JOURNAL OF CRITICAL REVIEWS

ISSN- 2394-5125 VOL 7, ISSUE 06, 2020

A solid profit transportation problem in a class of level 2 fuzzy environment with vehicle capacity

Sarbari Samanta [a], Dipak Kumar Jana [a] Sumana Mandal [a] B S Mahapatra [a] M B Bera [a]

Corresponding author Email: samantasarbari@gmail.com, Tel. +918637364785

[a]Department of Applied Sciences, Haldia Institute of technology, Haldia, Purba Midnapur-721657, West Bengal, India

Abstract: The goal of this paper is to come up with the best solutions to the solid profit transportation problem with vehicle capacity. This paper treated transportation costs, demands, supply, purchasing costs, and selling prices as level 2 fuzzy in nature to reflect the uncertainty in the practical decision environment. Due to not meeting the capacity, there is an extra cost (surcharge) under this model. Our goal is to make the most money possible. We develop a level 2 fuzzy solid profit transportation model with expected objectives and chance constraints, which we then convert to crisp counterparts. We also use the expected value model to arrive at a choice by maximising the expected objective values subject to expected constraints. A numerical example is provided to demonstrate the model. The Generalised Reduced Gradient approach yields a set of optimum solutions. There is also a graphical depiction of the profit as well as the various levels of necessity confidence levels.

Keywords: Solid transportation problem; Level 2 fuzzy; Fuzzy expected value model; Necessity measures.

1 Introduction

Haley [1] initially proposed the solid transportation problem (STP) in 1962, in which three types of constraints are considered instead of two (source and destination) as in the transportation problem (TP). This additional limitation is mainly related to transportation modes (conveyances). There have been a slew of articles published in this subject in recent years. Some papers reduce the total cost of transportation. For example, ojha et al. [2] analysed a STP for a fixed charge, transportation cost, and price discounted variable charge for an item. MOSTP was also created by Pramanik et al. [3], [4], [5].

Typically, a STP is simulated assuming that all conveyances' entire supply capacity is met for all source to destination routs, regardless of the amount to be conveyed. However, this may not always be the case in real-world transportation problems. This sort of real-world scenario prompted us to develop some viable solid transportation models.

Uncertainty, in general, is present in all real-life problems, such as unpredictability, fuzziness, and roughness. Since the introduction of the fuzzy set by Zadeh [6] in 1965, fuzzy set theory has been developed and applied to a wide range of real-world issues. Following that, Liu and Liu [7] created a new type of fuzzy ramdom optimization called expected value models. Maity et al. [8] created a production recycling model with a uncertain holding cost. Defective unit production is a normal part of the manufacturing process. Some academics have raised concerns about the limitations of utilising conventional fuzzy sets. Ordinary fuzzy set models fall short of completely explaining these complex uncertain optimal issues. Because standard fuzzy sets are unable to capture such uncertainty, advanced fuzzy sets must be used to compensate. However, we can see from market research data in supply chain network design challenges that simple fuzzy sets are insufficient to explain some variables, such as market demand and transportation costs. Assume a marketing agency is using analysis to approximate uncertain demand as a triangle fuzzy number (a, c, b), but the most promising value is unable to be established as a certain value when other sources of information are needed. That is, by adding to the left and right spreads (a, (l, c, r), b), the centre of the triangle fuzzy number may be modified. As a result, a level 2 fuzzy set should be used in this scenario. That is, the demand for a product should be better considered as a level 2 fuzzy number $(a, \tilde{z}, b)$ with $\tilde{z} = (l, c, r)$. To address the supply chain network design problem, it is necessary to expand fuzzy programming to level 2 fuzzy programming.

Zadeh presented two types of advanced fuzzy sets in 1971 and 1975: the level 2 fuzzy set [9] and the type 2 fuzzy set

[10], which were further developed by Gottwald [11], Mendel [12], Pandian et al. [13], and others. When the elements of a fuzzy number are also fuzzy numbers, the fuzzy number is classified as a level 2 fuzzy number; when the memberships of a fuzzy number are also fuzzy numbers, the fuzzy number is classified as a type 2 fuzzy number. In reality, level 2 fuzzy sets provide us more degrees of freedom in modelling mathematical optimization problems by allowing us to reflect uncertainties and fuzziness. As a result, they are far more accurate when attempting to characterise data.

We design a solid profit transportation problem with vehicle capacity and extra cost (penalty) that occurs when the vehicle capacity is not met in this work. All expenses, demands, and availabilities are treated as level 2 fuzzy variables here. The model is developed as a general level 2 fuzzy programming model with expected objectives and chance restrictions. A model with expected objectives and expected constraints is also constructed. In addition, a crisp equivalent transformation is performed. We also provide some numerical data.

2 Preliminary idea on Fuzzy sets, Level 2 fuzzy sets and Level 2 fuzzy EVM

We will review some fundamental understanding of fuzzy set theory, Level 2 fuzzy, and Level 2 fuzzy EVM in this part.

Definition 1 A fuzzy set is a group of objects in which there is no clear distinction between those that belong to a class and those that do not. If X is a set of objects and x is an element of X, then a fuzzy set $\widetilde{A}$ in X is a set of ordered pairs
$\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x))/x \in X\}$
where $\mu_{\widetilde{A}}(x)$ is the membership function or grade of membership of x in $\widetilde{A}$ and X is mapped to the membership space M, which is the closed interval [0,1].

Definition 2 A Triangular fuzzy number (TFN) denoted by $\widetilde{A}$ is defined as (r_1, r_2, r_3) where the membership function is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{x-r_1}{r_2-r_1} & \text{for } r_1 \le x < r_2 \\ \frac{r_3-x}{r_3-r_2} & \text{for } r_2 < x \le r_3 \\ 0 & \text{otherwise} \end{cases}$$

where r_1, r_2 and r_3 are real numbers.

Lemma 1 Let $\widetilde{A} = (r_1, r_2, r_3)$ be a triangular fuzzy number and ρ is a crisp number. The expected value of $\widetilde{A}$ is

$$E[\widetilde{A}] = \frac{1}{2}[(1-\rho)r_1 + r_2 + \rho r_3], 0 < \rho < 1.$$
$$= \frac{r_1+2r_2+r_3}{4}, \rho = 0.5. \quad (1)$$

Proof: The Proof of the lemma-1 is in reference in Liu and Liu [7].

Level 2 fuzzy set: A level 2 fuzzy set is a fuzzy set where the elements are also fuzzy sets, and the level 2 fuzzy variable is a fuzzy variable with fuzzy parameters. Level 2 fuzzy sets were presented by Zadeh [9]. Such sets are fuzzy sets whose elements themselves are ordinary fuzzy sets. They are very useful in circumstances where it is difficult to determine some elements for a fuzzy set.

Definition 3 In Gottwald [11] A level 2 fuzzy set $\widetilde{\widetilde{V}}$ over a universal set U is defined by

$$\widetilde{\widetilde{V}} = \{(\widetilde{V}, \mu_{\widetilde{\widetilde{V}}}(\widetilde{V})) | \forall \widetilde{V} \in F(U): \mu_{\widetilde{V}} > 0\}, \quad (2)$$

where each ordinary fuzzy set $\widetilde{V}$ is defined by

$$\widetilde{V} = \{(x, \mu_{\widetilde{V}}(x)) | \forall x \in U: \mu_{\widetilde{V}} > 0\}. \quad (3)$$

For convenience, the membership grades $\mu_{\widetilde{V}}(\widetilde{V})$ of the fuzzy sets $\widetilde{V} \in F(U)$ are called 'outer-layer' membership grades, whereas the membership grades $\mu_{\widetilde{V}}(\tilde{x})$ of the elements $x \in U$ are called 'inner-layer'¯membership grades. Since level 2 fuzzy sets are still fuzzy sets, their mathematical behavior is defined by the fuzzy set operators.

Example: $\tilde{\tilde{\xi}} = (s_L, \tilde{\rho}, s_R)$ with $\tilde{\rho} = (\rho_L, \rho_M, \rho_R)$ is called Fu-Fu variable,(cf. Fig. 1), if the outer-layer and inner-layer membership functions are as follows

$$\mu_{\tilde{\xi}}(x) = \begin{cases} (\frac{x-s_L}{\tilde{\rho}-s_L}) & \text{if } s_L \le x \le \tilde{\rho} \\ 0 & \text{otherwise} \\ (\frac{s_R-x}{s_R-\tilde{\rho}}) & \text{if } \tilde{\rho} \le x \le s_R \end{cases}$$

and

$$\mu_{\tilde{\rho}}(x) = \begin{cases} (\frac{x'-\rho_L}{\rho_M-\rho_L}) & \text{if } \rho_L \le x' \le \rho_M \\ 0 & \text{otherwise} \\ (\frac{\rho_R-x'}{\rho_R-\rho_M}) & \text{if } \rho_M \le x' \le \rho_R \end{cases}$$

where $\tilde{\rho}$ is the center of $\tilde{\tilde{\xi}}$, which is a triangular fuzzy variable, $s_L \in R$ and $s_R \in R$ are the smallest possible value and the largest possible value of $\tilde{\tilde{\xi}}$, $\rho_L \in R$, $\rho_M \in R$ and $\rho_R \in R$ are the the smallest possible value, the most promising value and the largest possible value of $\tilde{\rho}$ respectively.

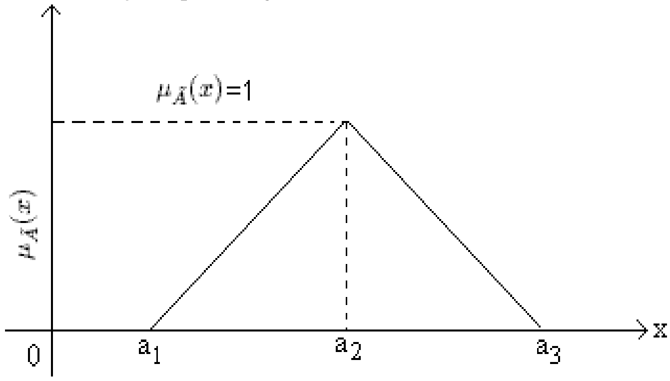

Figure 1: Triangular Level 2 fuzzy variable

Lemma 2 The expected value for the level 2 fuzzy variable $\tilde{\tilde{c}} = (l_1, \tilde{c}, r_1)$ with $\tilde{c} = (l_2, c, r_2)$ we obtain that

$$E[\tilde{\tilde{c}}] = c + \frac{(r_1+r_2)-(l_1+l_2)}{4} \tag{4}$$

Proof: Let $\tilde{\tilde{c}} = (l_1, \tilde{c}, r_1)$ where $\tilde{c} = (l_2, c, r_2)$. So

$$\begin{aligned} E[\tilde{\tilde{c}}] &= \frac{E(\tilde{c}-l_1)+2E(\tilde{c})+E(\tilde{c}+r_1)}{4} \\ &= \frac{E(\tilde{c})-l_1+2E(\tilde{c})+E(\tilde{c})+r_1}{4} \\ &= \frac{4E(\tilde{c})-l_1+r_1}{4} \\ &= E(\tilde{c}) + \frac{r_1-l_1}{4} \\ &= c + \frac{r_2-l_2}{4} + \frac{r_1-l_1}{4} \\ &= c + \frac{(r_1+r_2)-(l_1+l_2)}{4} \end{aligned}$$

Lemma 3 Assume that $\tilde{\tilde{\xi}}$ and $\tilde{\tilde{\eta}}$ are level 2 fuzzy variables with finite expected values. Then for any real numbers a and b, we have

$$E[a\tilde{\tilde{\xi}} + b\tilde{\tilde{\eta}}] = aE[\tilde{\tilde{\xi}}] + bE[\tilde{\tilde{\eta}}] \tag{5}$$

Proof: The proof of the Lemma-3 is in reference Xu and Zhou [14].

General Model for Level 2 fuzzy EVM: First we give the general model of Fu-Fu multi-objective decision making model as follows,

$$\begin{cases} Min \quad f_1(x,\tilde{\tilde{\xi}}),\ f_2(x,\tilde{\tilde{\xi}}),\ \cdots,\ f_m(x,\tilde{\tilde{\xi}}) \\ s.t \quad \begin{cases} g_r(x,\tilde{\tilde{\xi}}) \le 0, r = 1,2,\cdots,p \\ x \in X \end{cases} \end{cases} \tag{6}$$

If $\tilde{\tilde{\xi}}$ is a Fu-Fu vector, $x = (x_1, x_2, \cdots, x_n)$ is decision vector, then the objective function $f_i(x,\tilde{\tilde{\xi}})$ and constraint functions $g_r(x,\tilde{\tilde{\xi}})$ are also Fu-Fu variables, $i = 1,2,\cdots,m, r = 1,2,\cdots,p$. In order to rank Fu-Fu objective $f_i(x,\tilde{\tilde{\xi}})$, we may employ the expected value operator to deal with the objective functions and constraints, and we can get the following model (eq-7). For the expected value of the objective $E[f_i(x,\tilde{\tilde{\xi}})], i = 1,2,\cdots,m$, it means that the larger the expected returns $E[f_i(x,\tilde{\tilde{\xi}})]$, the better the decision x. The first type of Fu-Fu decision-making model is expected value multi-objective decision-making model in which the underlying philosophy is based on selecting the decision with maximum expected objective values as:

$$\begin{cases} Min \quad E[f_1(x,\tilde{\tilde{\xi}})],\ E[f_2(x,\ \tilde{\tilde{\xi}})],\ \cdots,\ E[f_m(x,\tilde{\tilde{\xi}})] \\ s.t \quad \begin{cases} E[g_r(x,\tilde{\tilde{\xi}})] \le 0, r = 1,2,\cdots,p \\ x \in X \end{cases} \end{cases} \tag{7}$$

Theorem 1 If $\alpha^e_{rj1}, \alpha^e_{rj2}, \beta^e_{rj1}, \beta^e_{rj2}$ are left and right spreads of $\tilde{\tilde{e}}_{rj}(\theta)$ and $\tilde{e}_{rj}(\theta)$, $\alpha^b_{r1}, \alpha^b_{r2}, \beta^b_{r1}, \beta^b_{r2}$ are left and right spreads of $\tilde{\tilde{b}}_r(\theta)$ and $\tilde{b}_r(\theta)$, the basis function $L, R: [0,1] \to [0,1]$ are monotone decreasing continuous function, and it satisfies $L(1) = R(1) = 0, L(0) = R(0) = 1$ and the LR fuzzy variable is specified as the triangular fuzzy variable and $R^{-1}(\theta_i) = 1 - \theta_i$, $R^{-1}(\eta_i) = 1 - \eta_i$. and if $\tilde{\tilde{e}}_{rj}(\theta)$ and $\tilde{\tilde{b}}_r(\theta)$ are independent fuzzy variables. Then

$$Nec\{\theta | Nec\{\tilde{\tilde{e}}_{rj}(\theta)^T x \le \tilde{\tilde{a}}_r(\theta)\} \ge \theta_r\} \ge \eta_r$$

is equivalent to

$$a_r - e_r^T x - L^{-1}(1-\eta_r)(\alpha^a_{r2} + \beta^{eT}_{r2} x) - L^{-1}(1-\theta_r)\alpha^a_{r1} - R^{-1}(\theta_r)\beta^{eT}_{r1} x \ge 0$$

Proof: The proof of the Theorem 1 is in reference Xu and Zhou [14] in page 257.

Theorem 2 Assume that the Fu-Fu variable $\tilde{e}_{ij}$ and $\tilde{b}_r$ is as same as the assumption in Theorem -1, $i = 1,2,\cdots,m, j = 1,2,\cdots,n$. For confidence level $\delta_i, \gamma_i \in [0,1], i = 1,2,\cdots,m$. Then

$$Nec\{\delta | Nec\{\tilde{\tilde{e}}_{rj}(\delta)^T x \ge \tilde{\tilde{b}}_r(\delta)\} \ge \delta_r\} \ge \gamma_r$$

is equivalent to

$$e_r^T x - b_r - L^{-1}(1-\gamma_r)(\alpha^b_{r2} + \alpha^{eT}_{r2} x) - \mathrm{L}^{-1}(1-\delta_r)\alpha^{eT}_{r1} x - R^{-1}(\delta_r)\beta^b_{r1} \ge 0$$

Proof: The proof of the Theorem 2 is in reference Xu and Zhou [14] in page 258.

3 Assumptions and Notations

3.1 Notations

To formulate the solid transportation model with vehicle capacity, the following notations are used:

- m = number of sources.
- n = number of demands.
- K = number of conveyances i.e., different modes of the transportation problem.
- z = the objective function.
- $\tilde{\tilde{p}}_i$ = the purchasing price of the product at the i^{th} origin.
- $\tilde{\tilde{s}}_j$ = the selling price of the product at the j^{th} destination.
- $\tilde{\tilde{c}}_{ijk}$ = the unit transportation cost from i^{th} origin to j^{th} destination by k^{th} conveyance.
- x_{ijk} = the decision variable which is the amount of product to be transported from i^{th} origin to j^{th} destination

ISSN- 2394-5125 VOL 7, ISSUE 06, 2020

by k^{th} conveyance.

- $\tilde{\tilde{a}}_i$ = the amount of the product available at the i^{th} origin.
- $\tilde{\tilde{b}}_i$ = the demand of the product at the j^{th} destination.
- z_{ijk} = the frequency of conveyance k for transporting products from the i^{th} origin to j^{th} destination.
- e_k = capacity of a single vehicle of k^{th} type conveyance.
- q_k = number of available vehicles of k^{th} type conveyance.
- ρ_{ijk} = total additional cost for $i-j-k$ route due to not fulfilling the vehicle capacity.

3.2 Assumptions

In this solid profit transportation problem, the following assumptions are made.
i)No damageability of the units occurs
ii)A single item of a homogeneous product should be transported from sources to destinations.

4 Model formulation

We assume that m origins (or sources) O_i $(i = 1,2,\cdots,m)$, n destinations(i.e. demands) $D_j\,(j = 1,2,\cdots,n)$ and K conveyances $E_k(k = 1,2,\cdots,K)$. K conveyances i.e., different modes of transport may be trucks, cargo flights, goods trains, ships, etc. Let $\tilde{\tilde{a}}_i$ be the amount of available product at i^{th} origin, $\tilde{\tilde{b}}_j$ be the demand at j^{th} destination and e_k represents the amount of product which can be carried by k^{th} conveyance. $\tilde{\tilde{c}}_{ijk}$ be the cost for transportation of a unit product from i^{th} source to j-th destination by means of the k^{th} conveyance. The variable x_{ijk} represents the unknown quantity to be transported from origin O_i to destination D_j by means of k^{th} conveyance.

Moreover, in a traditional solid transportation problem, the whole transportation capacity of conveyances is taken into account, and the problem is solved under the assumption that the total capacity may be used. However, in many real-world transportation systems, this does not occur. Vehicles must be hired in full, and the number of vehicles necessary is determined by the amount of merchandise to be delivered across a certain route. The challenge in this situation emerges when the amount of assigned product is insufficient to fill the vehicle's capacity, because additional costs are paid despite the unit transportation cost owing to the vehicle's capacity not being filled. So, to address this sort of issue, we develop a solid transportation model with vehicle capacity. Let e_k represent the capacity of single vehicle of k^{th} type, z_{ijk} represent the number of vehicles of k^{th} type conveyance necessary to carry the product from i^{th} origin to j^{th} destination via k^{th} conveyance and x_{ijk} represent the corresponding amount of goods. Here c_{ijk} is the unit transportation cost calculated based on the vehicle capacity e_k being fully used. So an additional cost ρ_{ijk} will be introduced if the capacity e_k is not fully utilised. To calculate the additional cost, at first deficit amount is to be calculated. Either deficit amount is to be calculated for each route as $(z_{ijk}\,e_k - x_{ijk})$ or by computing the empty ratio of each vehicle of k^{th} type conveyance for delivering goods from i^{th} origin to j^{th} destination as

$$d_{ijk} = \begin{cases} 0 & if\,\frac{x_{ijk}}{q_k} = [\frac{x_{ijk}}{q_k}] \\ 1 - (\frac{x_{ijk}}{q_k} - [\frac{x_{ijk}}{q_k}]) & Otherwise \end{cases}$$

Then the deficit amount for $i-j-k$ route is given by $e_k.d_{ijk}$. Now if u_{ijk} is the additional cost per unit amount of deficit for $i-j-k$ route, then additional cost for this route is

$\rho_{ijk} = u_{ijk}\ (z_{ijk}\ \ e_k - x_{ijk})$ or $\rho_{ijk} = u_{ijk}\,.\ e_k.\ d_{ijk}\,.$

As a result, the total extra expense is $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\rho_{ijk}$

So the solid transportation problem will be

$$max\quad \tilde{\tilde{Z}} = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}[(\tilde{\tilde{s}}_j - \tilde{\tilde{p}}_i - \tilde{\tilde{c}}_{ijk} - \rho_{ijk})\ \ x_{ijk}] \tag{8}$$
$$where\ \ \rho_{ijk} = u_{ijk}\ \ (z_{ijk}\ \ e_k - x_{ijk})$$

subject to

$$\begin{aligned} &\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \le \tilde{\tilde{a}}_i i = 1,2,3,\cdots,m \\ &\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \ge \tilde{\tilde{b}}_j j = 1,2,3,\cdots,n \\ &x_{ijk} \le z_{ijk}\,e_k i = 1,2,\cdots,m; j = 1,2,\cdots,n; k = 1,2,\cdots,K \\ &\sum_{i=1}^{m}\sum_{j=1}^{n} z_{ijk} \le q_k k = 1,2,\cdots,K \end{aligned} \tag{9}$$

5 Equivalent Crisp Problems

With unknown market needs and costs, the solid transportation problem is defined here.Let the level 2 fuzzy number $\tilde{\tilde{s}}_j$ be denoted as $\tilde{\tilde{s}}_j = (s_{jl1}, \tilde{s}_j, s_{ju1})_{LR}$ where $\tilde{s}_j$ is also a LR fuzzy vector with the left and right spread s_{jl2} and s_{ju2}. So $\tilde{s}_j = (s_{jl2}, s_j, s_{ju2})_{LR}$. Similarly the level 2 fuzzy numbers $\tilde{\tilde{p}}_i$, $\tilde{\tilde{c}}_{ijk}$, $\tilde{\tilde{a}}_i$ and $\tilde{\tilde{b}}_j$ are approximated to $\tilde{\tilde{p}}_i = (p_{il1}, \tilde{p}_i, p_{iu1})_{LR}$ with $\tilde{p}_i = (p_{il2}, p_i, p_{iu2})_{LR}$, $\tilde{\tilde{c}}_{ijk} = (c_{ijkl1}, \tilde{c}_{ijk}, c_{ijku1})_{LR}$ with $\tilde{c}_{ijk} = (c_{ijkl2}, c_{ijk}, c_{ijku2})_{LR}$, $\tilde{\tilde{a}}_i = (a_{il1}, \tilde{a}_i, a_{iu1})_{LR}$ with $\tilde{a}_i = (a_{il2}, a_i, a_{iu2})_{LR}$, and $\tilde{\tilde{b}}_j = (b_{jl1}, \tilde{b}_j, b_{ju1})_{LR}$ with $\tilde{b}_j = (b_{jl2}, b_j, b_{ju2})_{LR}$ respectively. The previous transportation model then has the following structure:

5.1 Model-1

Model with expected objective and chance constraints.

$$max \quad E[\tilde{\tilde{Z}}] = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} [(E[\tilde{\tilde{s}}_j] - E[\tilde{\tilde{p}}_i] - E[\tilde{\tilde{c}}_{ijk}] - \rho_{ijk}) \; x_{ijk}] \tag{10}$$
$$where \;\; \rho_{ijk} = u_{ijk}(z_{ijk} e_k - x_{ijk})$$

subject to

$$\begin{aligned}
&Nec\{\theta | Nec\{\textstyle\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq \tilde{\tilde{a}}_i\} \geq \theta_i\} \geq \eta_i, i = 1,2,\cdots,m \\
&Nec\{\phi | Nec\{\textstyle\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk} \geq \tilde{\tilde{b}}_j\} \geq \phi_j\} \geq \xi_j, j = 1,2,\cdots,n \\
&x_{ijk} \leq z_{ijk} e_k i = 1,2,\cdots,m; j = 1,2,\cdots,n; k = 1,2,\cdots,K \\
&\textstyle\sum_{i=1}^{m} \sum_{j=1}^{n} z_{ijk} \leq q_k k = 1,2,\cdots,K
\end{aligned} \tag{11}$$

Using Lemma-2, Theorem-1 and Theorem-2, the above equations (10) and (11) reduce to

$$\begin{aligned}
Max \quad E\left[\tilde{\tilde{Z}}\right] = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} [\{\{s_j + \tfrac{(s_{ju1}+s_{ju2})-(s_{jl1}+s_{jl2})}{4}\} + \\
\{p_i + \tfrac{(p_{iu1}+p_{iu2})-(p_{il1}+p_{il2})}{4}\} + \{c_{ijk} + \tfrac{(c_{ijku1}+c_{ijku2})-(c_{ijkl1}+c_{ijkl2})}{4}\} - \rho_{ijk}\} \;\; x_{ijk}]
\end{aligned} \tag{12}$$
$$where \;\; \rho_{ijk} = u_{ijk} \;\; (z_{ijk} \;\; e_k - x_{ijk})$$

subject to

$$\begin{aligned}
&\textstyle\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq a_i - L^{-1}(1-\eta_i) a_{il2} - L^{-1}(1-\theta_i) \, a_{il1}, i = 1,2,\cdots,m \\
&\textstyle\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk} \geq b_j + L^{-1}(1-\xi_j) b_{jl2} + R^{-1}(\phi_j) b_{jr1}, j = 1,2,\cdots,m \\
&x_{ijk} \leq z_{ijk} e_k i = 1,2,\cdots,m; j = 1,2,\cdots,n; k = 1,2,\cdots,K \\
&\textstyle\sum_{i=1}^{m} \sum_{j=1}^{n} z_{ijk} \leq q_k k = 1,2,\cdots,K
\end{aligned} \tag{13}$$

5.2 Model-2

Model with expected objective values subject to expected constraints (level 2 Expected Value Model(EVM)).

$$max \quad E[\tilde{\tilde{Z}}] = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} [(E[\tilde{\tilde{s}}_j] - E[\tilde{\tilde{p}}_i] - E[\tilde{\tilde{c}}_{ijk}] - \rho_{ijk}) \; x_{ijk}] \tag{14}$$
$$where \;\; \rho_{ijk} = u_{ijk}(z_{ijk} e_k - x_{ijk})$$

subject to

$$\begin{aligned}
&\textstyle\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq E[\tilde{\tilde{a}}_i], i = 1,2,\cdots,m \\
&\textstyle\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk} \geq E[\tilde{\tilde{b}}_j], j = 1,2,\cdots,n \\
&x_{ijk} \leq z_{ijk} e_k i = 1,2,\cdots,m; j = 1,2,\cdots,n; k = 1,2,\cdots,K \\
&\textstyle\sum_{i=1}^{m} \sum_{j=1}^{n} z_{ijk} \leq q_k k = 1,2,\cdots,K
\end{aligned} \tag{15}$$

Using lemma-2, the above equations reduces to

$$\begin{aligned}
Max \, E\left[\tilde{\tilde{Z}}\right] = \sum_{j=1}^{n} \sum_{k=1}^{K} [\{\{s_j + \tfrac{(s_{ju1}+s_{ju2})-(s_{jl1}+s_{jl2})}{4}\} + \\
\{p_i + \tfrac{(p_{iu1}+p_{iu2})-(p_{il1}+p_{il2})}{4}\} + \{c_{ijk} + \tfrac{(c_{ijku1}+c_{ijku2})-(c_{ijkl1}+c_{ijkl2})}{4}\} - \rho_{ijk}\} \, x_{ijk}]
\end{aligned} \tag{16}$$
$$where \;\; \rho_{ijk} = u_{ijk} \;\; (z_{ijk} \;\; e_k - x_{ijk})$$

subject to

$$\begin{aligned}
&\textstyle\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq \{a_i + \tfrac{(a_{iu1}+a_{iu2})-(a_{il1}+a_{il2})}{4}\}, i = 1,2,\cdots,m \\
&\textstyle\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk} \geq \{b_j + \tfrac{(b_{ju1}+b_{ju2})-(b_{jl1}+b_{jl2})}{4}\}, j = 1,2,\cdots,m \\
&x_{ijk} \leq z_{ijk} e_k i = 1,2,\cdots,m; j = 1,2,\cdots,n; k = 1,2,\cdots,K
\end{aligned} \tag{17}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} z_{ijk} \leq q_k k = 1,2, \cdots, K,$$

6 Numerical Experiment

We explore a solid profit transportation problem with two sources, six destinations, and two modes of transportation to highlight the issues. As a result, m=2,n=6, and K=2. The additional costs for unit deficit amount, the number of required vehicles of each type, and the capacity of each type are assumed as crisp numbers, while the selling prices, purchasing costs of products, availabilities of the corresponding origins (i.e. resources), demands of the destinations, and unit transportation costs are assumed as level 2 fuzzy numbers. Assume that the extra expenses for the unit deficiency amount are $u_{ijk} = 0.8 \cdot E[\tilde{\tilde{c}}_{ijk}]$ with $\tilde{\tilde{c}}_{ijk}$ being the unit transportation cost. The following assumptions are used for selling prices, product purchasing costs, availability of relevant origins (i.e. resources), destination needs, unit transportation costs, number of necessary vehicles of each kind, and capacity of each type:

Table 1. Input data for level 2 fuzzy unit selling price $\tilde{\tilde{s}}_j$

$\tilde{\tilde{s}}_1 = (0.2, \tilde{s}_1, 0.2)_{LR}$ with $\tilde{s}_1 = (42.8, 43, 43.2)$	$\tilde{\tilde{s}}_2 = (0.2, \tilde{s}_2, 0.2)_{LR}$ with $\tilde{s}_2 = (56.8, 57, 57.2)$	$\tilde{\tilde{s}}_3 = (0.1, \tilde{s}_3, 0.1)_{LR}$ with $\tilde{s}_3 = (49.9, 50, 50.1)$
$\tilde{\tilde{s}}_4 = (0.4, \tilde{s}_4, 0.4)_{LR}$ with $\tilde{s}_4 = (51.6, 52, 52.4)$	$\tilde{\tilde{s}}_5 = (0.3, \tilde{s}_5, 0.3)_{LR}$ with $\tilde{s}_5 = (54.7, 55, 55.3)$	$\tilde{\tilde{s}}_6 = (0.2, \tilde{s}_6, 0.2)_{LR}$ with $\tilde{s}_6 = (59.8, 60, 60.2)$

Table 2. Input data for level 2 fuzzy unit purchasing price $\tilde{\tilde{p}}_i$

$(0.1, \tilde{p}_1, 0.1)_{LR}$ with $\tilde{p}_1 = (31.9, 32, 32.1)$	$(0.1, \tilde{p}_2, 0.1)_{LR}$ with $\tilde{p}_2 = (45.9, 46, 46.1)$	$(0.2, \tilde{p}_3, 0.2)_{LR}$ with $\tilde{p}_3 = (44.8, 45, 45.2)$

Table 3. Input data for level 2 fuzzy unit transportation cost $\tilde{\tilde{c}}_{ijk}$ via conveyance k=1

$\tilde{\tilde{c}}_{111} = (0.5, \tilde{c}_{111}, 0.5)_{LR}$ with $\tilde{c}_{111} = (7.5, 8, 8.5)$	$\tilde{\tilde{c}}_{121} = (0.2, \tilde{c}_{121}, 0.2)_{LR}$ with $\tilde{c}_{121} = (6.8, 7, 7.2)$	$\tilde{\tilde{c}}_{131} = (0.1, \tilde{c}_{131}, 0.1)_{LR}$ with $\tilde{c}_{131} = (9.9, 10, 10.1)$
$\tilde{\tilde{c}}_{141} = (0.3, \tilde{c}_{141}, 0.3)_{LR}$ with $\tilde{c}_{141} = (8.7, 9, 9.3)$	$\tilde{\tilde{c}}_{151} = (0.3, \tilde{c}_{151}, 0.3)_{LR}$ with $\tilde{c}_{151} = (8.7, 9, 9.3)$	$\tilde{\tilde{c}}_{161} = (0.1, \tilde{c}_{161}, 0.1)_{LR}$ with $\tilde{c}_{161} = (8.9, 9, 9.1)$
$\tilde{\tilde{c}}_{211} = (0.5, \tilde{c}_{211}, 0.5)_{LR}$ with $\tilde{c}_{211} = (6.5, 7, 7.5)$	$\tilde{\tilde{c}}_{221} = (0.2, \tilde{c}_{221}, 0.2)_{LR}$ with $\tilde{c}_{221} = (9.8, 10, 10.2)$	$\tilde{\tilde{c}}_{231} = (0.3, \tilde{c}_{231}, 0.3)_{LR}$ with $\tilde{c}_{231} = (9.7, 10, 10.3)$
$\tilde{\tilde{c}}_{241} = (0.4, \tilde{c}_{241}, 0.4)_{LR}$ with $\tilde{c}_{241} = (7.6, 8, 8.4)$	$\tilde{\tilde{c}}_{251} = (0.2, \tilde{c}_{251}, 0.2)_{LR}$ with $\tilde{c}_{251} = (10.8, 11, 11.2)$	$\tilde{\tilde{c}}_{261} = (0.2, \tilde{c}_{261}, 0.2)_{LR}$ with $\tilde{c}_{261} = (9.8, 10, 10.2)$
$\tilde{\tilde{c}}_{311} = (0.3, \tilde{c}_{311}, 0.3)_{LR}$ with $\tilde{c}_{311} = (9.7, 10, 10.3)$	$\tilde{\tilde{c}}_{321} = (0.2, \tilde{c}_{321}, 0.2)_{LR}$ with $\tilde{c}_{321} = (8.8, 9, 9.2)$	$\tilde{\tilde{c}}_{331} = (0.1, \tilde{c}_{331}, 0.1)_{LR}$ with $\tilde{c}_{331} = (8.9, 9, 9.1)$
$\tilde{\tilde{c}}_{341} = (0.4, \tilde{c}_{341}, 0.4)_{LR}$ with $\tilde{c}_{341} = (9.6, 10, 10.4)$	$\tilde{\tilde{c}}_{351} = (0.2, \tilde{c}_{351}, 0.2)_{LR}$ with $\tilde{c}_{351} = (9.8, 10, 10.2)$	$\tilde{\tilde{c}}_{361} = (0.1, \tilde{c}_{361}, 0.1)_{LR}$ with $\tilde{c}_{361} = (8.9, 9, 9.1)$

Table 4. Input data for level 2 fuzzy unit transportation cost $\tilde{\tilde{c}}_{ijk}$ via conveyance k=2

$\tilde{\tilde{c}}_{112} = (0.1, \tilde{c}_{112}, 0.1)_{LR}$ with $\tilde{c}_{112} = (10.9, 11, 11.1)$	$\tilde{\tilde{c}}_{122} = (0.3, \tilde{c}_{122}, 0.3)_{LR}$ with $\tilde{c}_{122} = (7.7, 8, 8.3)$	$\tilde{\tilde{c}}_{132} = (0.2, \tilde{c}_{132}, 0.2)_{LR}$ with $\tilde{c}_{132} = (7.8, 8, 8.2)$
$\tilde{\tilde{c}}_{142} = (0.4, \tilde{c}_{142}, 0.4)_{LR}$ with $\tilde{c}_{142} = (6.6, 7, 7.4)$	$\tilde{\tilde{c}}_{152} = (0.5, \tilde{c}_{152}, 0.5)_{LR}$ with $\tilde{c}_{152} = (11.5, 12, 12.5)$	$\tilde{\tilde{c}}_{162} = (0.3, \tilde{c}_{162}, 0.3)_{LR}$ with $\tilde{c}_{162} = (12.7, 13, 13.3)$
$\tilde{\tilde{c}}_{212} = (0.5, \tilde{c}_{212}, 0.5)_{LR}$ with $\tilde{c}_{212} = (7.5, 8, 8.5)$	$\tilde{\tilde{c}}_{222} = (0.1, \tilde{c}_{222}, 0.1)_{LR}$ with $\tilde{c}_{222} = (6.9, 7, 7.1)$	$\tilde{\tilde{c}}_{232} = (0.2, \tilde{c}_{232}, 0.2)_{LR}$ with $\tilde{c}_{232} = (8.8, 9, 9.2)$
$\tilde{\tilde{c}}_{242} = (0.4, \tilde{c}_{242}, 0.4)_{LR}$ with $\tilde{c}_{242} = (9.6, 10, 10.4)$	$\tilde{\tilde{c}}_{252} = (0.3, \tilde{c}_{252}, 0.3)_{LR}$ with $\tilde{c}_{252} = (7.7, 8, 8.3)$	$\tilde{\tilde{c}}_{262} = (0.2, \tilde{c}_{262}, 0.2)_{LR}$ with $\tilde{c}_{262} = (6.8, 7, 7.2)$
$\tilde{\tilde{c}}_{312} = (0.4, \tilde{c}_{312}, 0.4)_{LR}$ with $\tilde{c}_{312} = (7.6, 8, 8.4)$	$\tilde{\tilde{c}}_{322} = (0.2, \tilde{c}_{322}, 0.2)_{LR}$ with $\tilde{c}_{322} = (8.8, 9, 9.2)$	$\tilde{\tilde{c}}_{332} = (0.2, \tilde{c}_{332}, 0.2)_{LR}$ with $\tilde{c}_{332} = (6.8, 7, 7.2)$
$\tilde{\tilde{c}}_{342} = (0.3, \tilde{c}_{342}, 0.3)_{LR}$ with $\tilde{c}_{342} = (7.7, 8, 8.3)$	$\tilde{\tilde{c}}_{352} = (0.2, \tilde{c}_{352}, 0.2)_{LR}$ with $\tilde{c}_{352} = (9.8, 10, 10.2)$	$\tilde{\tilde{c}}_{362} = (0.2, \tilde{c}_{362}, 0.2)_{LR}$ with $\tilde{c}_{362} = (10.8, 11, 11.2)$

Table 5. Input data for level 2 fuzzy availabilities $\tilde{\tilde{a}}_i$

$(0.3, \tilde{a}_1, 0.3)_{LR}$ with $\tilde{a}_1 = (29.5, 30, 30.5)$	$(0.8, \tilde{a}_2, 0.8)_{LR}$ with $\tilde{a}_2 = (32.9, 33, 33.1)$	$(0.4, \tilde{a}_3, 0.4)_{LR}$ with $\tilde{a}_3 = (22.7, 23, 23.3)$

Table 6. Input data for level 2 fuzzy demands $\tilde{b}_j$

$(0.1, \tilde{b}_1, 0.1)_{LR}$ with $\tilde{b}_1 = (14.7, 14.8, 14.9)$	$(0.2, \tilde{b}_2, 0.2)_{LR}$ with $\tilde{b}_2 = (16.5, 16.8, 17.1)$	$(0.2, \tilde{b}_3, 0.2)_{LR}$ with $\tilde{b}_3 = (13.6, 13.8, 14)$
$(0.3, \tilde{b}_4, 0.3)_{LR}$ with $\tilde{b}_4 = (17.4, 18, 18.6)$	$(0.2, \tilde{b}_5, 0.2)_{LR}$ with $\tilde{b}_5 = (9.6, 10, 10.4)$	$(0.2, \tilde{b}_6, 0.3)_{LR}$ with $\tilde{b}_6 = (5.2, 5.5, 5.8)$

Table 7. Input data for vehicle capacity e_k and number of available vehicles q_k

$e_1 = 2.48,\ e_2 = 3.78,\ q_1 = 15,\ q_2 = 22$

Table 8. Optimum results for Model-1 with different values of θ, η, ϕ, ξ

θ	η	ϕ	ξ	profit	x_{ijk}	z_{ijk}
0.5	0.5	0.5	0.5	368.15	$x_{121} = 11.15, x_{142} = 18.45,$ $x_{211} = 6.25, x_{222} = 5.9$ $x_{252} = 10.3, x_{262} = 10.1,$ $x_{312} = 8.65, x_{332} = 14$	$z_{121} = 4, z_{142} = 5,$ $z_{211} = 3, z_{222} = 2$ $z_{252} = 3, z_{262} = 3,$ $z_{312} = 2, z_{332} = 4$
0.4	0.5	0.5	0.5	369.48	$x_{121} = 11.2, x_{142} = 18.45,$ $x_{211} = 6.22, x_{222} = 5.85$ $x_{252} = 10.3, x_{262} = 10.19,$ $x_{312} = 8.68, x_{332} = 14$	$z_{121} = 5, z_{142} = 5,$ $z_{211} = 3, z_{222} = 2$ $z_{252} = 3, z_{262} = 3,$ $z_{312} = 2, z_{332} = 4$
0.3	0.5	0.5	0.5	370.81	$x_{121} = 11.25, x_{142} = 18.45,$ $x_{211} = 6.19, x_{222} = 5.85$ $x_{252} = 10.3, x_{262} = 10.28,$ $x_{312} = 8.71, x_{332} = 14$	$z_{121} = 5, z_{142} = 5,$ $z_{211} = 2, z_{222} = 2$ $z_{252} = 3, z_{262} = 3,$ $z_{312} = 2, z_{332} = 4$
0.2	0.5	0.5	0.5	372.14	$x_{121} = 11.3, x_{142} = 18.45,$ $x_{211} = 6.16, x_{222} = 5.75$ $x_{252} = 10.3, x_{262} = 10.37,$ $x_{312} = 8.74, x_{332} = 14$	$z_{121} = 5, z_{142} = 5,$ $z_{211} = 2, z_{222} = 2$ $z_{252} = 3, z_{262} = 3,$ $z_{312} = 2, z_{332} = 4$
0.1	0.5	0.5	0.5	373.47	$x_{121} = 11.35, x_{142} = 18.45,$ $x_{211} = 6.13, x_{222} = 5.7$ $x_{252} = 10.3, x_{262} = 10.46,$ $x_{312} = 8.77, x_{332} = 14$	$z_{121} = 5, z_{142} = 5,$ $z_{211} = 2, z_{222} = 2$ $z_{252} = 3, z_{262} = 3,$ $z_{312} = 2, z_{332} = 4$

Table 9. Remaining part of the optimum results for Model-1 with different values of $\vartheta, \eta, \phi, \xi$

0.5	0.4	0.5	0.5	369.62	$x_{121} = 11.18, x_{142} = 18.45,$ $x_{211} = 6.21, x_{222} = 5.87$ $x_{252} = 10.3, x_{262} = 10.25,$ $x_{312} = 8.69, x_{332} = 14$	$z_{121} = 5, z_{142} = 5,$ $z_{211} = 3, z_{222} = 2$ $z_{252} = 3, z_{262} = 3,$ $z_{312} = 2, z_{332} = 4$
0.5	0.3	0.5	0.5	371.09	$x_{121} = 11.21, x_{142} = 18.45,$ $x_{211} = 6.17, x_{222} = 5.84$ $x_{252} = 10.3, x_{262} = 10.4,$ $x_{312} = 8.73, x_{332} = 14$	$z_{121} = 5, z_{142} = 5,$ $z_{211} = 2, z_{222} = 2$ $z_{252} = 3, z_{262} = 3,$ $z_{312} = 2, z_{332} = 4$
0.5	0.2	0.5	0.5	372.56	$x_{121} = 11.24, x_{142} = 18.45,$ $x_{211} = 6.13, x_{222} = 5.81$ $x_{252} = 10.3, x_{262} = 10.55,$ $x_{312} = 8.77, x_{332} = 14$	$z_{121} = 5, z_{142} = 5,$ $z_{211} = 2, z_{222} = 2$ $z_{252} = 3, z_{262} = 3,$ $z_{312} = 2, z_{332} = 4$
0.5	0.1	0.5	0.5	374.03	$x_{121} = 11.27, x_{142} = 18.33,$ $x_{211} = 6.19, x_{222} = 5.72,$ $x_{252} = 10.3, x_{262} = 10.7,$ $x_{312} = 8.81, x_{332} = 14$	$z_{121} = 5, z_{142} = 5,$ $, z_{211} = 2, z_{222} = 2,$ $z_{252} = 3, z_{262} = 3,$ $z_{312} = 2, z_{332} = 4$
0.5	0.5	0.5	0.4	369.31	$x_{121} = 11.21, x_{142} = 18.39,$ $x_{211} = 6.22, x_{222} = 5.81,$ $x_{252} = 10.26, x_{262} = 10.26,$ $x_{312} = 8.67, x_{332} = 13.98,$	$z_{121} = 5, z_{142} = 5,$ $z_{211} = 3, z_{222} = 2,$ $z_{252} = 3, z_{262} = 3,$ $z_{312} = 2, z_{332} = 4$
0.5	0.5	0.5	0.3	369.31	$x_{121} = 11.27, x_{142} = 18.33,$ $x_{211} = 6.19, x_{222} = 5.72,$ $x_{252} = 10.22, x_{262} = 10.42,$ $x_{312} = 8.69, x_{332} = 13.96$	$z_{121} = 5, z_{142} = 5,$ $z_{211} = 3, z_{222} = 2,$ $z_{252} = 3, z_{262} = 3,$ $z_{312} = 2, z_{332} = 4$
0.5	0.5	0.5	0.2	369.31	$x_{121} = 11.33, x_{142} = 18.27,$ $x_{211} = 6.16, x_{222} = 5.63,$ $x_{252} = 10.18, x_{262} = 10.58,$ $x_{312} = 8.71, x_{332} = 13.94$	$z_{121} = 5, z_{142} = 5,$ $z_{211} = 2, z_{222} = 1,$ $z_{252} = 3, z_{262} = 3,$ $z_{312} = 2, z_{332} = 4$
0.5	0.5	0.5	0.1	369.31	$x_{121} = 11.39, x_{142} = 18.21,$ $x_{211} = 6.13, x_{222} = 5.54,$ $x_{252} = 10.14, x_{262} = 10.74,$ $x_{312} = 8.73, x_{332} = 13.92$	$z_{121} = 5, z_{142} = 5,$ $z_{211} = 2, z_{222} = 1,$ $z_{252} = 3, z_{262} = 3,$ $z_{312} = 2, z_{332} = 4$

Table 10. Optimum results for Model-2

profit	x_{ijk}	z_{ijk}

391.80	$x_{121} = 12, x_{142} = 18, x_{211} = 5.6, x_{222} = 4.8$ $x_{252} = 10, x_{262} = 12.6, x_{312} = 9.2, x_{332} = 13.8$	$z_{121} = 5, z_{142} = 5, z_{211} = 2, z_{222} = 1$ $z_{252} = 3, z_{262} = 3, z_{312} = 2, z_{332} = 4$

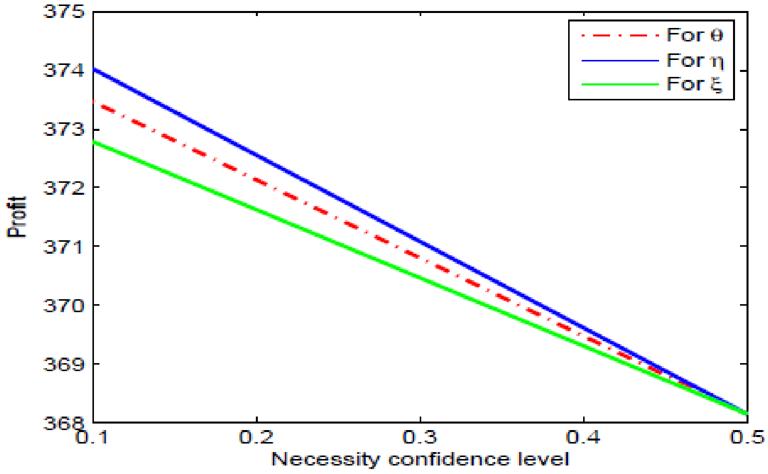

Figure 2: The graphical representation of profit and different necessity levels

7 Discussion

It is observed that the profit in model-1 is increasing as the value of θ, η and ξ is increasing respectively. The results are calculated (in Table -8,-9) by using Generalised Reduced Gradient (GRG) technique. A graphical representation of profit versus different necessity confidence levels is shown. Here it is seen that the profit is inversely dependent on the necessity confidence levels which is a natural Phenomenon.

8 Conclusions

In traffic engineering, the solid transportation problem is a critical network optimization problem. We focus on a level 2 fuzzy solid profit transportation issue with vehicle capacity in this research. We make an effort to utilize level 2 fuzzy set theory to describe the uncertain information because of its advantage of having additional degrees of freedom that make it possible to directly model and handle uncertainties. So the transportation costs, demands, supplies, purchasing costs, selling prices are supposed to be level 2 fuzzy variables due to the instinctive imprecision. Here the transportation problem is more realistic as an additional cost (penalty) is considered which is due to not fulfilling the vehicle capacity. We have developed a model with expected objectives and chance constraints. The objectives and constraints are converted into their crisp equivalents. We also develop a model with expected objectives and constraints and consequently that is converted into crisp. Finally the optimum solutions are obtained by Lingo 12. A graphical representation of the profit and the different necessity confidence levels is also shown. The present model can be extended for multi-objective STP for further research work.

References

[1] K.. Haley, The solid transportation problem, Operations Research, 10 (1962) 448-463.

[2] A. Ojha, B. Das, S. Mondal, M. Maiti, A solid transportation problem for an item with fixed charge, vechicle cost and price discounted varying charge using genetic algorithm, Applied Soft Computing, (2010) 100-110.

[3] S. Pramanik, D.K.. Jana, M. Maiti, Multi-objective solid transportation problem in imprecise environments, Journal of Transportation Security, 6 (2) (2013) 131-150.

[4] S. Pramanik, D.K.. Jana, K. Maiti, A multi objective solid transportation problem in fuzzy, bi-fuzzy environment via genetic algorithm, Int. J. Advanced Operations Management, 6 (2014) 4-26.

ISSN- 2394-5125 VOL 7, ISSUE 06, 2020

[5] S. Pramanik, D.K.. Jana, M. Maiti, A fixed charge multi-objective solid transportation problem in random fuzzy environment, Journal of Intelligent and Fuzzy Systems, 28(6) (2015) 2643-2654.
[6] L. Zadeh, Fuzzy sets, Information and Control, 8 (1965) 338-353.
[7] Y. Liu, B. Liu, A class of fuzzy ramdom optimization: expected value models, Information Science, 155 (2003) 89-102.
[8] A.K. Maity, K. Maity, M. Maiti, A production-recycling-inventory system with imprecise holding costs, Applied Mathematical Modelling, 32 (2008) 2241-2253.
[9] L. Zadeh, Quantitative fuzzy semantics, Information Sciences, 3(2) (1971) 177-200.
[10] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning, Information Sci, 8 (1975) 199-249.
[11] S. Gottwald, Set theory for fuzzy sets of higher level, Fuzzy Sets and Systems, 2(2) (1979) 125-151.
[12] J.M. Mendel, R.I.B. John, Type-2 Fuzzy Sets Made Simple, IEEE Transactions on Fuzzy Systems, 10(2) (2002) 117-127.
[13] P. Pandian, D. Anuradha, A New Approach for Solving Solid Transportation Problems, Applied Mathematical Sciences, 4 (2010) 3603 - 3610.
[14] J. Xu, X. Zhou, Fuzzy Link Multiple-Objective Decision Making, Springer- Verlag, Berlin, (2009).