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Combinatorial Optimization Using (Integer) Linear Programming and Metaheuristics

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Abstract:
There are a number of different strategies available for dealing with difficult optimization problems. Two particularly successful methodologies for dealing with combinatorial challenges are mathematical programming techniques, which include (integer) linear programming-based methods and metaheuristic approaches. These two organisations were created by different communities that were more or less isolated from one another. Building hybrids of mathematical programming techniques and metaheuristics has just recently gained widespread attention from academics, who have recognised the many advantages and enormous possibilities of doing so. When it comes down to it, many issues may be dealt with significantly more successfully by using synergy between these different methodologies than by using "pure" classical algorithms. How mathematical programming methods and metaheuristics should be coupled to get these benefits is the central question. In the last several years, a slew of new procedures have been introduced. In this chapter, after providing a brief introduction to the basics of integer linear programming, we review well-known solutions for such combinations and divide them into ten different methodological groups.

1 Introduction

Combinatorial optimization problems (COPs) are frequent in a broad variety of highly important and practical disciplines, and their solution is notoriously difficult due to their computational complexity. Timetable creation, setting optimal schedules for operations that will be handled on a production line, developing efficient communication networks, and containerization are all examples of jobs that fall into this category.loading, determining the most cost-effective truck routes, and a multitude of other difficulties that emerge in the transportation industryComputational biology and artificial intelligence are only a few such examples. This includes setting values for discrete variables in such a manner that an optimal solution in terms of the constraints is produced.It is established whether or not a certain goal function exists under the limits of a specific job. Constraints.

The bulk of COPs are quite difficult to settle. For example, the fact that many such problems are NP-hard [38], which is captured in theoretical computer science, is an excellent illustration of this. NP-hard COPs are frequently referred to as "hard COPs" because of their inherent complexity as well as their enormous practical relevance.In the literature, there has been a plethora of solutions for addressing difficulties that are comparable to those that have been discussed.the last couple decades The techniques available for resolving COPs may be divided into the following categories:Algorithms are separated into two types: precise algorithms and heuristic algorithms. Precision algorithms are the most exact algorithms. Precise algorithms are guaranteed to discover the optimal solution while also demonstrating that it is in fact the best response.for each and every instance in which a COP occurs Running time increases dramatically as a problem instance grows in size; yet, only small or moderately-sized issues are often impacted by this phenomenon.Cases may be treated in a realistic manner in order to attain proven maximum efficiency. In the event of more serious circumstancesMost of the time, the only choice available is to use heuristic algorithms, which trade off optimality for speed, meaning that they are intended to provide outstanding results but not necessarily the best results.providing the best possible replies in a reasonable length of timeWhen it comes to exact approaches, the methods listed below have been shown to be successful.These

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approaches, including branch-and-bound algorithms, dynamic programming, constraint programming, and, in particular, the vast class of integer (linear) algorithms, have achieved significant success.The use of approaches such as linear programming and other relaxation-based methods is common in this field (ILP).Techniques such as branch-and-cut, cutting plane and column generating processes, and others are available.

When it comes to the building sector, the expressions "branch and price" and "branch and cut and price" are often heard. Check out [52 and 59] for more broad information on the subject.the mathematical programming principles mentioned in this part are introduced in the following sections:Metaheuristics (MHs) have been demonstrated to be highly successful in the area of application research, which is a heuristic approach to problem solving. In this field, some of the problem-solving methodologies that have been used include, for example,Various population-based models, such as evolutionary algorithms, memetic algorithms, and scatter search, as well as estimation of distributions, are used, amongst other things.An algorithm is shown by the ant colony optimization technique. Further information may be found in Chapter 1 of this book as well.More general introductions to metaheuristics may be found in, for example, [41], [48], and [49].When examining the benefits and drawbacks of ILP techniques and metaheuristics, it is feasible to see that the approaches are very complementary to one another in terms of their effectiveness. As an example,It seems to be a natural step to combine notions from both schools of thinking, at least for the time being.Despite this, hybrid tactics such as these have just lately acquired favour in the United States of America.years. There are a number of recent publications that explore different sorts of phobias that have been published recently.

Hybrid optimizers, which are typically much more effective in terms of performance than traditional optimizers.Synergy allows them to both benefit in terms of running time and/or solution quality as a consequence of their collaboration. The Hybrid Metaheuristics workshop series is one example of a series of international scientific conferences and workshops that are organised annually.Beginning in 2004, the First Workshop on Mathematical Modelling and Analysis was held, as was the First Workshop on Mathematical Modelling and Analysis.Among the many contributions to metaheuristics, Metaheuristics 2006 emphasises the significance of heuristics.It is said that hybrid systems such as these carry the potential of a better future. In reality, the fabrication was created.The latter instance is credited with popularising the word "mat heuristics" for referring to heuristics, which was previously used only informally.The methodologies of metaheuristics and mathematical programming are employed in cooperation with one another to solve problems.In the next part, we will provide a rapid recap of the issues that have been previously addressed.According to the authors' recommendations, strategies for incorporating metaheuristics have been categorised structurally.additionally to certain optimization methodologies Section 3 presents a high-level overview of the basics.There are many well-known ILP techniques, as well as the notations that are often used in the field. There are a variety of different sorts of objects.and tactics for combining ILP approaches into metaheuristics as well as vice versa, as well as

Throughout Sections 4 through 13, you will find a detailed assessment of the literature, as well as annotated references to successful cases. Finding high-quality incumbents and bounds in branch-and-bound is made easier with the use of machine learning (MHs); ILPs are made easier with the help of machine learning (ILPs).

Relaxations are employed to steer metaheuristic search in order to improve performance.In this instance, the primal-dual relationship in MHs is used.Following the spirit of local search in branch-and-bound search is important.

• Methods for examining large areas of land utilising ILP methods in large neighbourhoods

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• a fusion of potential solutionsUsing ILP approaches, it is possible to decode representations that are indirect or incomplete.

• problem-solving techniques that include many stages

Cut and column data are generated by the application of metaheuristics.

• setting a strategic direction for the search and collaboration process

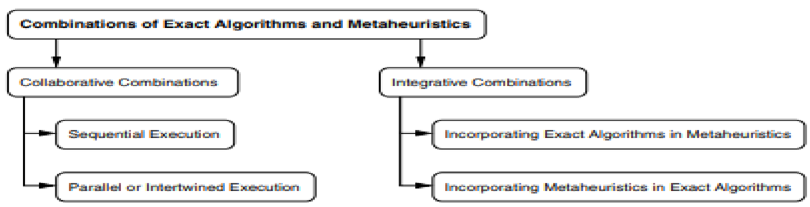
Figure 2 depicts structural models for the use of many metaheuristics in conjunction with one another.

2. With the Aid of Exact Approaches and Methods

The following are overviews of several structural models that were built by combining precise approaches with a range of additional methodologies.Metaheuristics are shown in [25, 67, and 75] in various ways.

According to Dumitrescu and Stutzle [25], there are currently accessible combination tactics that are mostly focused on local search approaches that are boosted by the use of exact match. These strategies are described as follows:

algorithms. Rather of concentrating on obvious combinations such as preprocessing, they place emphasis on integration in their survey results.According to [67], contemporary approaches for combinatorial optimization that use both precise and metaheuristic algorithms in combination with one another should be classified in a broader manner than they are now.It is possible to differentiate between the following two primary categories, according to this classification:Collaboration results in the formation of new combinations. It is more efficient to use algorithms while working in a group setting, than than working alone.They exchange information, however they are not affiliated with the same organisation. It is necessary to employ both accurate and heuristic approaches.Using this method, it is feasible to execute algorithms sequentially, interleaved, or simultaneously. It is advised that you execute your code in a sequential manner.Integrative Combinations are a kind of combination in which two or more components are combined together. Another method that is used in connection with integration models has a subordinate embedded component of another technique that is used in conjunction with it. As a result, there is a separate master algorithm that, depending on the scenario, may be either accurate or metaheuristic in nature.algorithm, as well as at least one slave that has been put into the systemDanna and Le Pape [21] give a classification of hybrid algorithms that is similar to the preceding one, but also includes constraint programming as a subset of hybrid algorithms. From the data they gathered, the authors derive a decomposition scheme that corresponds to integrative combinations and a multiple search approach that corresponds to collaborative combinations. There are four different kinds.There are many optimization techniques taken into account, including polynomial optimization algorithms.Operational research algorithms, constraint programming, mixed integer programming, as well as various forms of local search and metaheuristics are just a few of the topics addressed in this course. The most crucial thing to keep in mind is the fact thatIn a section of their work, they provide examples from the literature to demonstrate six arguments they have made.



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According to [67], the primary structural categorization of exact/metaheuristic combinations is shown in Fig. 1 as a result of the classification scheme.Numerous collaborative schemes are constructed from the combination of two of the algorithm types discussed above.The authors have developed an overall taxonomy of hybrid metaheuristics, which they have presented.Author Talbi developed Talbi [82] as a fictitious character in his novel Talbi. Cotti [19] examines a variety of hybridization schemes, including evolutionary algorithms (EAs) in particular, and how they may be used. Two of the most brilliant musicians on the planet, El-Abd [26] and Kamel [26] are two of the most skilled musicians on the planet.An emphasis was placed on cooperative parallel architectures, which received special attention.Raidl [75] attempts to bring together earlier classifications and taxonomies of hybrid plants in an effort to unify the field.There are many sorts of metaheuristics, and the primary distinction is between (a) the types of algorithms that are utilised and (b) the types of algorithms that are not employed.hybridised, (b) the intensity of hybridization (high- or low-level), and (c) the sequence in which they were hybridised are all specified.Execution in batches, interleaved, or parallel mode), and (d) the control approach that will be employed during the whole process.Integration and collaboration are two alternative methods to issue resolution that are denoted by the terms "integrative" and "collaborative."

3 Linear and Integer Programming at a Glance

We recommend reading the books on linear optimization by Bertsimas and Tsitsiklis [11] and on combinatorial and integer optimization by Nemhauser and Wolsey [59] and Wolsey [88] for a more in-depth treatment of the subject; for more information on integer programming, see the section on integer programming. When dealing with integers, an integer (linear) programme is used to solve an optimization issue.variables, an objective function that is linearly dependent on the variables, and a collection of variableslinear (in)equalities are limitations that are stated as linear (in)equalities We take the form into consideration.

$$z_{\mathrm{ILP}} = \min\{cx \mid Ax \geq b, x \geq 0, x \in \mathbb{Z}^n\}, \qquad (1)$$

The sign of the related coefficients of equalities may be changed to greater-than-or-equal form, and inequalities can be turned into pairs of inequalities by changing the sign of their associated coefficients. By performing the necessary transformations, we are able to investigate any kind of linear constraint. As a result, for the sake of generality, we will confine the scope of the following considerations to concerns of minimization of the kind described above (1).MIP stands for mixed integer (linear) programme, and it is a combination of integer and linear programmes.In contrast to real-valued variables, however, it is otherwise defined in the same way as real-valued variables.Section 3.1 Relaxations and the Ideology of DualityIt is important to note that relaxations are one of the most important concepts in integer programming, and they are defined as follows:circumstance in which some or all constraints on a particular subject are loosened or abolished Generally speaking, relaxations are used to acquire related, smaller concerns that may be resolved with the use of regular approaches.in a timely way, creating boundaries and approximate (but not necessarily actual) repliesin order to resolve the underlying problemIn order to accomplish the ILP (1) linear programming relaxation, the ILP (1) linear programming relaxation must first be relaxed.In order to achieve the linear programme, the integrality requirement must be satisfied (LP)

$$z_{\mathrm{LP}} = \min\{cx \mid Ax \geq b, x \geq 0, x \in \mathbb{R}^n\}. \qquad (2)$$

3.1 The Approach Using a Cutting Plane

One of the most significant goals when modelling COPs as ILPs is to create a strong formulation for which the LP relaxation offers a solution that is not too far away from the integer optimum in general.

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It is feasible to get COPs for several of them.By incorporating more inequalities, it is possible to greatly improve an existing ILP formulation. A common occurrence is that the number of such limitations increases exponentially with the number of people.the scope of the issue This, on the other hand, implies that we have already solved the LP relaxation byBecause of the exponential growth of the population, typical procedures may be too expensive in reality.In order to do this, Dantzig et al. [23] suggested the cutting plane technique, which typically only analyses a small portion of all constraints explicitly. sized LP.However, it is capable of determining an ideal solution for the whole LP.Beginning with a restricted subset of starting inequalities, this cutting plane technique may be used.and finds a solution to the decreased LP Afterwards, it attempts to detect disparities that are not already present.nonetheless, they are valid for the original issue (i.e., they are fulfilled by the derived solution).included inside the whole LP). Cuts or cutting are the terms used to describe these violations of limitations.planes. This is done by adding them to the existing reduced LP and then resolving the LP.

The whole procedure is repeated until no further cuts can be identified. If the algorithm is successfulIf able to offer evidence that there is no additional breached inequity, the last step is reached.In addition, the derived solution is optimum when compared to the original entire LP. TheThe separation issue is a subproblem of the problem of finding cuts, and it is very important.It is critical to resolve it swiftly since there are often numerous occurrences of it.the problem is addressed till the cutting plane method is effectively completedIt should be noted that, from a theoretical standpoint, every ILP can be solvedMaking use of a pure cutting plane method and a variety of relevant cut classes ThereFortunately, there are general forms of cuts, such as the Chvatal-Gomory cuts [88], that ensure such an outcome. In actuality, though, it may take a lengthy time for such a procedure to be completed.Using a cutting plane strategy to converge to the optimum is advantageous in part because it is common.It is difficult to distinguish between successful cuts in this subproblem. The cutting plane technique is a kind of cutting plane.As we shall see later, this strategy is often used in conjunction with other strategies.

4 Metaheuristics for Finding High-Quality Incumbents and Bounds in B&B

Almost every effective B&B technique is reliant on some heuristic for generating a promising initial solution, whose objective value is then used as the original upper limit in the optimization process, and this is true for almost all of them. As previously mentioned, heuristics are often used in the process of making decisions in a given situation.with the objective of attaining success applied to any or all of the subproblems included inside the B&B treeGet better upper bounds on the present solutions, as well as better alternatives to the existing solutions. InSolid upper bounds are required in order to keep the B&B tree as compact as feasible while yet preserving its modest size.interest. Metaheuristics are often used in order to attain these objectives.When a somewhat expensive metaheuristic is applied at each phase, the cost of the process rises.Performing the additional processing effort needed to process each node of a large B&B tree in a clear, independent manner is not always necessary or beneficial. There have been several phone calls from theDepending on the context, the metaheuristic may do more or fewer repeated searches in similar regions than necessary.spanning the whole search area B&B tree nodes for which a rigorous selection process has been carried outThe performance of the metaheuristic, as well as the amount of effort put into each call, are both monitored and measured.As a consequence, it is quite important.Woodruff [89] gives an instance of this by demonstrating a selection approach based on chunking.in order to decide whether or not reactive tabu search should be done at each node of the B&B tree.called. It is possible to compute the distance between two locations in time using the chunking-based approach.a node or a set of nodes that have been previously explored by the metaheuristic in order to skew the findingsdecision to go in the direction of faraway destinations The stated computational results are consistent with the following:By including the metaheuristic, the B&B is able to improve its overall performance.

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5 Relaxations for Using Metaheuristic Search to Guide Your Decisions

An optimum solution for a relaxation of the original issue often reveals which portions of the original problem's search space are excellent or even ideal in terms of the original problem's search spaceIt's possible that solutions exist. As a result, solutions to relaxations are commonly used in practise.in the field of (meta-)heuristics In the next section, we will look at many options for such a situation.approaches.

5.1 Developing Initial Solutions That Look Promising

It is possible to repair an optimum solution to a relaxation by using a problemspecific approach in order to make it viable for the original problem and to make it feasible for other problems.Make advantage of it as a potential beginning point for a further metaheuristic evaluation (or exact)search. This is often accomplished via the use of linear programming (LP) relaxation.and just a simple rounding approach is required.For example, Raidl and Feltl [73] offer a hybrid genetic algorithm that incorporates both genetic and evolutionary elements (GA)the generalised assignment issue, where the LP relaxation is used to solve the problemThe issue has been solved, and the result has been exploited by a randomised rounding technique in order to generate an initial population of potential integral solutions for the problem. TheseSolutions, on the other hand, are often impractical; as a result, randomised repair and improvement operators are employed in addition, resulting in an even more relevant beginning population for the GA.Plateau and colleagues [64] use a combination of interior point approaches and metaheuristics to solve problems.finding a solution to the multidimensional knapsack issue (MKP). An first stage entailsA technique called inner point method is used, and it has an early end. By rounding and squaringUsing a variety of alternative ascent strategies, a population of various possible solutions is generated.It is possible to produce potential solutions. This collection of solutions is then utilised as a starting point.path relinking/scatter search population for use in path relinking/scatter search The obtained data demonstrate that theThecombination that has been provided is an interesting research path.

5.2 Repairing, local improvement, and variation in accordance with the guidelines

OperatorsIn addition to initiation, the optimality of LP relaxations is often used for guiding.Local improvement or the mending of infeasible candidate solutions are examples of local improvement. Examples include [74], where the MKP is taken into account, and variables are ordered by importance.Increasing the LP-values with time. A greedy repair technique takes into account all of the factors in this situation.It follows the sequence and takes objects out of the backpack until all limitations are met.In a greedy improvement technique, elements are reviewed in reverse order and the process is repeated until the desired result is achieved.As long as no limitation is broken, the item may be included in the bag.There are other analogous instances for exploiting LP solutions, as well as a biassing example.In EAs, there are a variety of variation operators, such as recombination and mutation.

5.3 Using Dual Variables to Your Advantage

On rare occasions, the values of dual variables are also utilised. By computing so-called pseudoutility ratios for the primal variables and employing them in the same methods as were explained above for the primal solution values, Chu and Beasley [15] utilise them in their GA for the MKP, and they are discussed in more detail below. These pseudo-utility ratios have a tendency toin order to provide more accurate indicators of the likelihood of the associated things being purchasedIncorporated into an optimum solution; see [76] for further information on GA techniquesin the case of the MKP

5.4 Fixing Variables: Reducing Issues to Their Fundamentals

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The best solution of an LP relaxation may also be used in another way.The next procedure is more direct and restrictive: Some of the choice variables with integral values in the LP-optimum are set at these values, and the subsequent optimization examines just the remaining variables. Occasionally, such procedures are used.Core approaches are also referred to as such since the original issue is minimised and just a few variables are involved.Its "hard core" is subjected to further processing. Obviously, the variables were chosen in a certain way.It is necessary to start with the fundamentals.The fundamental notion was first introduced for the 0–1 knapsack problem [9], and it has since been used to numerous other very effective accurate algorithms, such as [63].The MKP technique was further developed by Puchinger et al. [72], who studied numerous different options for selecting approximation cores. When binary decision variables x1,...,xn are considered, the fundamental procedure first sorts all variables according to whether they are 0 or 1.defines the so-called split-interval in relation to a certain efficiency metricwhich is the subsequence of the variables beginning with the first and finishing with the lastwith regard to the last fractional variable Various efficiency methods are being investigated, andIt is shown that the above-mentioned pseudo-utility ratios, which are considered to bedefined by the values of two variables, are generally considered to be a suitable option for theMKP. The split interval is eventually stretched to an approximate core by adding an additional split period.

There are a total of 0 additional variables on each side of the split-center. interval's According to empirical experiments in [72], excellent quality results are already achieved with n = 0.1n.When solving optimization problems, it is possible to produce solutions with average optimality gaps smaller than 0.1 percent.addressing the remaining fundamental issue to the best of one's ability under established optimum conditions Using an EA to your advantageAddressing the reduced issue using relaxation-guided variable neighbourhood searchWhen these metaheuristics are applied to the original examples, they provide much better solutions in significantly less time than when they are applied to the original instances.Continuing with the MKP, here is another example of how to take use of the LP relaxation.The hybrid tabu search method developed by Vasquez and colleagues falls under the category of metaheuristics.Hao [86] is a character in the Chinese novel Xiao Long [86]. In this case, the search space is narrowed and partitioned by the use of additional criteria.limitations that limit the overall number of objects that may be packed There are limits for them.By solving modified LP relaxations, it is possible to determine constraints. Tabu search is performed individually for each remaining portion of the search space, beginning with a solution obtained from the LP relaxation of the partial issue and progressing to the next section of the search space.Additional variable fixing has been added to the technique in [87] to make it even better.

5.5 Taking Advantage of Lagrangian Relaxation

Other relaxations, in addition to the LP relaxation, have been effectively used in combination with metaheuristics on a number of occasions. The most important approaches forSuch combos are comparable to one another. The hybrid Lagrangian is a successful illustration of this.For the prize-collection Steiner tree problem from Haouaria and Siala [47], GA is required.A Lagrangian decomposition is performed on a minimal spanning tree formulation of the issue and the volume approach is used for the Lagrangian dual solution to be found by these researchers. Following termination, the genetic algorithm is launched and exploits the data.

There are numerous ways to interpret the results of the volume algorithm:

• Graph reduction: The volume method generates a series of intermediary nodes in the graph.As a by-product, spanning trees are produced. All of the edges that exist in these intermediatesTrees are tagged, and only this reduced edge set is taken into consideration by the algorithm.Using the intermediate primal findings, we may deduce the GA; that is, a core of edges for the GA.while attempting to solve the Lagrangian dual.

• Beginning population: A Lagrangian heuristic is used to construct a subset of varied initial solutions by greedily generating solutions based on the reduced set of constraints.The costs that occur as intermediate outcomes in the volume algorithm are referred to as costs.

• Alternative objective function: In place of the original goal function, an alternate objective function is used.a method is used, which is based on the lowered costs that are ultimately realisedby the method for calculating volume The objective is to direct the search into specific areas of interest.the search space, where there are also superior options in comparison to what was originally proposedIt is likely that an objective function may be discovered.According to Pirkwieser and colleagues [62], a similar combination of Lagrangian decomposition and a GA was used to compute the greatest spanning tree with a knapsack constraint.issue. The issue is divided into two parts: a minimal spanning tree and a 0–1 knapsack problem, thanks to Lagrangian relaxation. Once again, the volume algorithm is in play.In order to solve the Lagrangian dual, it is necessary to use While the process of graph reduction is underway

The goal function stays the same as it was before. as an alternative, final decreasedThe costs of initialization, recombination, and mutation are used to skew the results of these operations.operators. Furthermore, the best possible solution derived from the volume was considered.The algorithm is used as a seed in the GA's initial population, and the results are shown on the screen. As a result of the findings,that the volume algorithm alone is already capable of identifying solutions to highly complex problemsHigh-quality service is available even for huge instances. The GA polishes these solutions to make them more appealing.Therefore, in the vast majority of situations, the most optimum solutions are eventually discovered.

6 Using the Primal-Dual Relationship in Metaheuristics

use the complementing slackness conditions (5) to your advantage, or (9). In order to identify a primal feasible solution x that meets these characteristics with regard to the feasible dual solution u, we start with the feasible dual solution u. On the other hand, if one searches in both the dual and primal spaces, it is possible to come up with relevant results.Performance guarantees for primal viable solutions that have been discovered by heuristics.

6.1 Creating Tight Bounds in the Code

A primal-dual variable neighbourhood search is presented by Hansen and colleagues [44]. (VNS)in the case of the straightforward plant placement issue (SPLP). Because the situations that have been dealt with areThe authors suggest that linear programming methods be used to handle problems that are too large to be solved by linear programming approaches.to first run a variable neighbourhood decomposition search on the SPLP before moving onresulting in the discovery of a primary practicable solution The complimentary slackness requirements are used to develop an initial, perhaps infeasible, dual solution, which is subsequently improved upon.The use of variable neighbourhood descent improves the local quality of this solution.(VND), which also lessens the likelihood of a possible impossibility. An exact dual solution is one that is accurate and precise.It is necessary to determine a valid lower limit for the SPLP in order to calculate the upper bound. It is obtained by the process ofUsing the newly discovered sliding simplex approach, which was just introduced The writers go on to saymake advantage of the produced constraints to improve the performance of a B&B algorithm that solves the problem preciselySPLP. The computer studies given here demonstrate the effectiveness of thesuggested technique, which is capable of resolving situations that were previously unsolvableoptimality that has been shown

6.2 Integrating Primal and Dual Solution Approaches in a Single Framework

In his paper, Rego [77] offers a metaheuristic framework that he refers to as relaxation adaptable.RAMP is a kind of memory programming that combines the ideas of Lagrangian and heuristic programming.The effects of surrogate relaxation were compared to those of adaptive memory programming (AMP).PD-RAMP is further extended to include a primal-dual extension. He also offers a particular implementation of PD-RAMP that is based on Lagrangian and surrogate constraint relaxation on the dual side, as well as scatter search and path-relinking on the primal side, to further improve performance.side.A cross-parametric model is constructed by combining lagrangian and surrogate relaxation.The relaxation approach, which makes use of subgradient optimization to obtain acceptable surrogate constraints, is described in detail below. The use of constructive and improvement heuristics allows for the projection of dual solutions into the primal space. The technique produces primordial results.solutions as well as dual limits and, as a result, may be able to demonstrate optimality or provide performance guarantees for the solutions that are developed. AMP stands for Accelerated Mobile Pages.It is possible to project solutions from the dual space to the primal space, which results in the RAMPframework. The authors suggest to utilise frequency-based tabu search or a frequency-based tabu search and replace it with aA strategy in which tabu search and path-relinking are used in conjunction. The primal-dual is a pair of opposites.

The RAMP methodology alternates between a relaxing strategy and a stimulation method.both updating the same reference set in the primordial space, and path-relinking in the primal space ThePD-RAMP is the subject of preliminary computer studies, which the author discusses.is outperforming even the most well-known approaches from the literature in terms of performanceIn regard to various variations of the generalised assignment issue

7 Keeping with the ethos of local search in bed and breakfast

The majority of metaheuristics are founded on the notion of local search, which means that, beginning with an initial solution, a specific neighbourhood surrounding it is searched, and so on and so forth.The incumbent solution is replaced with a superior one if one can be located.This procedure is performed many times. As a result, the key concept is to concentrate the search forEnhanced solutions on previously defined parts of the search space in the vicinity of the problem, goodsolutions.Most B&B algorithms, on the other hand, pick the next B&B tree node to visit.be handled in accordance with the best-first strategy: A node with the lowest lower bound is designated asalways chosen because it is deemed to be the most likely to contain an antigenthe best possible option This is often the most effective way for limiting the risk.the total number of nodes that must be examined before an optimal solution is discovered

as well as demonstrating its superiority. Good comprehensive solutions, on the other hand, and hence alsoDuring this search, it is common for tight upper limits to be discovered late in the process. The best-first approachIn most cases, node selection technique "hops about" on the search tree and in the node selection algorithmIt searches the whole search space and does not remain focused on subregions. When there are no powerful primordialA heuristic is used to determine the most promising full solutions, as well as the best possible solution.A first method is often used in conjunction with an initial dive, in which a depth-first approach is used.Starting from the outset, a strategy is pursued until a workable solution is discovered.When using depth-first search, the next node to be analysed is always the one that contains the most information.Branching has been used to generate the most current versions of things.In recent years, a number of more advanced notions have been put forth.with the intention of focusing B&B-search efforts in the early phase on nearby communitiesIt is necessary to swiftly discover high-quality heuristics among prospective incumbents.solutions. Essentially, we might consider these techniques to be "virtual" executions in a way.a metaheuristic is a heuristic In the next section, we will go through some of these tactics.

7.1 Dives with a Guide

Danna and colleagues [22] discuss guided dives, which are a modest, but successful modification of the previously stated basic diving by momentarily switching to a more advanced mode of operation.Searching in deep initially. Consider the difference between a traditional branching in LP-based B&B and aThe fractional variable, as indicated in Section 3.4. The subproblem that has to be addressed.In the case of guided dives, the dive immediately following is invariably the one in which the branching variable is present.permitted to retain the value it has accrued as a result of the present incumbent solution Scuba diving is a sport.As a result, the incumbent's neighbourhood is favoured over other neighbourhoods. as an alternative toFollowing a single dive at the start of the optimization, guided dives are applied regularly at regular intervals throughout the duration of the optimization. Despite the fact that this methodAlthough it is simple to put into practise, experimental findings show that it has substantial benefits.in comparison to conventional node selection algorithms

8 ILP Techniques for Exploring Large Neighborhoods

A typical strategy in more complex local search based metaheuristics is to search neighbourhoods using smart exact algorithms, which is a popular approach in local search based metaheuristics. If the neighbourhoods are selected properly, they might be rather big, making it possible to conduct an efficient search for the best neighbour while yet remaining within normal limits. Techniques of this kind includeVLSN search [3] is an abbreviation for very large-scale neighbourhood search. Probably the majority ofCombinations of local search-based metaheuristics and ILP approaches used today are described in detail below.This is the method to take. We'll go through a few instances in the next section.In Dynasearch [17, 18], exponentially huge regions are investigated by using a computer algorithm.Programming in a dynamic environment It is feasible to find all conceivable combinations of basic search steps that are mutually independent in a neighbourhood where the search is carried out.one Dynasearch move refers to a collection of individual steps that are combined into one moveUsing ILP Techniques and Metaheuristics in Combination 47run in parallel during a single iteration of the local search In the context of Dynasearch, independence refers to the fact that the individual motions do not interact with one another.Dynamic programming may be used to identify the optimum combination of independent movements in this situation, among others. Dynasearch is limited to cases in which there is asingle search steps are independent of one another, and to our knowledge, it has only been used once.

In issues where solutions are represented by permutations, this has been implemented.Ergun and Orlin [28] conducted an investigation on numerous such communities, with a special focus onThe difficulty of the travelling salespersonThompson et al. [84, 85] propose the following algorithm for a class of partitioning problems:The notion of a cyclic exchange neighbourhood is built on the transfer of goods and services.The movement of single elements between an unlimited number of subsets is done in a cyclic fashion. A 2-exchange move may be thought of as the most basic example of a cyclic exchange.having a total length of two In order to efficiently discover the optimum cyclic exchange for a current solution, a weighted, directed graph is formed, in which each arc represents a cyclic exchange for the current solution.The arc's weight relates to the induced difference in the objective value of the solution, and it indicates a potential transfer of a single element. AThe cyclic exchange with the highest performance may then be determined by identifying the lowest negative-cost option.In this graph, there is a subset-disjoint cycle. The writers take into consideration both accurate and heuristic methods.approaches that may be used for this reasonPuchinger and colleagues [71] offer a hybrid GA/B&B strategy for tackling a combinatorial optimization problem.A real-world difficulty with glass cutting. The GA employs an order-based representation of the data.This is decrypted with the help of a greedy heuristic The B&B algorithm is used in conjunction withan increase in the likelihood of boosting the decoding phase via the generation of locally optimum subpatterns According to the reported findings, the technique of only infrequentlySolving subpatterns to the best of one's ability often improves the overall solution quality.

Budenbender and colleagues [14] describe a tabu search hybrid for tackling a real-world direct flight network design issue using the tabu search algorithm. Neighborhoods are formed as a result ofa substantial subset of the integer variables corresponding to the accomplished operations is fixed.Allowing the other variables to be modified while on the flights CPLEX is a programme that is used to solve problems.the decreased difficulties that are associated with these neighbourhoods

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DiversificationThis is accomplished by closing flights that are commonly encountered in previously established systems.Using an automobile, Prandtstetter and Raidl [65] employ variable neighbourhood search techniques.CPLEX is used for scanning huge neighbourhoods and for solving the sequencing issue.A subset of the planned automobiles is chosen, they are withdrawn from the timetable, and they are replaced with new cars.reinserted in the most efficient manner The techniques used in the different communities are different.to choose the automobiles and their numbers The results show that this method is effective.compete well with top algorithms from a competition hosted by the National Science FoundationIn 2005, the French Operations Research Society ROADEF was founded.Hu and colleagues [49] offer a VNS metaheuristic for determining the generalised minimal set.The issue of the spanning tree The technique makes use of two different forms of dual representations.and the increasingly enormous neighbourhoods that are linked with them Dynamic programming techniques are used to identify the best neighbours, and – in the case of the best neighbours –By solving an ILP formulation with CPLEX, we may achieve the so-called global subtree optimization neighbourhood. The findings of the experiments reveal that each of the variables under consideration.

9. ILP Techniques as Decoders for Indirect or Incomplete Representations

When potential solutions are either indirectly or incompletely represented in metaheuristics, a "intelligent" decoding function is employed in order to determine an actual, full solution to the problem at hand. This is especially true for a large number of GAs.For the decoding stage, it is possible to apply ILP algorithms effectively in certain cases.It is generally simple to approach a MIP by breaking it down into its constituent parts.There are two parts: the integer and the continuous variable. After that, one may use a metaheuristic to optimise just the integer component of the problem; before assessing a solution, one can use a linear regression.The programming solver is used in order to supplement the integer component with an arithmetic expression.The most optimum selection of continuous variable values Such techniques are discussed in detail.ILP Techniques and Metaheuristics in Combination 51when used in combination with GRASP by Net and Pedroso [60] and when used in conjunction with GRASPPedroso [61] does a tabu search.

Glover [40] proposes a parametric tabu search for heuristically solving a system of equations.MIPs. In addition, this technique takes use of an underlying LP-solver to produce the results.candidates for a full solution The current search point is represented indirectly by the LP relaxation of the MIP as well as extra objective criteria that have been established.confine the domains of a subset of the integer variables to a certain range of values These are the desired outcomes.When using the framework, however, they are not immediately recognised as hard limits.Although they use the LP-solver, they are relaxed and incorporated into the objective function in a similar way.as in the case of Lagrangian relaxation It is also possible to use this method in this manner.challenge in which finding any possible integer answers (under certain constraints) is difficultchallenges with customer satisfaction). Glover proposes a number of intensification and diversification tactics that are based on adaptive tabu memory and may be used to make the heuristic decision.make your search more efficientStaggemeier et al. [80] introduced a hybrid GA for addressing a lot-sizing and scheduling issue with the goal of decreasing the total number of lots produced in a year.On parallel machines, inventory and backlog expenses for numerous items are calculated. Solutions are given as product subsets for each machine and for each time in which they are implemented.When the answer is decrypted, the optimum lot sizes that correspond to it are established.by resolving a linear programming problem The technique outperforms a MIP formulation in terms of performance.CPLEX has provided a straightforward solution to the issue.

10. Multi-Stage Approaches to Problem Solving

Some optimization techniques include a series of successively executed optimizations.There are many stages, and various strategies are used at each of the stages.In many real-world applications, the issue spontaneously decomposes into two or more components.If there are many levels, and if the decision

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factors associated with the lower level are different from the higher level (s)given that they have a substantially less influence on the objective value than higher-level factors, optimising the individual levels seems like a viable strategyas part of a rigidly sequential process Techniques such as metaheuristics and ILP may be used.considered, and then used at the individual level in conjunction with one anotherWhen dealing with such a challenge, multi-stage procedures are sometimes used.The process of breakdown is not immediately apparent. Section 9 discussed ways where a metaheuristic is utilised to produce a collection of heuristic answers, such as the one described in Section 9.and the blending of them is accomplished by the application of a precise approach Variable fixing techniques, such as those discussed in Section 5.4, are another example.Tamura and colleagues [83] take on a scheduling challenge at a work shop and start from the ground up.Its ILP formulation may be found here. They determine the probable values for each variable by taking the range of values.and then divide it into a number of subranges, each of which is subsequently indexed. The encoded versionIn a GA, the solutions are specified in such a way that each location represents a variable, andThe index of one of the subranges is represented by the value of this variable. The suitability of suchIn order to acquire a chromosome, Lagrangian relaxation is used to compute its length.limit on the optimum solution under the restrictions imposed by the values of the variables52 Günther R. Raidl and Jakob Puchinger are co-authors of the paper.

The variables are contained within the ranges that have been displayed. When the GA comes to an end, anAn extensive search of the area that has been chosen as the most promising is conducted.in order to arrive at the ultimate solutionLin and colleagues [54] present a precise technique for producing the smallest possible collection of variables.the set of affine functions that characterises the value function of the finite horizon to a significant extentMarkov decision process has been observed. The first stage involves the usage of a GA to createThe largest feasible collection of witness points should be considered. The second stage entailsIn order to remove a component, a component-wise dominance technique is undertaken.

There are repetitive points in. In general, the collection that has been constructed so far does not completely representexplain the value function in detail As a result, a MIP is solved in order to create the missing data.points at the conclusion of the third stage of the algorithm According to the reported outcomes,This method takes less time to complete than some other numerical techniques.When dealing with a two-machine flowshop scheduling issue, Nagar et al. [58] discuss another kind of sequential combination of B&B and a GA that they have developed.Candidates for the solution are represented as permutations of jobs in this case. Prior to executing the GA, B&B is carried out down to a predefined depth k and then stopped.At each node of the clearly defined graph, appropriate boundaries are determined and stored.B&B tree that has been saved. During the course of the GA's execution, each partial answer up toThe tree node corresponding to point k is mapped to position k. In the event that the relatedThe boundaries suggest that there is no route below this node that may lead to an ideal solution; hence,It is treated to a mutation operator that has been particularly designed for this purpose.intended to make a positive alteration to the first section of the permutation's sequence

11. Conclusion

We have examined a large number of situations in which more powerful optimization systems have been built by combining mathematical programming approaches with metaheuristics and other techniques. There are a plethora of very varied methods for achieving such hybridizations, which we have organised into 10 key methodological groups.The most common strategy is to employ a metaheuristic for giving high-quality incumbents and boundaries to a B&B-based exact method, which is perhaps the most classic way.Relaxations that can be addressed rapidly, or the primal-dual connection, on the other hand,In metaheuristics, they are often used for the purpose of directing or restricting the search. Those approaches in which B&B is used are a relatively fresh and extremely promising field of research.adjusted in some manner in order to adhere to the spirit of local search-based metaheuristics, for example, The number of approaches that are now regularly and effectively used is considerable.ILP methods are used to do a neighbourhood search. When this notion is

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extended to include searching for the neighbourhood defined by the common and disjunct elements,When we combine the characteristics of two or more parental solutions, we call this solution merging.approaches. After that, we looked at ILP algorithms as decoders for indirect data transmission.or erroneous or insufficient representations Furthermore, multi-stage techniques are a logical way to address certain issues since they are more efficient. Although it is significantly less often used, yet in theIn the authors' perspective, the most promising hybrid techniques are those that combineFor cut separation and column formation, metaheuristics are used inside more complicated branch-and-cut and branch-and-price algorithms, which are both based on branching and cutting.Last but not least, we looked at collaborative hybrid systems, which are systems that work together.One approach may serve as a type of strategic advice for another, or it may even be the other's source of inspiration.It is possible to establish mutual guiding. As previously stated, certain techniques from the literature are being considered.may be deemed to fit within a number of the methodological categories that we have discussed thus far.identified.

Despite the fact that a great deal of expertise with hybrid systems currently exists,It is frequently a difficult issue to answer the algorithms and types of combinations to use.are the most promising for a new issue that has been presented. Despite the many successful cases,Combine ILP Techniques and Metaheuristics for the Best Results 57In addition to hybrid instances, the reader should bear in mind that a more complicated hybrid is possible.There is no guarantee that the system will perform better than a simpler "pure" method. There have been several unsuccessful attempts to combine mathematical programming.Techniques and metaheuristics are also available, although they are often not disclosed since they are proprietary.The most important piece of advice the authors can provide for designing better hybrid systems is to thoroughly read the literature in search of the most successful examples.methods for dealing with comparable difficulties and ultimately adopting (hybridising) some of their fundamental characteristics. We hope that this chapter serves as an excellent starting point.a starting point as well as some resources for this reason.

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