A STUDY ON COMPLEX NEUTROSOPHIC GRAPH

[1]Dr. Veeramani. V*,[2] Dr. Said Jaboob**

[1]Assistant Professor, Department of General Requirements, University of Technology and Applied Sciences, Salalah, Sultanate of Oman.

* veeramani.sal@cas.edu.om, **saidj.sal@cas.edu.om

ABSTRACT:

This paper introduces various types of Complex Neutrosophic Graphs(CNG) called complete complex neutrosophic graph, bipartite complex neutrosophic graph, spanning complex neutrosophic graph and path-forest-tree of complex neutrosophic graph. Further, it proposes and identifies new and innovative concepts of dominations in complex neutrosophic graph.

KEYWORDS:

Complex fuzzy sets; Complex intuitionistic fuzzy sets; Complex Neutrosophic Graphs; Path; Forest; Tree; Bipartite; Domination

1. INTRODUCTION:

Fuzzy set was introduced by Zadeh [9] whose basic component is only a membership function. In Zadeh's fuzzy set, the sum of membership degree and a non-membership degree is equal to one. Complex fuzzy set (CFS) [4]-[5] is a new development in the theory of fuzzy systems in [10,11]. The concept of CFS is an extension of fuzzy set, by which the membership for each element of a complex fuzzy set is extended to complex-valued state. The first definition of fuzzy graphs was proposed by Kauffmann in 1973, from the Zadeh's fuzzy relations from [9] [10] [11]. The concepts of complex neutrosophic set was introduced by Mumtaz Ali et.al defined in [1]. The first definition and applications of a complex fuzzy graph defined in [8].Moreover, Regular complex neutrosophic graph is the new and an innovative area from complex fuzzy graph which was defined in [7]. Neutrosophic graph is totally different from intuitionistic fuzzy graph. Degree and order of intuitionistic fuzzy graph defined in [2]. In Neutrosophic Graph(NG), complete NG, Bipartite and complex bipartite NG, Spanning NG, Path-Forest-Tree of NG and Dominative set of NG is defined and derived by Nikfar[3]. We deduce an innovative theoretical concepts of Complex Neutrosophic Graph(CNG) which will be helpful to analyze the structure of complex neutrosophic graph.

In this paper, section 1 describes about introduction and informative collection of existing concepts about complex fuzzy set. The definition and properties of complex fuzzy set, complex fuzzy graph with example is discussed in section 2. In section 3, concept of complex neutrosophic set, complex neutrosophic graph [6], degree of vertex in complex neutrosophic graph are discussed. New and innovative concepts of complete CNG, Bipartite and complex bipartite CNG, Spanning CNG are defined in section 4. Path-Forest-Tree of CNG and Dominative set of CNG is derived in section 5. Further work and concluded of this paper in section 6 is given.

2. COMPLEX FUZZY SETS:

In a complex fuzzy set, membership values are complex numbers in the unit disc of the complex plane [4]-[5]. Although the introductory theory of the CFS has been presented [4], the research on complex fuzzy system designs and applications using the concept of CFS is found rarely. Since the seminal paper in 1965 by Zadeh proposed *Fuzzy Sets* [9], a huge amount of literature has appeared on different aspects of fuzzy sets and their applications [9-11].

DEFINITION: 2.1. Ramot et al. [4] proposed an important extension of these ideas, the *Complex Fuzzy Sets (CFS)*, where the membership function of a CFS is complex-valued, different from fuzzy complex numbers developed in Buckley et.al. The membership function to characterize a CFS consists of an amplitude function and a phase function. In other words, the membership of a CFS is in the two-dimensional complex-valued unit disc space, instead of one-dimensional real-valued unit interval space. Thus, CFS can be much richer in membership description than traditional fuzzy set.

Assume that there is a complex fuzzy set *S* whose membership function $\mu_S(h)$ is given as follows,

$$\mu_S(h) = r_S(h)e^{j\omega_S(h)} = \mathrm{Re}\left(\mu_S(h)\right) + j\,\mathrm{Im}\left(\mu_S(h)\right)$$
$$= r_S(h)\cos\left(\omega_S(h)\right) + jr_S(h)\sin\left(\omega_S(h)\right)$$

where $j = \sqrt{-1}$, *h* is the base variable for the complex fuzzy set, $r_s(h)$ is the amplitude function, $\omega_s(h)$ is the phase function of the complex membership function.

The property of sinusoidal waves appears obviously in the definition of complex fuzzy set. In the case that $\omega_s(h)$ equals to 0, a traditional fuzzy set is regarded as a special case of a complex fuzzy set.

DEFINITION 2.2. Let V be a nonempty set. A fuzzy graph is a pair of functions G:(σ, μ) where σ is a fuzzy subset of V and μ is a symmetric fuzzy relation on σ. i.e. $\sigma: V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$ such that $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all u,v in V.

DEFINITION 2.3.[8]A complex fuzzy graph G is a quadruple of the form G=(V, σ, E, ϕ) , where V is a complex fuzzy set referred to as the set of vertices and E$\subseteq V \times V$ is a complex fuzzy set of edges . Also, σ is a mapping from V to $[0,1] \times [0,1]$. i.e., σ is the assignment of the complex degrees of membership to members of V, $\phi: E \rightarrow [0,1] \times [0,1]$ is a function that maps element of the form $e \in E: (u, v) \rightarrow [0,1] \times [0,1]$,where u $\in V, v \in V$, we assume thatV$\cap$ E =ϕ . In general, we use the form e= (a,b) to denote a specific edge that is said to connect the vertices a and b. For an undirected graph both e_1=(u,v) and e_2=(v,u) are in the domain of φ .

EXAMPLE:2.4

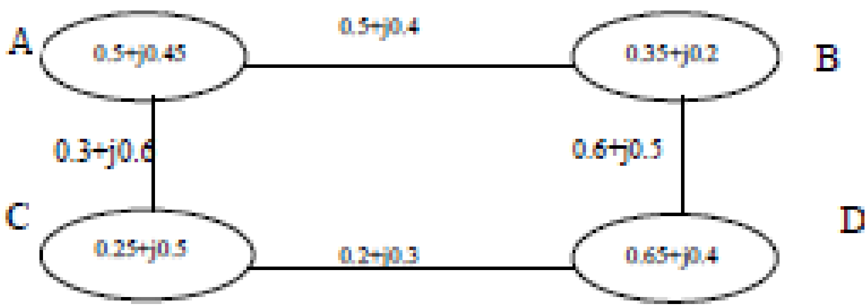

Figure1. Complex Fuzzy Graph

3. COMPLEX NEUTROSOPHIC GRAPH:

DEFINITION 3.1 COMPLEX NEUTROSOPHIC SET.

A complex neutrosophic set as defined in [1], i.e. A complex neutrosophic set defined on a universe of discourse X, which is charaterized by a truth membership function $T_s(x)$,an indeterminacy membership function $I_s(x)$,and a falsity membership function $F_s(x)$ that assigns a complex-valued grade $(j = \sqrt{-1})$ of $T_s(x)$, $I_s(x)$ and $F_s(x)$ in S for any $x \in X$.The values $T_s(x)$, $I_s(x)$, $F_s(x)$ and their sum may all within the unit circle in the complex plane and so is of the following form, $T_s(x) = p_S(x)e^{j\mu_S(x)}$, $I_s(x) = q_S(x)e^{j\vartheta_S(x)}$ and $F_s(x) = r_S(x)e^{j\omega_S(x)}$,where $p_S(x), q_S(x), r_S(x)$ and $\mu_S(x), \nu_S(x), \omega_S(x)$ are amplitude term and phase term of $T_s(x)$, $I_s(x)$, $F_s(x)$ respectivelyand real valued and $p_S(x), q_S(x), r_S(x) \in [0,1]$ such that $0 \leq p_S(x) + q_S(x) + r_S(x) \leq 3$ and $0 \leq \mu_S(x) + \nu_S(x) + \omega_S(x) \leq 2\pi$. The complex neutrosophic set S can be represented in set form as

$$S = \{(x, T_S(x) = a_T, I_S(x) = a_I, F_S(x) = a_F): x \in X\},$$

where , $T_S : X \rightarrow \{a_T : a_T \in C, |a_T| \leq 1\}$, $I_S : X \rightarrow \{a_I : a_I \in C, |a_I| \leq 1\}$,

$F_S : X \rightarrow \{a_F : a_F \in C, |a_F| \leq 1\}$, and $|T_S + I_S + F_S| \leq 3$.

Definition 3.2 [6] Complex Neutrosophic Graph or CNG

Let V be a vertex set. Two functions are consideredas follows: $\rho = (\rho_T, \rho_I, \rho_F): V \to [0,1]^3$ and $\omega = (\omega_T, \omega_I, \omega_F): V \times V \to [0,1]^3$. We suppose

$$A = \{(\rho_T(x), \rho_T(y)) \mid \omega_T(x,y) \geq 0\},$$

$$B = \{(\rho_I(x), \rho_I(y)) \mid \omega_I(x,y) \geq 0\},$$

$$C = \{(\rho_F(x), \rho_F(y)) \mid \omega_F(x,y) \geq 0\},$$

and we have considered ω_T, ω_I and $\omega_F \geq 0$ for all set A,B,C, since it is possible to have edge degree equal to0 (for T, or I , or F). The triad (V, ρ, ω) is defined to be complex neutrosophic graph of type 1 **(CNG),** if $\omega_T(x,y) \leq Min(\rho_T(x), \rho_T(y))$, $\omega_I(x,y) \geq Max(\rho_I(x), \rho_I(y))$ and $\omega_F(x,y) \geq Max(\rho_F(x), \rho_F(y))$, $x, y \in V$. Here $\rho(x) = (\rho_T(x), \rho_I(x), \rho_F(x)), x \in V$ are the complex truth membership, complex indeterminate-membership and complex false-membership of the vertex *x* and $\omega(x,y) = (\omega_T(x,y), \omega_I(x,y), \omega_F(x,y)), x, y \in V$ are the complex truth membership, complex indeterminate-membership and complex false-membership values of the edge (*x*,*y*). For the numerical calculation, we consider only the amplitude term.

EXAMPLE 3.3

Membership function of Vertex set	a	b	C	d
ρ_T	$0.3e^{j\frac{2\pi}{3}}$	$0.4e^{j\frac{\pi}{2}}$	$0.2e^{j\frac{\pi}{3}}$	$0.6e^{\frac{j\pi}{3}}$
ρ_I	$0.2e^{j\frac{4\pi}{3}}$	$0.8e^{j\frac{\pi}{3}}$	$0.7e^{j\frac{\pi}{4}}$	$0.5e^{j\frac{\pi}{3}}$
ρ_F	$0.5e^{j\frac{\pi}{3}}$	$0.3e^{j\frac{2\pi}{3}}$	$0.6e^{j\frac{3\pi}{4}}$	$0.8e^{j\frac{\pi}{4}}$

Table 1: **Complex truth-membership, complex indeterminate-membership and complex false-membership of the vertex set.**

Membership function of edge set	(a,b)	(a,c)	(b,d)	(b,c)
ω_T	$0.3e^{j\frac{2\pi}{3}}$	$0.1e^{j\frac{\pi}{3}}$	$0.3e^{j\frac{\pi}{2}}$	$0.2e^{j\frac{\pi}{3}}$
ω_I	$0.9e^{j\frac{4\pi}{3}}$	$0.7e^{j\frac{4\pi}{3}}$	$0.8e^{j\frac{\pi}{3}}$	$0.9e^{j\frac{\pi}{3}}$
ω_F	$0.5e^{j\frac{2\pi}{3}}$	$0.7e^{j\frac{2\pi}{3}}$	$0.9e^{j\frac{2\pi}{3}}$	$0.7e^{j\frac{3\pi}{4}}$

Table 2: **Complex truth-membership, complex indeterminate-membership and complex false-membership of the edge set.**

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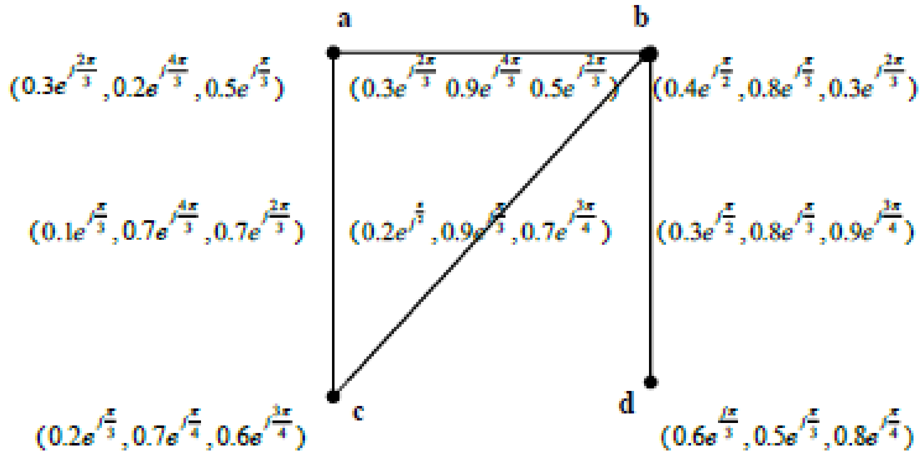

Figure : 2 Complex Neutrosophic Graph

DEFINITION 3.4

The degree of a vertex in complex neutrosophic graph defined as in [6]. Let $G=(V,\rho,\omega)$ be complex neutrosophic graph. The degree of a vertex x is denoted by

$D(x)=(\omega_T(x,y),\omega_I(x,y),\omega_F(x,y))$, where $\omega_T(x,y)=\sum_{y\in\rho,y\neq x} p_T(x,y)e^{j\sum_{y\in\rho,y\neq x}\mu_T(x,y)}$,

$\omega_I(x,y)=\sum_{y\in\rho,y\neq x} q_I(x,y)e^{j\sum_{y\in\rho,y\neq x}\vartheta_I(x,y)}$, and $\omega_F(x,y)=\sum_{y\in\rho,y\neq x} r_F(x,y)e^{j\sum_{y\in\rho,y\neq x}\omega_F(x,y)}$.

4. COMPLETE, BIPARTITE AND SPANNING COMPLEX NEUTROSOPHIC GRAPH:

DEFINITION 4.1 (COMPLETE CNG)

A complex neutrosophic graph of type 1 (CNG1) is complete if $\omega_T(x,y)=Min\,(\rho_T(x),\rho_T(y))$, $\omega_I(x,y)=Max\,(\rho_I(x),\rho_I(y))$ and $\omega_F(x,y)=Max\,(\rho_F(x),\rho_F(y))$, $x,y\in V$, here $\rho(x)=(\rho_T(x),\rho_I(x),\rho_F(x)), x\in V$ are the complex truth membership, complex indeterminate-membership and complex false-membership of the vertex x and $\omega(x,y)=(\omega_T(x,y),\omega_I(x,y),\omega_F(x,y))$, $x,y\in V$ are the complex truth membership, complex indeterminate-membership and complex false-membership values of the edge (x,y).

DEFINITION 4.2 (BIPARTITE CNG)

A complex neutrosophic graph $G=(V,\rho,\omega)$ on vertex set V is said bipartite if the set V can be partitioned into two non-empty sets V_1 and V_2 such that $\omega_T(x,y)=\omega_I(x,y)=\omega_F(x,y)=0$. For all $\{x,y\}\in V_1$ or $\{x,y\}\in V_2$.

DEFINITION 4.3 (COMPLETE BIPARTITE CNG)

A bipartite complex neutrosophic graph $G=(V,\rho,\omega)$ on vertex set V is said complete, if $\omega_T(x,y)=\min\{\rho_T(x),\rho_T(y)\}, \omega_I(x,y)=\min\{\rho_I(x),\rho_I(y)\}, \omega_F(x,y)=\max\{\rho_F(x),\rho_F(y)\}$ for all $\{x,y\}\in V$

DEFINITION 4.4 (SPANNING COMPLEX NEUTROSOPHIC GRAPH)

Let $G=(V,\rho,\omega), G_1=(V_1,\rho_1,\omega_1)$ be a complex neutrosophic graph on a given set V. Then G_1 is called the spanning complex neutrosophic graph of G if $V=V_1$ but $E_1\subseteq E$

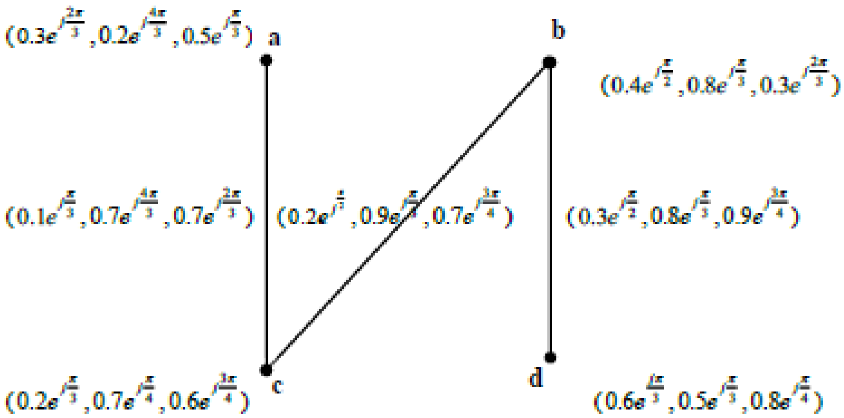

Figure: 3: Spanning Complex Neutrosophic Graph of Figure 2

5. PATH, FOREST, TREE AND DOMINATIVE SET OF COMPLEX NEUTROSOPHIC GRAPH:

DEFINITION 5.1 (PATH)

Let $G=(V,\rho,\omega)$ be a complex neutrosophic graph on V and $v_0, v_1, ...v_n$ in G is called a path of length n from v_0 to v_n, if $\omega_T(v_i, v_{i+1}) > 0$, $\omega_I(v_i, v_{i+1}) > 0$, and $\omega_F(v_i, v_{i+1}) > 0$, for $i = 0,1,2,..., n-1$. The $\min_{i=0}^{n-1} \omega_T(v_i, v_{i+1})$, $\min_{i=0}^{n-1} \omega_I(v_i, v_{i+1})$ and $\min_{i=0}^{n-1} \omega_F(v_i, v_{i+1})$ are called the strength of the path in Truth, Indeterminate and False membership function of edges respectively.

DEFINITION 5.2. (FOREST)

Let $G=(V,\rho,\omega)$ be a complex neutrosophic graph on a given set V. Then G is called the Forest, if G was acyclic and there is a spanning complex neutrosophic graph *F* such that for all edge *xy* out of F, there is a path P from *x* to *y*, whose strength greater than $\omega_T(x,y)$, $\omega_B(x,y)$ and $\omega_F(x,y)$.

DEFINITION 5.3 (TREE)

Let $G=(V,\rho,\omega)$ be a complex neutrosophic graph on a given set *V*. Then G is called a tree, if G was a forest such that there is a path P from *x* to *y*, for all $x, y \in V$.

REMARK: 5.4

$\mu^{\infty}_{G-\{xy\}}(x,y)$ is the strength between *x* and *y* in the complex neutrosophic graph obtained from G by deleting the edge *xy*. This is as the same for truth, indeterminate and falsity membership function also.

DEFINITION 5.5 (EFFECTIVE EDGES)

Let $G=(V,\rho,\omega)$ be a complex neutrosophic graph on a given set *V*. Then an edge xy in G is called the effective edge if $\omega_T(x,y) > \mu^{\infty}_{G-\{xy\}}(x,y)_T$ *(T-effective)*,

$\omega_I(x,y) > \mu^{\infty}_{G-\{xy\}}(x,y)_I$ *(I-effective)* and $\omega_F(x,y) > \mu^{\infty}_{G-\{xy\}}(x,y)_F$ *(F-effective)*.

5.6 DOMINATION IN COMPLEX NEUTROSOPHIC GRAPH

a. A subset V_1 of V is called the *T-effective* dominative set in G, if for every $v \in V - V_1$, there is $u \in V_1$ such that *u* dominates *v* as *T-effective* and weight of x is defined by $$\omega_v(x)_T = \rho_T(x) + \frac{\sum_{xy} (\text{ is a T - effective edge } \omega_T(xy))}{\sum_{xy} (\text{ is a edge } \omega_T(xy))}$$

b. A subset V_1 of V is called the *I-effective* dominative set in G, if for every $v \in V - V_1$, there is $u \in V_1$ such that *u* dominates *v* as *I-effective* and weight of x is defined by $\omega_v(x)_I = \rho_I(x) + \frac{\sum_{xy} (\text{ is a I - effective edge } \omega_I(xy))}{\sum_{xy} (\text{ is a edge } \omega_I(xy))}$

c. A subset V_1 of V is called the *F-effective* dominative set in G, if for every $v \in V - V_1$, there is $u \in V_1$ such that *u* dominates *v* as *F-effective* and weight of x is defined by $$\omega_v(x)_F = \rho_F(x) + \frac{\sum_{xy} (\text{ is a F - effective edge } \omega_F(xy))}{\sum_{xy} (\text{ is a edge } \omega_F(xy))}$$

d. We say that x dominates y in G as effective, if the edge xy is either *T*-effective, *I*-effective and *F*-effective. A subset V_1 of *V* is called the effective dominating set in G, if for every $v \in V - V_1$, there is $u \in V_1$ such that u dominates v as effective. The weight of V_1 is defined by $\omega_v(V_1) = \min\{\omega_v(V_1)_T, \omega_v(V_1)_I, \omega_v(V_1)_F\}$

6. CONCLUSION:

This work offers new insights into complex neutrosophic graphs, which ultimately is leading to new concepts and definitions. The paper discusses the vital concepts of a complex neutrosophic set which is defined by a complex-valued truth membership function, complex-valued indeterminate membership function and a complex-valued falsehood membership function. In particular, Complex neutrosophic graphs are dealing with uncertainty with both periodicity and determinacy. Thus, complex neutrosophic graph captures and assists in handling the redundant nature of uncertainty, incompleteness, indeterminacy, inconsistency, etc. The important concepts of complex neutrosophic graph are also derived and these include Path, Bipartite CNG, Complete CNG, Forest, Tree, Effective and Dominating set in complex neutrosophic graph for analyzing its structure. Future work will further comprehensively and rigorously investigate the concepts of the definitions which can lead to more suggested findings in terms of the coloring, chromatic number, cycles of complex neutrosophic graph and its properties.

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