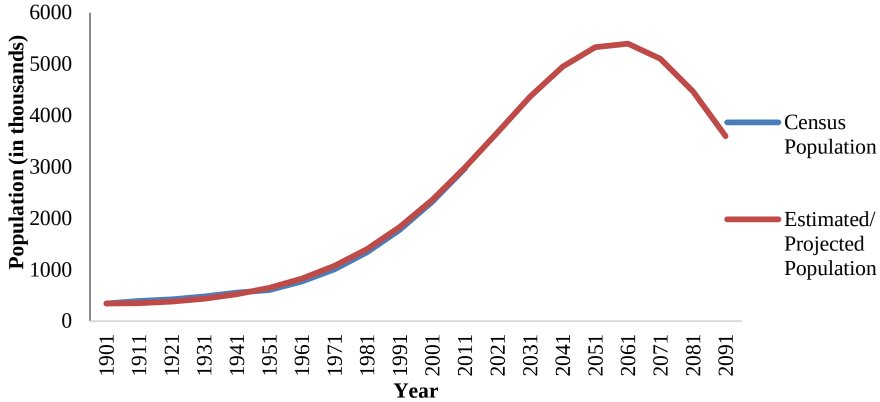


Fig. 11 Census, Estimated and Projected Population of Mizoram

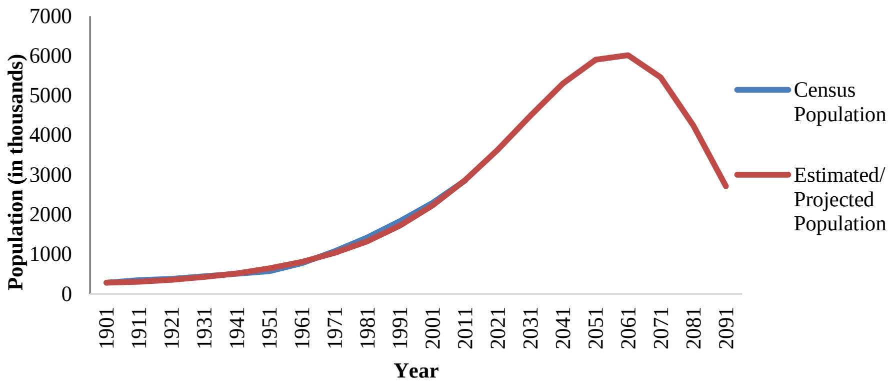
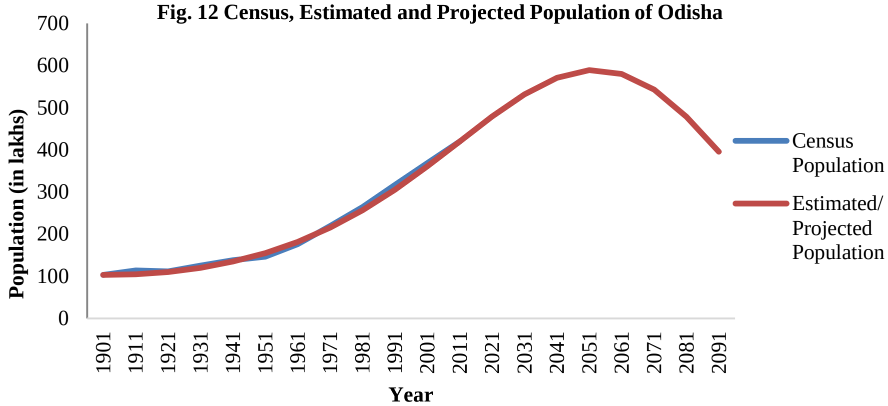


Fig. 12 Census, Estimated and Projected Population of Odisha



In the same manner, the proposed model fits to the decadal census data of Rajasthan, Tamil Nadu, Uttarakhand and Uttar Pradesh as shown in Figures 13 to 16 and population estimates against the census data are appreciably precise, shown in Table 4. Tamil Nadu and Uttar Pradesh are relatively higher density states and are predicted to attain culmination population growth in the relatively earlier years. The proposed model has predicted possible peak points of population of Rajasthan, Tamil Nadu, Uttarakhand and Uttar Pradesh as 1201.68, 914.25, 173.78 and 3232.59 lakhs which may be attained in the years 2053, 2049, 2058, 2048 respectively and then decline. When we observe overall population saturation of Indian states, it may happen in between 2048 to 2058. The model has clearly predicted that Indian population may stabilise by 2048.

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Table 4. Population estimation/projection of Rajasthan, Tamil Nadu, Uttarakhand and Uttar Pradesh

States	Rajasthan (Population in lakhs)		Tamil Nadu (Population in lakhs)		Uttarakhand (Population in lakhs)		Uttar Pradesh (Population in lakhs)	
Parameters' estimate	r =0.000183009 a=1.723697784 c=0.11861457 d=152		r =0.00042665 a=1.140898 c=0.45886 d=148		r =0.000200999 a=1.50255953 c=0.236789708 d=157		r =0.000157016 a=2.273035872 c= 0.010659022 d=147	
Years	Census Population	Estimated/ Projected Population	Census Population	Estimated/ Projected Population	Census Population	Estimated/ Projected Population	Census Population	Estimated/ Projected Population
1901	102.94	102.94	192.53	192.53	19.80	19.80	466.48	466.48
1911	109.84	104.07	209.03	196.61	21.42	20.07	460.13	467.80
1921	102.93	109.08	216.29	208.90	21.16	21.10	445.56	476.42
1931	117.48	119.42	234.72	229.34	23.01	23.07	474.79	498.13
1941	138.64	136.55	262.68	258.31	26.15	26.15	539.21	538.18
1951	159.71	162.47	301.19	296.51	29.46	30.59	602.74	602.51
1961	201.56	199.93	336.87	344.64	36.11	36.69	701.44	698.63
1971	257.66	252.52	411.99	403.14	44.93	44.82	838.49	836.14
1981	342.62	324.42	484.08	471.81	57.26	55.38	1051.37	1026.61
1991	440.06	419.77	558.59	549.34	70.51	68.68	1320.62	1281.92
2001	565.07	541.15	624.06	632.84	84.89	84.80	1661.98	1610.17
2011	685.48	687.19	721.47	717.50	100.86	103.41	1998.12	2007.61
2021		849.30		796.50		123.56		2446.92
2031		1008.87		861.39		143.42		2865.15
2041		1137.03		903.10		160.38		3161.39
2051		1199.73		913.52		171.31		3218.28
2061		1169.04		887.38		173.30		2954.09
2071		1037.38		823.97		164.61		2385.43
2081		826.44		728.08		145.54		1650.52
2091		582.57		609.51		118.75		951.59

Fig. 13 Census, Estimated and Projected Population of Rajasthan

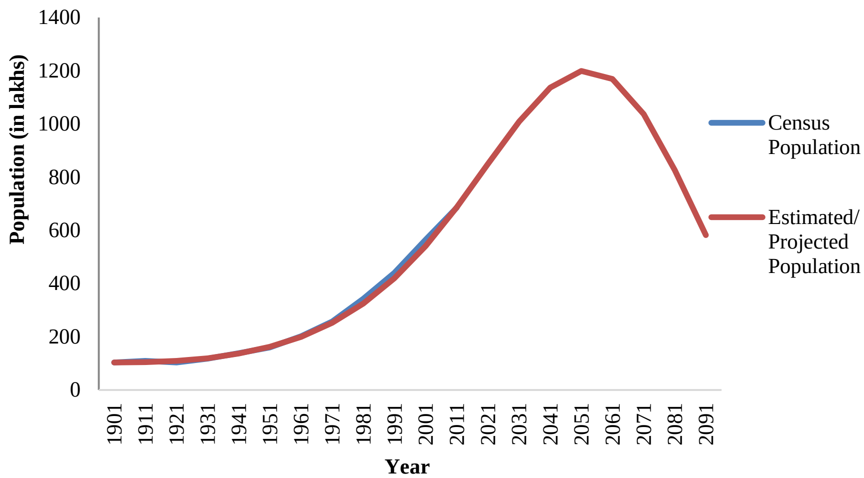


Fig. 14 Census, Estimated and Projected Population of Tamil Nadu

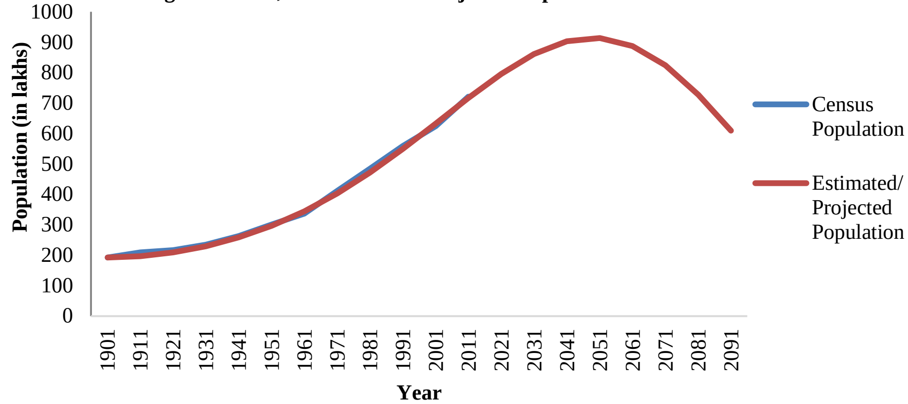


Fig. 15 Census, Estimated and Projected Population of Uttarakhand

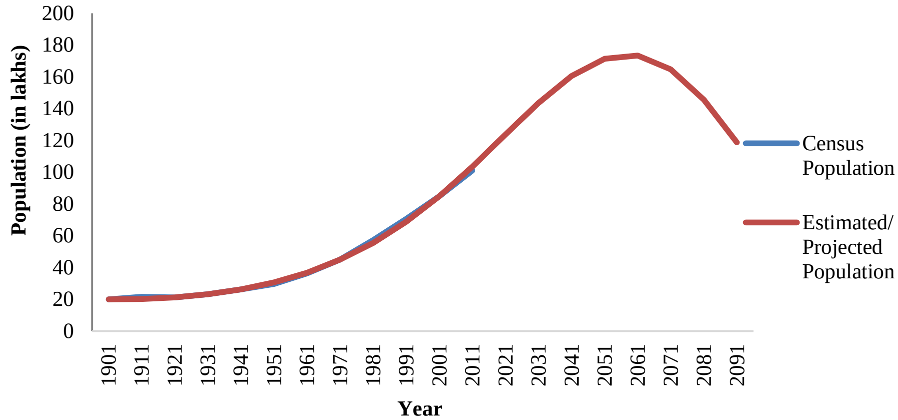
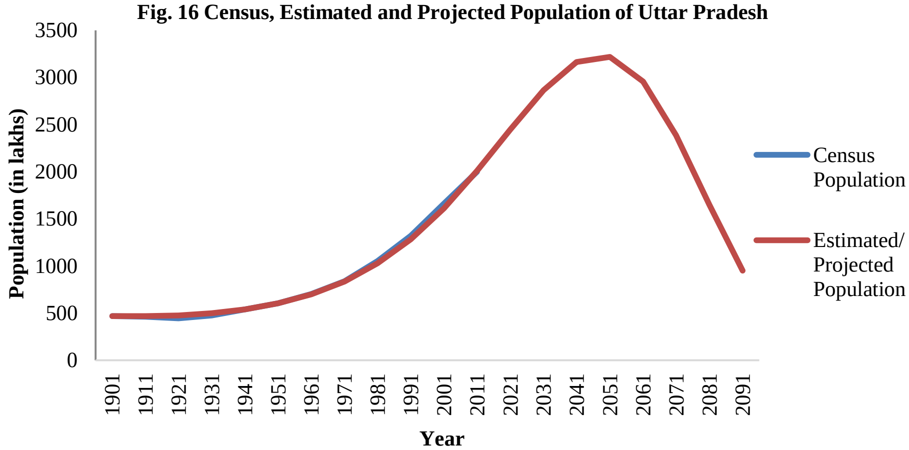


Fig. 16 Census, Estimated and Projected Population of Uttar Pradesh



4. Conclusion

The population projection by using the proposed growth model is purely based on past knowledge of observed population. However, the model is constructed strictly to explain the typical behavior of RGR of observed populations of census data with a preconceived idea of

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approximate population carrying capacity. This model too, like earlier models, does not take into account of many uncertain factors of future accidents such as war, major natural calamities, flood, pandemic, etc., that can give huge impact to change population dynamics enormously. However, under the existing trend of population dynamics of India and Indian states, our proposed model has done an efficacious exercise in estimating past populations, predicting future population and enlightening nature of population growth, stabilisation and decay in future. The projected populations of India and fifteen Indian states may provide useful information to the concerned government departments and private agencies which work on making future plans and policies based on population.

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