

Fuzzy Linear Programming Solved via Solving Linear Programming

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ABSTRACT

Engineering design is typically plagued with inaccuracies due to the complexity of many real-world engineering systems. Fuzzy linear programming issues play an important part in fuzzy modelling, which is able to express uncertainty in the real world. Dubois and Prade's LR fuzzy number is one of the most practical themes in recent research, with several useful and simple approximation arithmetic operators on it. Fuzzy vectors occur as a vector of triangular fuzzy integers in various vector calculations. To begin, we are looking for a nonnegative fuzzy vector \tilde{x} in this situation fuzzy numbers. Here, our main scope is finding some nonnegative fuzzy vector \tilde{x} in which maximizes the objective function $\tilde{z} = c \tilde{x}$ so that $\tilde{A} \tilde{x} = \tilde{b}$, where A and \tilde{b} are a real matrix and a fuzzy vector respectively, and $n \times 1$ c is a real vector too.

Keywords: Fuzzy arithmetic, Fuzzy linear programming, Fuzzy number

1 Introduction

A wide range of fields have benefited from fuzzy set theory, including control theory, management science, mathematical modelling, and industrial applications. Tanaka et al. [6] initially suggested the idea of fuzzy linear programming (FLP) on a general-level. This was followed by a large number of writers considering different FLP difficulties and coming up with a variety of solutions. Fuzzy numbers may be compared using ranking functions [1,4,5]. In particular, these approaches are the most convenient. Many writers employ this approach by defining an analogous FLP issue and then using the optimum solution of that solution as the FLP solution. To solve the linear programming issue with fuzzy variables and its dual, fuzzy number linear programming problem directly, we used a generic linear ranking function in [4]. A linear programming issue using triangular fuzzy integers is the focus of this research. New methods for addressing FLP issues without ranking functions have been developed by our team. In addition, we provide an example to demonstrate our strategy.

2 Preliminary

In this section we review some necessary backgrounds of the fuzzy theory in which will be used in this paper. Below, we give definitions and notations taken from [2].

Fuzzy numbers

Definition 2.1. A fuzzy number \tilde{A} is a convex normalized fuzzy set on the real line R such that:

- 1) There exists at least one $x_0 \in R$ with $\mu_{\tilde{A}}(x_0)=1$.
- 2) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Let us assume that the membership function of any fuzzy number \tilde{A} is as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \frac{m^A - x}{\alpha^A}, & m^A - \alpha^A \leq x < m^A \\ 1 - \frac{x - m^A}{\beta^A}, & m^A \leq x \leq m^A + \beta^A \\ 0, & \text{otherwise} \end{cases}$$

where m^A is the mean value of \tilde{A} and α^A and β^A are left and right spreads, respectively and it is termed as **triangular fuzzy number**. We show any triangular fuzzy number by $\tilde{A} = (m^A, \alpha^A, \beta^A)$. Let $F(R)$ be the set of all triangular fuzzy numbers.

Definition 2.2. A fuzzy number $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in R\}$ is nonnegative if and

only if $\mu_{\tilde{A}}(x) = 0$ for all $x < 0$. Then a triangular fuzzy number

$\tilde{A} = (m^A, \alpha^A, \beta^A)$ is nonnegative if $m^A - \alpha^A \geq 0$.

Definition 2.3. Two triangular fuzzy numbers $\tilde{A} = (m^A, \alpha^A, \beta^A)$ and $\tilde{B} = (m^B, \alpha^B, \beta^B)$ are said to be equal if and only if $m^A = m^B, \alpha^A = \alpha^B$ and $\beta^A = \beta^B$.

Definition 2.4. A fuzzy number $\tilde{A} = (m^A, \alpha^A, \beta^A)$ is called symmetric, if $\alpha^A = \beta^A$.

2.2. Arithmetic on triangular fuzzy numbers

Let $\tilde{A} = (m^A, \alpha^A, \beta^A)$ and $\tilde{B} = (m^B, \alpha^B, \beta^B)$ be two triangular fuzzy numbers, then arithmetic on them is defined as [2]:

Addition: $\tilde{A} \oplus \tilde{B} = (m^A + m^B, \alpha^A + \alpha^B, \beta^A + \beta^B)$

Scalar multiplication: For any scalar λ , we have

$$\lambda \tilde{A} = \lambda(m^A, \alpha^A, \beta^A) = \begin{cases} (\lambda m^A, \lambda \alpha^A, \lambda \beta^A), & \text{if } \lambda \geq 0 \\ (\lambda m^A, \lambda \beta^A, \lambda \alpha^A), & \text{if } \lambda \leq 0 \end{cases}$$

Subtraction: $\tilde{A} - \tilde{B} = (m^A - m^B, \alpha^A + \beta^B, \beta^A + \alpha^B)$

2.3. Nonnegative matrix and nonnegative fuzzy vector

Definition2.5. A matrix A is called nonnegative and denoted by $A \geq 0$ if each element of A be a nonnegative number.

Definition2.6. A fuzzy vector $\tilde{b} = (\tilde{b}_i)_{m \times 1}$ is called nonnegative and denoted by $\tilde{b} \geq 0$, if each element of \tilde{b} be a nonnegative fuzzy, that is $\tilde{b}_i \geq 0$.

3 Fuzzy Linear System of Equations

Definition3.1. Consider the $m \times n$ linear system as:

$$A\tilde{x} = \tilde{b}, \tag{1}$$

where $A = [a_{ij}]_{m \times n}$ is a nonnegative crisp matrix and $\tilde{x} = (\tilde{x}_j)$, $\tilde{b} = (\tilde{b}_i)$ are nonnegative fuzzy vectors and $\tilde{x}_j, \tilde{b}_i \in F(R)$ for all $1 \leq j \leq n, 1 \leq i \leq m$, is called a fuzzy linear system with nonnegative triangular numbers.

Definition3.2. We say a nonnegative fuzzy vector \tilde{x} is the solution of $A\tilde{x} = \tilde{b}$,

where A and \tilde{b} are defined in (1), if \tilde{x} satisfies in system.

Now since $\tilde{x} \in F^n(R)$ and $\tilde{b} \in F^m(R)$, we may let $\tilde{x} = (x^m, x^\alpha, x^\beta)$, $\tilde{b} = (b^m, b^\alpha, b^\beta)$ where $x^m, x^\alpha, x^\beta \in R^n$ and $b^m, b^\alpha, b^\beta \in R^m$

Then, we may rewrite the system $A\tilde{x} = \tilde{b}$ as:

$$A(x^m, x^\alpha, x^\beta) = (b^m, b^\alpha, b^\beta), \quad x^m - x^\alpha \geq 0. \tag{2}$$

In other hand, \tilde{b} and \tilde{x} are two nonnegative fuzzy vectors, hence by use of Definition 2.3 and arithmetic on nonnegative triangular fuzzy number, it is enough to solve the following crisp system:

$$Ax^m = b^m, \quad Ax^\alpha = b^\alpha, \quad Ax^\beta = b^\beta. \tag{3}$$

Note that if we use from the symmetric triangular fuzzy numbers, then last system $Ax^\beta = b^\beta$ is not necessary to solved, because it is equal to system $Ax^\alpha = b^\alpha$.

3 Fuzzy Linear Programming

Definition 3.1. Consider the following linear programming problem:

$$\begin{aligned} \max \quad & \tilde{z} = c \tilde{x} \\ \text{s.t.} \quad & \begin{cases} A \tilde{x} = \tilde{b} \\ \tilde{x} \geq 0 \end{cases} \end{aligned} \tag{4}$$

where the coefficient matrix $A = [a_{ij}]_{m \times n}$ and the vector $c = (c_1, \dots, c_n)$ are a nonnegative crisp matrix and vector respectively, and $\tilde{x} = (\tilde{x}_j)$, $\tilde{b} = (\tilde{b}_i)$, are nonnegative fuzzy vectors such that $\tilde{x}_j, \tilde{b}_i \in F(R)$ for all $1 \leq j \leq n, 1 \leq i \leq m$, is called a fuzzy linear programming (FLP) problem.

Definition 3.2. We say that a fuzzy vector x is a fuzzy feasible solution of $Ax = \tilde{b}, x \geq 0$ where A and \tilde{b} are defined in (4), if \tilde{x} satisfies in system.

Now since $\tilde{x} \in F^n(R)$ and $\tilde{b} \in F^m(R)$, we may let $\tilde{x} = (x^m, x^\alpha, x^\beta)$ $\tilde{b} = (b^m, b^\alpha, b^\beta)$, where $x^m, x^\alpha, x^\beta \in R^n$ and $b^m, b^\alpha, b^\beta \in R^m$. Then, we may

rewrite the system $A\tilde{x} = \tilde{b}$ as:

$$A(x^m, x^\alpha, x^\beta) = (b^m, b^\alpha, b^\beta). \tag{5}$$

In other hand, \tilde{b} and \tilde{x} are two nonnegative fuzzy vectors, hence by use of Definition 2.3 and arithmetic on nonnegative triangular fuzzy numbers, it is equivalent to the following crisp system:

$$Ax^m = b^m, Ax^\alpha = b^\alpha, Ax^\beta = b^\beta, x^m - x^\alpha \geq 0. \tag{6}$$

Now we define an operator "max" for a fuzzy linear function which is defined as:

$$\tilde{z} = f(\tilde{x}_1, \dots, \tilde{x}_n) = \sum_{j=1}^n c_j \tilde{x}_j = c_1 \tilde{x}_1 \oplus \dots \oplus c_n \tilde{x}_n,$$

where $c_j, j = 1, \dots, n$, are real numbers and $\tilde{x}_j \in F(R), j = 1, \dots, n$.

Definition 3.3. Let $\tilde{x} = (\tilde{x}_1, \dots, \tilde{x}_n)^T$ and $\tilde{b} = (\tilde{b}_1, \dots, \tilde{b}_m)^T$ be two nonnegative fuzzy vectors, where $\tilde{x}_j = (x_j, \underline{x}_j, \overline{x}_j)$ and $\tilde{b}_i = (b_i, \underline{b}_i, \overline{b}_i) \in F(R)$. A fuzzy vector \tilde{x} maximizes the linear function $\tilde{z} = f(\tilde{x}_1, \dots, \tilde{x}_n)$, such that

$$\begin{aligned} A\tilde{x} &= \tilde{b} \\ \tilde{x} &\geq 0, \end{aligned} \tag{7}$$

where $x_j = (x_j, \underline{x}_j, \bar{x}_j), j = 1, \dots, n$, and $x_j = (x_j, \underline{x}_j, \bar{x}_j), j = 1, \dots, n$, are nonnegative real numbers, if and only if $x = (x_1, \underline{x}_1, \bar{x}_1, x_2, \underline{x}_2, \bar{x}_2, \dots, x_n, \underline{x}_n, \bar{x}_n) \in R^{3n}$ maximizes the below real function:

$$z = \sum_{c_j > 0} c_j (x_j + \frac{1}{2}(\bar{x}_j - \underline{x}_j)) + \sum_{c_j < 0} c_j (x_j + \frac{1}{2}(\underline{x}_j - \bar{x}_j)) \quad (8)$$

such that

$$Ax = b, A\underline{x} = \underline{b}, A\bar{x} = \bar{b}, x - \underline{x} \geq 0, \bar{x} - x \geq 0. \quad (9)$$

Here we give an illustrate example.

Example 4.1. Assume that a corporation produces two different items. Product P1 has a profit of \$40 per unit, whereas product P2 has a profit of \$30 per unit. In order to produce one unit of product P1, it takes twice as long as it does to produce one unit of P2. The overall number of labour hours available each day is close to 500, however this number is subject to vary if special overtime arrangements are made. Both P1 and P2 supplies are nearly 400 units each day, however this may alter based on past experience. When it comes to maximising profits, how many units of goods P1 and P2 should be produced each day? For the sake of simplicity, we'll call x_1 and x_2 the daily production rates for Products P1 and P2. The following linear programming problem with triangular fuzzy variables may therefore be used to outline the issue.

$$\begin{aligned} \max \quad & z = 40 x_1 \oplus 30 x_2 \\ \text{s.t.} \quad & \begin{cases} \tilde{x}_1 \oplus \tilde{x}_2 \leq 400, \\ 2 \tilde{x}_1 \oplus \tilde{x}_2 \leq 500, \\ \tilde{x}_1, \tilde{x}_2 \geq 0. \end{cases} \end{aligned} \quad (10)$$

The supply of material and the available labor hours are close to 400 and 500, and hence are modeled as $(400,5,5)$ and $(500,7,7)$, respectively. Now the current fuzzy linear programming model may be written in the standard form as follows:

$$\begin{aligned} \max \quad & z = 40x_1 \oplus 30x_2 \\ \text{s.t.} \quad & \begin{cases} \tilde{x}_1 \oplus \tilde{x}_2 \oplus \tilde{x}_3 = (400, 5, 5), \\ 2\tilde{x}_1 \oplus \tilde{x}_2 \oplus \tilde{x}_4 = (500, 7, 7), \\ \tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geq 0. \end{cases} \end{aligned} \quad (11)$$

Where ~ 3 x and ~ 4 x are two slack variables.

Hence, the equivalent fuzzy linear programming problem as follows:

$$\begin{aligned} \max \quad & z = 40(\underline{x}_1, \underline{x}_1, \overline{x}_1) \oplus 30(\underline{x}_2, \underline{x}_2, \overline{x}_2) \\ \text{s.t.} \quad & \begin{cases} (\underline{x}_1, \underline{x}_1, \overline{x}_1) \oplus (\underline{x}_2, \underline{x}_2, \overline{x}_2) \oplus (\underline{x}_3, \underline{x}_3, \overline{x}_3) = (400, 5, 5), \\ 2(\underline{x}_1, \underline{x}_1, \overline{x}_1) \oplus (\underline{x}_2, \underline{x}_2, \overline{x}_2) \oplus (\underline{x}_4, \underline{x}_4, \overline{x}_4) = (500, 7, 7), \\ (\underline{x}_1, \underline{x}_1, \overline{x}_1), (\underline{x}_2, \underline{x}_2, \overline{x}_2), (\underline{x}_3, \underline{x}_3, \overline{x}_3), (\underline{x}_4, \underline{x}_4, \overline{x}_4) \geq 0. \end{cases} \end{aligned}$$

Since the fuzzy numbers are symmetric, therefore it is enough to solve the following fuzzy linear programming problem:

$$\begin{aligned} \max \quad & z = 40x_1 + 30x_2 \\ \text{s.t.} \quad & \begin{cases} (\underline{x}_1, \underline{x}_1, \overline{x}_1) \oplus (\underline{x}_2, \underline{x}_2, \overline{x}_2) \oplus (\underline{x}_3, \underline{x}_3, \overline{x}_3) = (400, 5, 5), \\ 2(\underline{x}_1, \underline{x}_1, \overline{x}_1) \oplus (\underline{x}_2, \underline{x}_2, \overline{x}_2) \oplus (\underline{x}_4, \underline{x}_4, \overline{x}_4) = (500, 7, 7), \\ (\underline{x}_1, \underline{x}_1, \overline{x}_1), (\underline{x}_2, \underline{x}_2, \overline{x}_2), (\underline{x}_3, \underline{x}_3, \overline{x}_3), (\underline{x}_4, \underline{x}_4, \overline{x}_4) \geq 0. \end{cases} \end{aligned}$$

Now we can obtain an optimal fuzzy solution for problem (10) by solving the following linear programming:

$$\begin{aligned} \max \quad & z = 40x_1 + 30x_2 \\ \text{s.t.} \quad & \begin{cases} x_1 + x_2 + x_3 = 400, \\ 2x_1 + x_2 + x_4 = 500, \\ \underline{x_1} + \underline{x_2} + \underline{x_3} = 5, \\ 2\underline{x_1} + \underline{x_2} + \underline{x_4} = 7, \\ \underline{x_i} - \underline{x_i} \geq 0, x_i \geq 0, \underline{x_i} \geq 0, i = 1, \dots, 4. \end{cases} \end{aligned}$$

The optimal solution of the above linear programming is: $x_1 = 100, x_2 = 300, x_3 = 0, x_4 = 0, \underline{x_1} = 2, \underline{x_2} = 3, \underline{x_3} = 0, \underline{x_4} = 0$. Therefore, the optimal fuzzy solution of the problem (10) is: $\tilde{x}_1 = (100, 2, 2), \tilde{x}_2 = (300, 3, 3), \tilde{x}_3 = (0, 0, 0), \tilde{x}_4 = (0, 0, 0)$, and the optimal fuzzy value of the objective function is: $z = 40\tilde{x}_1 \oplus 30\tilde{x}_2 = (13000, 170, 170) = 13000$.

5 Conclusion

A novel strategy for tackling FLP issues has been suggested in this study by solving classical linear programming problems. As a result of this article, a novel approach for solving fuzzy linear programming without ranking functions has been developed. The findings presented in this article will also serve as a foundation for future research on fuzzy linear programming and will be used to investigate certain essential ideas of fuzzy linear programming, such as duality results and sensitivity analysis.

References

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