

# **Determination of Linear Far field Pattern in Triangular Index Optical fiber for the Fundamental Transmission Mode**

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## **ABSTRACT**

In this paper we present the far field pattern for the fundamental mode in triangular index fiber. The said profile is of extreme importance in determining the spot size leading to appropriate coupling of the fiber on long distance optical fiber communication system using single mode fiber. The estimation of Far field profile is a cumbersome and tedious task employing the finite element technique, however using a simple power series expansion method we have obtained an excellent approximation of the same. Henceforth the method is of extreme convenience to field engineers in the domain of optical fiber communication.

**Keywords** – Far field pattern, Finite element method, Long haul communication, Optical fiber communication, , Power series expansion.

## **I. INTRODUCTION**

Single mode optical fibers occupy an indispensable position in long distance communication owing to their inherent properties of low loss and high bandwidth [1]. It is extremely important to accurately determine the far field pattern of the fiber before going for any sort of practical implementation, as the same is required for establishing various other parameters of the fiber like modal field, numerical aperture etc [2]. The study of far field is also important for fiber Bragg grating sensor [3]. Further, in the domain of smart structure characterization, far field pattern analysis of sensor is made [4]. Furthermore, one can estimate the modal field provided far field is accurately determined. [5]. Different types of refractive index profiles in a fiber provide us the flexibility of controlling various propagation parameters in long distance optical communication. In this regard it is worth mentioning that step, parabolic and triangular indexed fibers; all poses their inherent advantages and disadvantages owing which different profiles are selected for different types of signal transmissions [6]. In this paper we have focused on triangular index fibers; as the same are extensively emerging in long haul communication systems because of their simplicity of parametric estimations.

In general, the finite element method has been widely used by various researchers and scientists to determine the propagation parameters of different types of fibers. However the said method is not only cumbersome and complex implement but also is not handy for the field engineers to utilise in the real time. In this paper we present a much simple approach to determine one of the fiber parameters i.e the Far field pattern by using the power series expansion technique and compare our results with the one obtained by conventional method to establish its accuracy and simplicity taking triangular index fibre as an example.

**II. THORY**

For a fiber with circular core, the RI profile is presented as

$$n^2(R) = \begin{cases} n_1^2(1 - 2\delta f(R)), & R \leq 1 \\ n_2^2, & R > 1 \end{cases} \quad (1)$$

The radius of the core is represented by a,  $R = r/a$  and  $\delta = (n_1^2 - n_2^2)/2n_1^2$

Here,  $n_1$ ,  $n_2$  and  $f(R)$  represent the core refractive index, cladding refractive index and the profile of the refractive index respectively. Therefore

$$f(R) = R^q, \quad R \leq 1 \quad (2)$$

Here, q stands for the profile exponent, the value of which is  $\infty$  for step index fibers.

Further,  $\eta_0 = \left(\frac{\mu_0}{\epsilon_0}\right)^{\frac{1}{2}}$  where  $\mu_0$  and  $\epsilon_0$  is the permeability and permittivity respectively

The fundamental modal field  $\Psi(R)$  is given by the following differential equation [7]

$$\frac{d^2\Psi(R)}{dR^2} + \frac{1}{R} \frac{d\Psi(R)}{dR} + [V^2(1 - f(R)) - W^2]\Psi(R) = 0 \quad (3)$$

Here,  $W = a(\beta^2 - n_2^2 k_0^2)^{1/2}$  is the cladding decay parameter and  $V = k_0 a(n_1^2 - n_2^2)^{1/2}$  is the normalised frequency.

Further,  $k_0$  and  $\beta$  are the well known constants having usual meaning.

The relation for the fundamental modal field obtained from the approximation of the power series expression in graded index fiber [8] is given by

$$\begin{aligned} \Psi(R) &= (a_0 + a_2 R^2 + a_4 R^4 + a_6 R^6), & R \leq 1 \\ &= (a_0 + a_2 + a_4 + a_6) \frac{K_0(WR)}{K_0(W)}, & R > 1 \end{aligned} \quad (4)$$

Where  $a_i$  ( $i = 0 \dots 3$ ) are constants.

$$R_m = \cos\left(\frac{2m-1}{2M-1} \frac{\pi}{2}\right) \text{ where } [m = 1, \dots, (M-1)] \quad (5)$$

From (4) we choose the value of M as 4 and thus three values of R are obtainable from (5) and those are

$$R_i = 0.9749, 0.7818 \text{ and } 0.4338 \text{ (for } i=1,2 \text{ and } 3) \quad (6)$$

$$a_0 [V^2(1 - f(R_i)) - W^2 + V^2 g \psi^2(R_i)] + a_2 [4 + R_i^2 \{V^2(1 - f(R_i)) - W^2 + V^2 g \psi^2(R_i)\}] + a_4 [16R_i^2 + R_i^4 V^2 - f R_i - W^2 + V^2 g \psi^2 R_i] + a_6 [6V^2 - f R_i - W^2 + V^2 g \psi^2 R_i] = 0$$

$$\text{Where } i = 1, 2, 3 \quad (7)$$

Using Least square fitting technique and choosing appropriate values of  $\alpha$  and  $\beta$  we can write

$$a_0(\alpha W + \beta) + a_2(\alpha W + 2 + \beta) + a_4(\alpha W + 4 + \beta) + a_6(\alpha W + 6 + \beta) = 0$$

The solution of (11) and (13) should be non trivial for the constants and must satisfy the following

$$\begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 & \delta_1 \\ \alpha_2 & \beta_2 & \gamma_2 & \delta_2 \\ \alpha_3 & \beta_3 & \gamma_3 & \delta_3 \\ \alpha_4 & \beta_4 & \gamma_4 & \delta_4 \end{bmatrix} = 0 \quad (8)$$

Where,

$$\alpha_i = V^2[1 - f(R_i)] - W^2$$

$$\beta_i = 4 + R_i^2[V^2(1 - f(R_i)) - W^2]$$

$$\gamma_i = 16R_i^2 + R_i^4[V^2(1 - f(R_i)) - W^2]$$

$$\delta_i = 36R_i^4 + R_i^6[V^2(1 - f(R_i)) - W^2] \tag{9}$$

with  $i = 1, 2, 3$  and

$$\alpha_4 = \alpha W + \beta; \beta_4 = 2 + \alpha_4; \gamma_4 = 4 + \alpha_4; \delta_4 = 6 + \alpha_4$$

Considering cylindrically symmetrical structure of the fiber, the far field of the fiber can be expressed as [1]

$$u(\theta) = 2\pi Ca^2 \int_0^\infty \psi(R) J_0(k_0 a R \sin \theta) R dR \tag{10}$$

In the above expression,  $\theta$  represents the angle between the axis of the core and the direction of observation while  $k_0$  denotes the wave no in free space and  $C$  is a constant. Using (4) in (10), we obtain  $u(\theta)$  as follows

$$\frac{u(\theta)}{2\pi Ca^2} = J_1(m) \left[ \frac{1}{m} - 4 \frac{A_2}{m^3} + \frac{A_2}{m} + 64 \frac{A_4}{m^5} - 16 \frac{A_4}{m^3} + \frac{A_4}{m} - 2304 \frac{A_6}{m^7} + 576 \frac{A_6}{m^5} - 36 \frac{A_6}{m^3} + \frac{A_6}{m} \right] + J_0(m) \left[ 2 \frac{A_2}{m^2} - 32 \frac{A_4}{m^4} + 4A_4m^2 + 1152A_6m^6 - 144A_6m^4 + 6A_6m^2 + 1 + A_2 + A_4 + A_6 W J_0 m K_1 W - m K_0 W J_1 m W^2 + m^2 K_0 W \right] \tag{11}$$

one can find  $u(0)$  which is given below

$$\frac{u(0)}{2\pi Ca^2} = \left[ 1 + \frac{A_2}{4} + \frac{A_4}{6} + \frac{A_6}{8} \right] + \frac{[1 + A_2 + A_4 + A_6][WK_1(W) - K_0(W)]}{[W^2 K_0(W)]} \tag{12}$$

Where, Bessel and modified Bessel function are represented by  $J$ 's and  $K$ 's respectively. The normalized far field intensity for different values of  $\theta$  is given by

$$I(\theta) = \left| \frac{u(\theta)}{u(0)} \right|^2$$

Therefore Eqn (11) and (12) can be utilized to evaluate the above equation for normalized value of intensity.

### III. RESULTS AND DISCUSSION

In this paper we have taken the  $V$  number 3.0 to determine the variation in intensity  $I(\theta)$  with the angle  $\theta$  for a typical triangular index optical fiber. Table 1 shows the normalized values of  $I(\theta)$  with variation in  $m (= k_0 a \sin \theta)$  where  $k_0 = 2\pi/\lambda$ . Figure 1 corresponds to the graphical representation of the far field pattern of the triangular index fiber corresponding to Table 1.

Table 1: Variation of normalized far field intensity in dB with 'm'

m	I(θ)
1	-5.3936
2	-17.5340
3	-27.6343
4	-38.0033
5	-51.1499
6	-99.7957
7	-66.5728
8	-73.2337
9	-104.1710
10	-82.5693

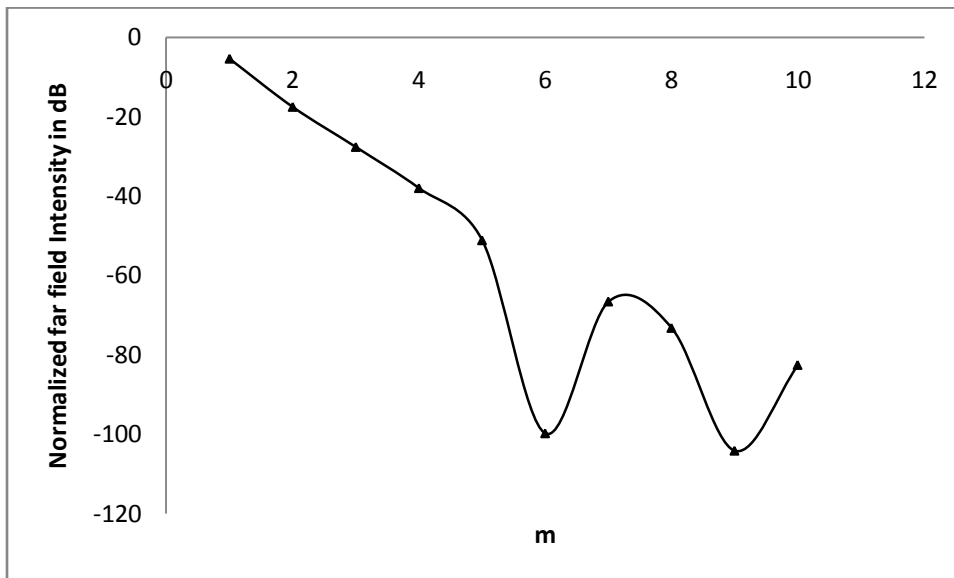


Figure 1: Variation of normalized far field intensity (dB) with m for triangular index fiber having  $V=3.0$ . (Our results are shown with ▲ and simulated exact results are denoted by solid line — )

Since the far field pattern is much better comprehended by illustrating the variation with  $\theta$ ; in Table 2 we have provided the normalized intensity values with respect to  $\theta$ . We have also shown the illustrative variation of the far field with  $\theta$  in Fig 2.

Table 2: Variation of normalized far field intensity in dB with ‘ $\theta$ ’

$\theta$	$I(\theta)$
3.2416	-5.3936
6.8555	-17.5340
10.3144	-27.6343
13.8117	-38.0033
17.3625	-51.1499
20.9835	-99.7957
24.6946	-66.5728
28.5199	-73.2337
32.4897	-104.1710
36.6433	-82.5693

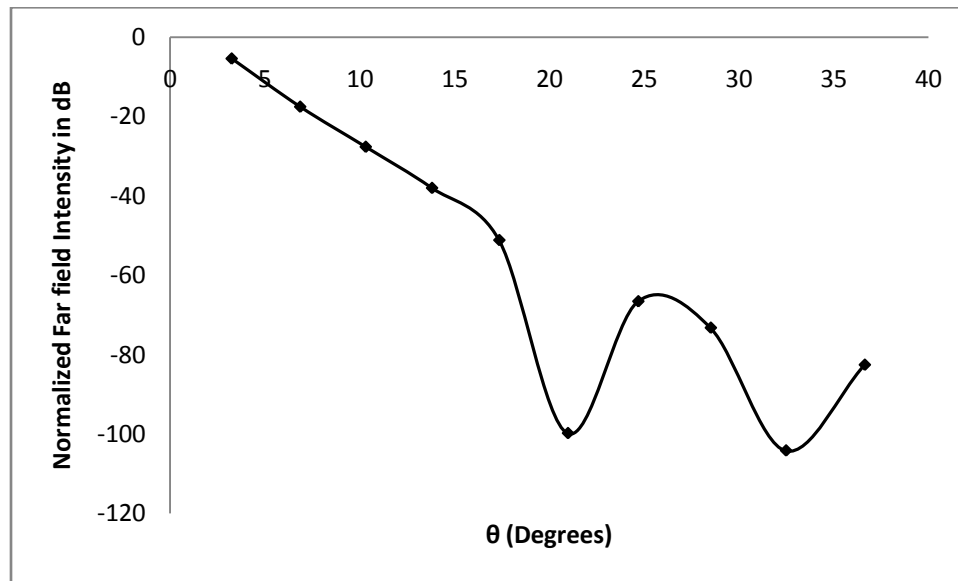


Figure 2: Variation of normalized far field intensity (dB) with  $\theta$  for triangular index fiber having  $V=3.0$ . (Our results are shown with  $\blacktriangle$  and simulated exact results are denoted by solid line  $\text{---}$  )

For calculation of the above values, we have taken the practical values of the fiber radius ‘a’ and the light wavelength ‘ $\lambda$ ’ as  $4\mu\text{m}$  and  $1.5\mu\text{m}$  respectively. Thereby using the relation  $m = k_0 a \sin\theta$ , the values of the normalized intensity is calculated for different values of  $m$  and as shown in Figure 1 and 2.

**IV. CONCLUSION**

Here we conclude that the power series method which has been utilized for determining the Far field pattern of triangular index fibers is very simple to formulate and provides the results which are in excellent agreement with the conventional technique. Furthermore the method is very well suited for calculating other fiber parameters like modal field using the same simple technique. Therefore is ideal for field engineers working in the field of optical communication systems and integrated optics.

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