

# Numerical simulation of vehicle dynamics problems

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## Abstract

The tasks of a traction engine with suspension elements and additional devices for converting movement (DCM) are considered. The object of protection, the dynamic state of which is estimated, is a solid with mass  $M$  and moment of inertia  $J$  relative to the center of gravity. To take into account the rheological properties of the suspension material, the Boltzmann-Volterra principle is used. Mathematical models of the problem under consideration are obtained, which are described by the systems of integro - differential equations. A solution method based on the use of quadrature formulas is developed and a computer program is compiled on its basis, the results of which are reflected in the graphs. The influence of DCM and rheological properties of the suspension material on the shape of the vertical vibrations of the object is investigated.

**Keywords:** dynamics, dynamic model, vibrations, body, viscosity, viscoelasticity, unevenness model, wheel, perturbations, equilibrium equations, stiffness, integral operator, relaxation kernel, integro-differential equations, trapezoid formula.

## 1. Introduction

The dynamic vibration dampers, as some additional devices introduced into the original design schemes of vibration protection systems, can be considered as one of the means of controlling the state of the protection object. It is shown that mathematical models of oscillatory systems in the form of structural schemes are dynamically equivalent to automatic control systems, which have certain advantages compared to conventional approaches based on the use of differential equations. The dynamic damping in structural models is interpreted as the introduction of additional negative feedback circuits. Such chains are formed on the basis of structural transformations of the original model according to the rules of parallel and serial connection of the spring [1-3].

Many tasks of assessing the dynamic properties of technical objects under the influence of the vibrational loads on them are solved by using the design schemes in the form of mechanical oscillatory systems with several degrees of freedom, which gives a certain opportunities in assessing the forms of dynamic interactions and determining the requirements for the corresponding structural solutions, which are predetermined by the properties of the constituent elements. A number of issues of constructing design schemes of technical objects, the features of their elemental base and the possibilities of forming and evaluating the dynamic states of technical objects were considered in the works [4-7].

## 2. Statement and mathematical model of the problem

The design scheme of the technical objects with a so-called suspension for protection against the external influences is considered. Such tasks are typical for vehicles for various purposes, in particular for the protection of traction engines of locomotives for various purposes [7].

On fig. 1 shows the design scheme of the traction engine with viscoelastic suspension elements and additional devices for converting movement (DCM). The object of protection, the dynamic state of which is estimated, is a solid with mass  $M$  and moment of inertia  $J$  relative to the center of gravity. The object ( $M, J$ ) with the center of gravity  $O$  ( $y_0, \varphi$ ), rests at points  $A$  and  $B$  on the supporting surfaces  $I$  and  $II$ , which perform harmonic in-phase oscillations specified by the laws  $z_1(t)$  and  $z_2(t)$ , respectively. The components of the supporting devices are viscoelastic elements in the form of linear springs with stiffness coefficients  $k_1$  and  $k_2$  having contacts with the supporting surfaces at points  $A$  and  $B$ , respectively. In addition, parallel to the springs are introduced DCM with correspondingly reduced masses  $l_1$  and  $l_2$ . Such devices can be implemented in various structural and technical versions, in particular, based on non-self-braking screw mechanisms, where  $L$  corresponds to the reduced mass of the flywheel nut [6; 9]. At points  $A_1$  and  $B_1$ , suspension elements are attached to the object of protection. The parallel connection of the spring and the control valve, which perceive external disturbances from the side of the supporting surface at points  $A$  and  $B$ , transfers the force at points  $A_1$  and  $B_1$ , respectively. In this case, the transformation of effort occurs. The peculiarity of the problem under consideration is that the object ( $M, J$ ) perceives at the same time a disturbance at two points with the supporting surfaces  $I$  and  $II$  (Fig. 1).  $l_1$  and  $l_2$  are the distances

to the center of gravity. It is assumed that the system has linear properties with vanishingly small resistance forces. The motion is considered in the coordinate systems  $y_1$  and  $y_2$  and  $y_0$  and  $\varphi$  associated with a fixed basis. The objective of the study is to assess the dynamic states, the peculiarity of which is formed by the presence in the structure of the system of additional elements in the form of DCM, as well as by the fact that the simultaneous joint action of two external force factors is realized in the system [7].

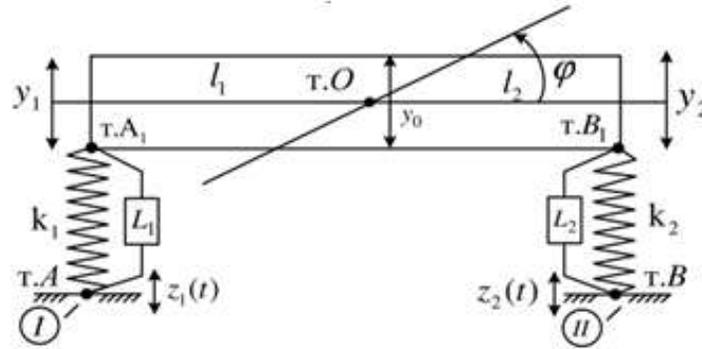


FIGURE 1. Schematic diagram of the suspension of the traction engine.

Using the Boltzmann-Volterra principle [8], we obtain a system of integro-differential equations describing the problem under consideration in a viscoelastic formulation:

$$\begin{cases} \ddot{y}_1(Ma^2 + Jc^2 + L_1) + k_1(1 - R_1^*)(y_1 - z_1) + \ddot{y}_2(Mab - Jc^2) = L_1 \dot{z}_1 \\ \ddot{y}_2(Mb^2 + Jc^2 + L_2) + k_2(1 - R_2^*)(y_2 - z_2) + \ddot{y}_1(Mab - Jc^2) = L_2 \dot{z}_2 \end{cases} \quad (1)$$

Where  $a = \frac{l_2}{l_1+l_2}$ ,  $b = \frac{l_1}{l_1+l_2}$ ,  $c = \frac{1}{l_1+l_2}$ ,  $R_p^*w = \int_0^t R_p(t-\tau)w(\tau)d\tau$  - integral operators

with relaxation kernels  $R_p(t) = \epsilon_p t^{\alpha p - 1} e^{-\beta p t}$ ;  $p = 1, 2$ .

Let's pretend that:

$$y_1(0) = \dot{y}_1(0) = y_2(0) = \dot{y}_2(0) = 0. \quad (2)$$

### 3. Solution methods

By entering the dimensionless parameters  $\frac{y_1}{y_0}, \frac{y_2}{y_0}, \frac{z_1}{y_0}, \frac{z_2}{y_0}, \frac{M}{M_0}, \frac{Jc^2}{M_0}, \frac{L_1}{M_0}, \frac{L_2}{M_0}, \frac{k_1}{k_0}, \frac{k_2}{k_0}, \frac{t}{t_0}$ ;  $t_0 = \sqrt{\frac{M_0}{k_0}}$  and while maintaining the previous notation we have:

$$\begin{cases} \ddot{y}_1 + c_1(1 - R_1^*)(y_1 - z_1) + c_2(1 - R_2^*)(y_2 - z_2) = c_3 \dot{z}_1 + c_4 \dot{z}_2 \\ \ddot{y}_2 + c_5(1 - R_1^*)(y_1 - z_1) + c_6(1 - R_2^*)(y_2 - z_2) = c_7 \dot{z}_1 + c_8 \dot{z}_2 \end{cases} \quad (3)$$

where  $c_1 = \frac{p_2}{p_7}$ ;  $c_2 = -\frac{p_1 p_5}{p_7}$ ;  $c_3 = \frac{p_3}{p_7}$ ;  $c_4 = -\frac{p_1 p_6}{p_7}$ ;  $c_5 = -\frac{p_2 p_4}{p_7}$ ;  $c_6 = \frac{p_5}{p_7}$ ;  $c_7 = -\frac{p_3 p_4}{p_7}$ ;  
 $c_8 = \frac{p_6}{p_7} p_1 = \frac{Mab - Jc^2}{Ma^2 + Jc^2 + L_1}$ ;  $p_2 = \frac{k_1}{Ma^2 + Jc^2 + L_1}$ ;  $p_3 = \frac{L_1}{Ma^2 + Jc^2 + L_1}$ ;  $p_4 = \frac{Mab - Jc^2}{Mb^2 + Jc^2 + L_2}$ ;  
 $p_5 = \frac{k_2}{Mb^2 + Jc^2 + L_2}$ ;  $p_6 = \frac{L_2}{Mb^2 + Jc^2 + L_2}$ ;  $p_7 = 1 - p_1 p_4$ ;

System (3) is solved by the method based on the use of the quadrature formula [10]. Integrating system (3) over  $t$  in the interval  $[0; t]$  twice and setting  $t = t_n = n \cdot \Delta t$ ,  $n = 1, 2, 3, \dots$  ( $\Delta t$  time step) we have:

$$\left\{ \begin{aligned} y_1(t_n) &= \int_0^{t_n} (t_n - s) \{c_3 \ddot{z}_1(s) + c_4 \ddot{z}_2(s) - c_1 [y_1(s) - z_1(s)] - c_2 [y_2(s) - z_2(s)]\} ds + \\ &+ c_1 \int_0^{t_n} G_1(t_n - s) [y_1(s) - z_1(s)] ds + c_2 \int_0^{t_n} G_2(t_n - s) [y_2(s) - z_2(s)] ds; \\ y_2(t_n) &= \int_0^{t_n} (t_n - s) \{c_7 \ddot{z}_1(s) + c_8 \ddot{z}_2(s) - c_5 [y_1(s) - z_1(s)] - c_6 [y_2(s) - z_2(s)]\} ds + \\ &+ c_5 \int_0^{t_n} G_1(t_n - s) [y_1(s) - z_1(s)] ds + c_6 \int_0^{t_n} G_2(t_n - s) [y_2(s) - z_2(s)] ds; \end{aligned} \right. \quad (4)$$

where

$$G_j(t_n - s) = \int_0^{t_n - s} (t_n - s - \tau) R_j(\tau) d\tau; \quad j = 1, 2.$$

Replacing the integrals with the trapezoid quadrature formulas in the system (4), we have the following recurrence relations for determining the vertical displacements  $y_{1n} = y_1(t_n)$  and  $y_{2n} = y_2(t_n)$  of the object (M, J):

$$\left\{ \begin{aligned} y_{1n} &= \sum_{j=0}^{n-1} A_j (t_n - t_j) [c_3 \ddot{z}_{1j} + c_4 \ddot{z}_{2j} - c_1 (y_{1j} - z_{1j}) - c_2 (y_{2j} - z_{2j})] + \\ &+ \sum_{j=0}^{n-1} A_j [G_1(t_n - t_j) c_1 (y_{1j} - z_{1j}) + G_2(t_n - t_j) c_2 (y_{2j} - z_{2j})]; \\ y_{2n} &= \sum_{j=0}^{n-1} A_j (t_n - t_j) [c_7 \ddot{z}_{1j} + c_8 \ddot{z}_{2j} - c_5 (y_{1j} - z_{1j}) - c_6 (y_{2j} - z_{2j})] + \\ &+ \sum_{j=0}^{n-1} A_j [G_1(t_n - t_j) c_5 (y_{1j} - z_{1j}) + G_2(t_n - t_j) c_6 (y_{2j} - z_{2j})]; \end{aligned} \right. \quad (5)$$

where  $A_0 = \frac{\Delta t}{2}$ ;  $A_j = \Delta t, j = \overline{1, n-1}$ .

#### 4. Results and conclusions

Based on the developed algorithm, a computer program was compiled in which the results are reflected in graphs. The computational experiments were carried out. The dimensionless distances to the center of gravity are  $l_1 = 0.6$  and  $l_2 = 0.4$ . The supporting surfaces I and II perform harmonic oscillations according to the given laws  $z_1(t) = z_2(t) = 0.02 \sin(0.7 + 0.5\pi t)$ .

For the suspension elasticity ( $\varepsilon_1 = \varepsilon_2 = 0$ ), the vertical displacements  $y_1(t)$  and  $y_2(t)$  of the object (M, J) without taking into account the DCM (solid line) and taking into account the DCM (dashed line) are respectively shown in Figures 2 and 3. The results show that taking into account the DCM leads to a decrease in the amplitude of oscillations of the object (M, J) and it is significant.

For a viscoelastic suspension ( $\varepsilon_1 = \varepsilon_2 = 0.01$ ;  $\alpha_1 = \alpha_2 = 0.25$ ;  $\beta_1 = \beta_2 = 0.05$ ), the vertical displacements  $y_1(t)$  and  $y_2(t)$  of the object (M, J) without taking into account solid line) and taking into account the DCM (dashed line) are respectively shown in Figures 4 and 5. From the graph, it can be seen that taking into account the viscosity of the suspension material leads to a decrease in the amplitude of the vertical vibration of the object (M, J) and the phase of the vibrations.

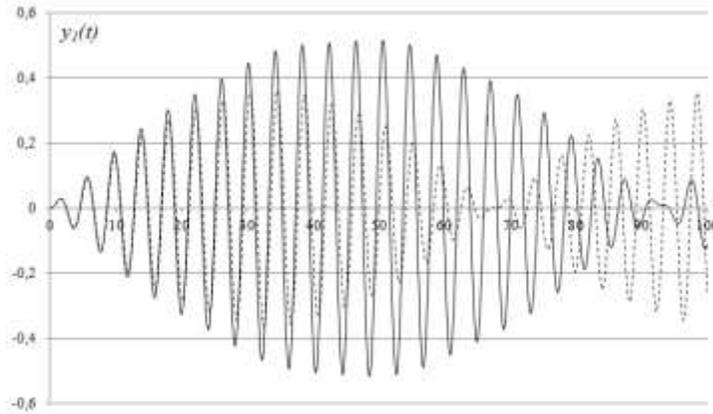


FIGURE 2. Change  $y_1(t)$ : without taking into account the DCM ( $c_1 = -2,2$ ;  $c_2 = c_5 = -0,2$ ;  $c_6 = -1,5333$ ;  $c_3 = c_4 = c_7 = c_8 = 0$ ) and taking into account the DCM ( $c_1 = -2,1292$ ;  $c_2 = c_5 = -0,1893$ ;  $c_3 = 0,0319$ ;  $c_4 = c_7 = 0,0028$ ;  $c_6 = -1,4983$ ;  $c_8 = 0,0225$ ).

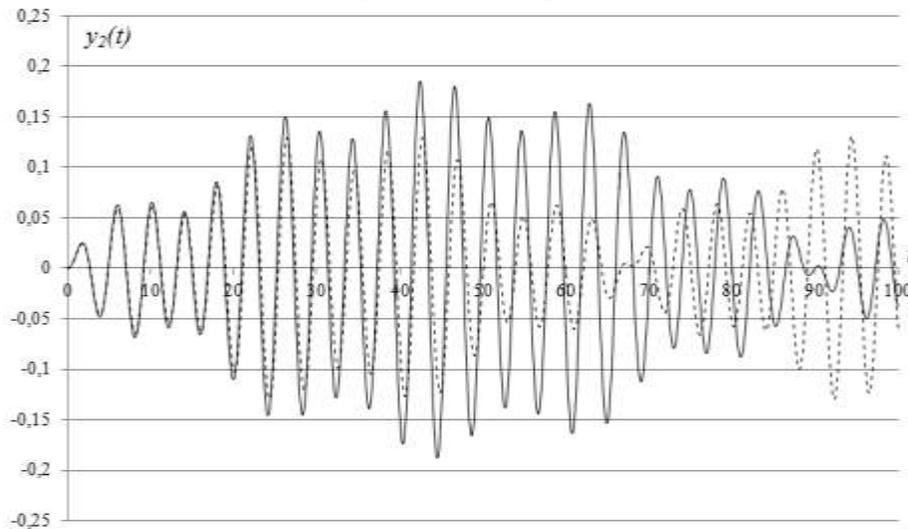


FIGURE 3. Change  $y_2(t)$ : without taking into account ( $c_1 = -2,2$ ;  $c_2 = c_5 = -0,2$ ;  $c_6 = -1,5333$ ;  $c_3 = c_4 = c_7 = c_8 = 0$ ) and taking into account ( $c_1 = -2,1292$ ;  $c_2 = c_5 = -0,1893$ ;  $c_3 = 0,0319$ ;  $c_4 = c_7 = 0,0028$ ;  $c_6 = -1,4983$ ;  $c_8 = 0,0225$ ).

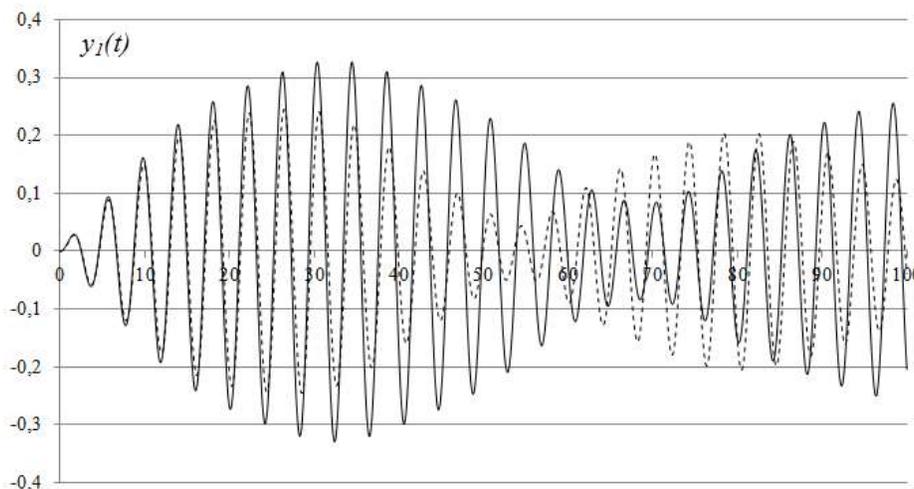


FIGURE 4. Change  $y_1(t)$ : without taking into account ( $c_1 = -2,2$ ;  $c_2 = c_5 = -0,2$ ;  $c_6 = -1,5333$ ;  $c_3 = c_4 = c_7 = c_8 = 0$ ) and taking into account ( $c_1 = -2,1292$ ;  $c_2 = c_5 = -0,1893$ ;  $c_3 = 0,0319$ ;  $c_4 = c_7 = 0,0028$ ;  $c_6 = -1,4983$ ;  $c_8 = 0,0225$ ).

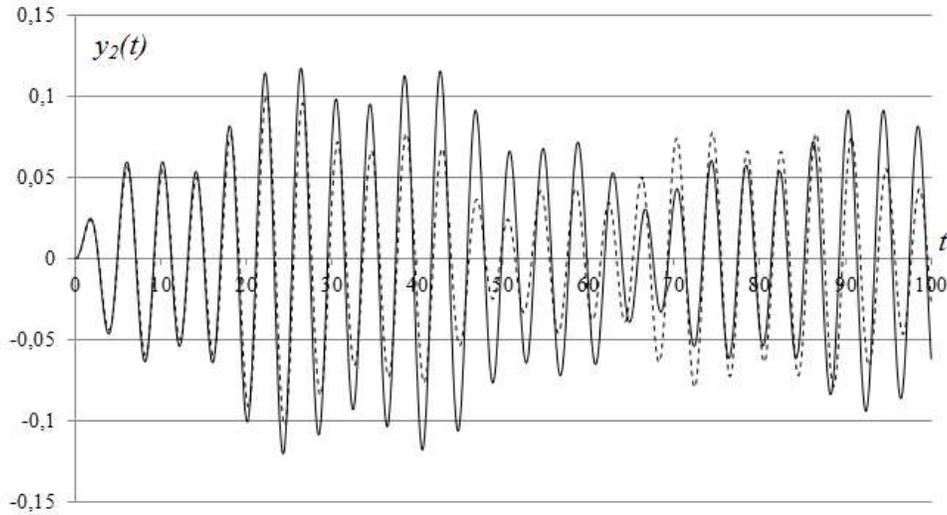


FIGURE 5. Change  $y_2(t)$ : without taking into account ( $c_1 = -2,2$ ;  $c_2 = c_5 = -0,2$ ;  $c_6 = -1,5333$ ;  $c_3 = c_4 = c_7 = c_8 = 0$ ) and taking into account ( $c_1 = -2,1292$ ;  $c_2 = c_5 = -0,1893$ ;  $c_3 = 0,0319$ ;  $c_4 = c_7 = 0,0028$ ;  $c_6 = -1,4983$ ;  $c_8 = 0,0225$ ).

The influence of the location of the center of gravity on the vertical displacement  $y_1(t)$  and  $y_2(t)$  of the object (M, J) was studied taking into account the drag coefficient and the rheological properties of the suspensions ( $\varepsilon_1 = \varepsilon_2 = 0.01$ ;  $\alpha_1 = \alpha_2 = 0.25$ ;  $\beta_1 = \beta_2 = 0.05$ ). The changes  $y_1(t)$  and  $y_2(t)$  are shown in Fig. 6 and Fig. 7, respectively.

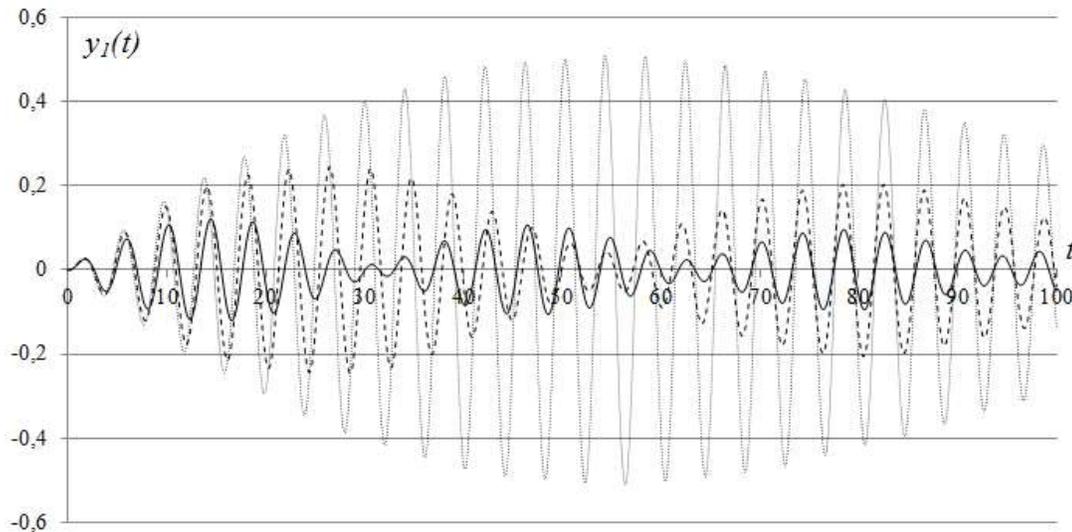


FIGURE 6. Change  $y_1(t)$ :  $l_1 = 0,5$ ;  $l_2 = 0,5$  (solid);  $l_1 = 0,65$ ;  $l_2 = 0,35$  (dashed);  $l_1 = 0,7$ ;  $l_2 = 0,3$  (point).

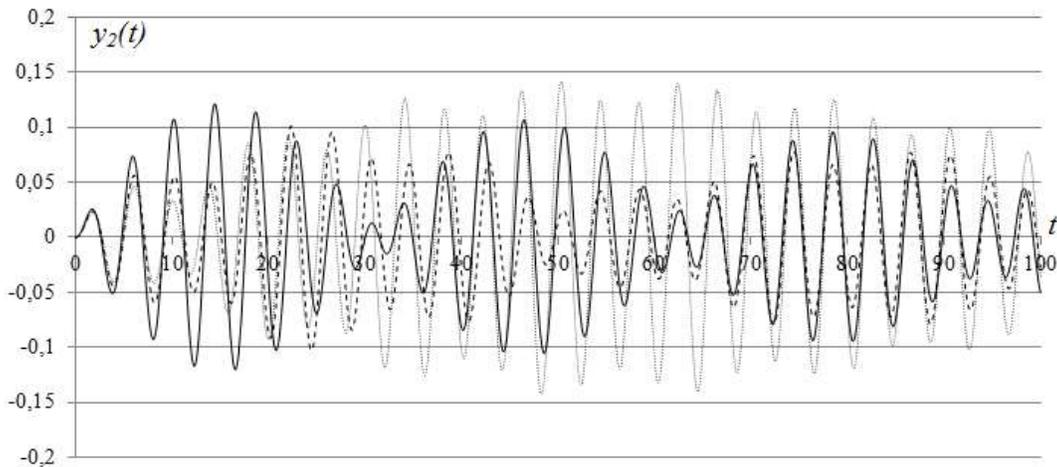


FIGURE 7. Change  $y_2(t)$ :  $l_1 = 0,5$ ;  $l_2 = 0,5$  (solid);  $l_1 = 0,65$ ;  $l_2 = 0,35$  (dashed);  $l_1 = 0,7$ ;  $l_2 = 0,3$  (point).

### Acknowledgement

The solution of the vehicle dynamics problems is associated with the need for multiple calculations in the process of optimizing the performance control parameters. Therefore, in some cases, it is advisable to carry out preliminary calculations according to simplified calculation schemes to determine the approximate efficiency and parameters of the control unit. The use of schemes that allow obtaining a solution in a closed form or using algorithms of type (5) is of great interest. The results allow us to conclude that it is advisable to use the DCM to reduce the amplitude of oscillation, both in perfectly elastic and in hereditarily deformable systems during transients.

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