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SBM-RANDOM SPACES

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ABSTRACT: The first aim of this work is to introduce measurable SB-random spaces (SBM-random spaces) by mixing between the space of measurable SBM-functions, probability theory and topological spaces. SBM-random neighborhood, SBM-random interior and SBM-random closure are introduced and studied. The second aim of this work is to present SBM-random separation axioms.

KEYWORDS: Topological spaces, Function spaces, Probability theory, Separation axioms.

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I. INTRODUCTION

There are abundant different forms of function spaces, and there are diverse various topologies that can be configured on a given collection of functions [2]. A function space is an valuable example of a topological space. There are numerous researchers studied various kinds of function spaces via placing various topologies on the collection of functions. In 1951 R. Arens and J. Dugundji discussed topologies for function spaces [8]. In 1981 P. Lambrions introduced the bounded-open topology on function spaces [9]. In 1996 K. Porter introduced the regular open-open topology by using a collection of continuous maps between two topological spaces. In 2016 R. Saadati introduced the quasicompact-open topology by using continuous real-valued maps on space X. In this paper introduced a new type of unfamiliar space is measurable SB-random spaces (SBM-random spaces) by mixing between the space of measurable SB-functions, probability theory and topological spaces**.**

II. PRELIMINARIES

Definition 2.1 [6]. A topological space $(\mathfrak{X},\mathcal{T})$ is said to be a T_1-space iff for each two point $x_1, x_2 \in \mathfrak{X}$ such that $x_1 \neq x_2$ there is two open sets U_1, U_2 such that $x_1 \in U_1, x_2 \notin U_1$ and $x_2 \in U_2, x_1 \notin U_2$.

Definition 2.2 [5]. Let $(\mathfrak{X},\mathcal{T})$ be a topological space. The Borel space associated to $(\mathfrak{X},\mathcal{T})$ is the pair $(\mathfrak{X},\Sigma_{\mathcal{T}})$, where $\Sigma_{\mathcal{T}}$ is the σ-algebra generated by the open sets of $\mathfrak{X}$ i.e. the smallest σ-algebra of subsets of $\mathfrak{X}$ containing the open sets of $\mathfrak{X}$. The elements of Borel σ-algebra are called Borel measurable sets or Borel sets.

Definition 2.3 [5]. Let $\left(\mathfrak{X},\sum_{\mathcal{T}_{\mathfrak{X}}}\right)$ and $\left(\mathfrak{U},,\sum_{\mathcal{T}_{\mathfrak{U}}}\right)$ be two Borel measurable spaces. A function $F:\mathfrak{X}\longrightarrow P(\mathfrak{U})$ is said to be set-valued Borel measurable function (SBM-function), iff $F^{-1}(U)\in\sum_{\mathcal{T}_{\mathfrak{X}}}, \forall\, U\in\mathcal{T}_{\mathfrak{U}}$. The collection of SBM-functions denoted by $SBM(\mathfrak{X},P(\mathfrak{U}))$ where $SBM(\mathfrak{X},P(\mathfrak{U}))=\{F;F:\mathfrak{X}\longrightarrow P(\mathfrak{U})\ni F \text{ is a SBM} - \text{function}\}$ and is said to be SBM-space.

III.SBM-RANDOM SPACES

Definition 3.1. Let $\left(\mathfrak{X},F_{\mathcal{T}_{\mathfrak{X}}}\right)$ and $\left(\Omega,,F_{\mathcal{T}_{\Omega}}\right)$ be two Borel measurable spaces such that $\left(\mathfrak{X},\mathcal{T}_{\mathfrak{X}}\right)$ is a T_1-space and P_r is a probability function on $\left(\mathfrak{X},F_{\mathcal{T}_{\mathfrak{X}}}\right)$, which assigns each subset in $F_{\mathcal{T}_{\mathfrak{X}}}$ a probability, which is a number between 0 and 1. Then the triple (Ω,M,P_r) is said to be a SBM- random space where $M=\{F;F:\mathfrak{X}\longrightarrow P(\Omega)\ni F \text{ is a SBM} - \text{function}\}$. The elements of M is said to be SBM-random set.

Definition 3.2. A SBM-random set F is said to be a be SBM-random open (closed) set if F is an open (closed) SBM-function.

Remark 3.3. Let (Ω, M, P_r) be a SBM-random space and $X_1, X_2 \subseteq X$. Then by laws of probability, the following statements are hold:

- **a)** $0 \leq P_r(X_*) \leq 1$ for each $X_* \subseteq X$.
- **b)** $P_r(X) = 1$.
- **c)** $P_r(\emptyset) = 0$.
- **d)** $X_1 \subseteq X_2 \Rightarrow P_r(X_1) \subseteq P_r(X_2)$.

Remark 3.4. Let (Ω, M, P_r) be a SBM-random space and $\omega \in \Omega$, $F \in M$ such that $F: X \longrightarrow P(\Omega)$ then if $\omega \in F(x), \forall x \in X$ denoted by $\omega \in F$, if $\omega \in F(x)$, for some $x \in X$ denoted by $\omega \Subset F$.

Remark 3.5. Let (Ω, M, P_r) be a SBM-random space and $\Omega_* \subseteq \Omega$, $F \in M$ such that $F: X \longrightarrow P(\Omega)$ then if $\Omega_* \subseteq F(x), \forall x \in X$ $(F(x) \subseteq \Omega_*, \forall x \in X)$ denoted by $\Omega_* \subseteq F$ $(F \subseteq \Omega_*)$, if $\Omega_* \subseteq F(x)$, for some $x \in X$ $(F(x) \subseteq \Omega_*$, for some $x \in X)$ denoted by $\Omega_* \Subset F$ $(F \Subset \Omega_*)$.

Example 3.6. Let $X = \{x_1, x_2, x_3\}, \Omega = \{\omega_1, \omega_2, \omega_3\}$ such that

$\mathcal{T}_X = \{X, \{\Phi\}, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}\}$,
$\mathcal{T}_\Omega = \{\Omega, \{\Phi\}, \{\omega_2\}, \{\omega_1, \omega_2\}, \{\omega_2, \omega_3\}\}$ then $\mathcal{T}_X$ is a T_1-space and $F_{\mathcal{T}_X} = P(X)$, $F_{\mathcal{T}_\Omega} = P(\Omega)$. Consider
$F_1(x_1) = \{\omega_2\},\ F_1(x_2) = \{\omega_1\}, F_1(x_3) = \{\omega_1, \omega_2\}$
$F_2(x) = \{\omega_2, \omega_3\}, \forall x \in X$
$F_3(x) = \{\omega_1, \omega_3\}, \forall x \in X$
$F_4(x_1) = \Omega,\ F_4(x_2) = \{\omega_1, \omega_3\}, F_4(x_3) = \{\omega_1, \omega_3\}$
$F_5(x) = \{\omega_3\}, \forall x \in X$
$F_6(x_1) = \{\omega_2\}, F_6(x_2) = \{\omega_2, \omega_3\}, F_6(x_3) = \{\omega_2, \omega_3\}$
$F_7(x) = \{\omega_1\}, \forall x \in X$
$F_8(x_1) = \Omega, F_8(x_2) = \Omega, F_8(x_3) = \{\omega_1, \omega_3\}$
Then $M = \{F_1, \ldots, F_8\}$ is a SBM-space.
Consider $P_r: F_{\mathcal{T}_X} \longrightarrow [0,1]$ is a probability function then (Ω, M, P_r) is a SBM-random space.

Definition 3.7. Let (Ω, M, P_r) be a SBM-random space and T is a subset of M. Then (Ω, T, P_r) is called a SBM-random subspace of (Ω, M, P_r).

Example 3.8. In the above example (3.6), consider $T = \{F_1, F_2, F_3, F_4\}$ then (Ω, T, P_r) is a SBM-random subspace of M.

Definition 3.9. Let (Ω, M, P_r) be a SBM-random space and $\omega_* \in \Omega$. A random set N is a SBM-random neighborhood of ω_* if there exists a SBM-random open set $\mathcal{U} \in M$ such that $P_r(\{x : \omega_* \in \mathcal{U}(x) \subseteq N(x)\}) = 1$. The collection R-$N[\omega_*]$ denoted of all SBM-random neighborhoods of ω_* is said to be SBM-random neighborhoods system at ω_*.

Definition 3.10. Let (Ω, M, P_r) be a SBM-random space and $\Omega_* \subseteq \Omega$. A random set N is a SBM-random neighborhood of Ω_* if there exists a SBM-random open set $\mathcal{U} \in M$ such that $P_r(\{x : \Omega_* \subseteq \mathcal{U}(x) \subseteq N(x)\}) = 1$.

Example 3.11. In the above example (3.6) a SBM-random space is (Ω, M, P_r) such that $\Omega = \{\omega_1, \omega_2, \omega_3\}$, $M = \{F_1, \ldots, F_8\}$. Then

- **a)** F_4 is a SBM-random neighborhood of ω_3 since there exists a confine SBM-random open set F_5 such that $P_r(\{x : \omega_3 \in F_5(x) \subseteq F_4(x)\}) = 1$.
- **b)** F_8 is a SBM-random neighborhood of $\Omega_* = \{\omega_1, \omega_3\}$ since there exists a confine SBM-random open set F_8 such that $P_r(\{x : \Omega_* \subseteq F_3(x) \subseteq F_8(x)\}) = 1$.

Proposition 3.12. A SBM-random set $\mathcal{U}$ of M is a SBM-random open set iff it a SBM-random neighborhood of each $\omega \in \mathcal{U}$.

Proof: Let $\mathcal{U}$ be a random open set and $\omega \in \mathcal{U}$ then $P_r(\{x: \omega \in \mathcal{U}(x) \subseteq \mathcal{U}(x)\}) = 1$ thus $\mathcal{U}$ is a SBM-random neighborhood of each $\omega \in \mathcal{U}$. Now let $\mathcal{U} \in M$ such that $\mathcal{U}$ a SBM-random neighborhood of each $\omega \in \mathcal{U}$. Then there exists a SBM-random open set $\mathcal{U}_\omega \in M$ such that $P_r(\{x: \omega \in \mathcal{U}_\omega(x) \subseteq \mathcal{U}(x)\}) = 1$. For each $x_* \in \{x: \omega \in \mathcal{U}_\omega(x) \subseteq \mathcal{U}(x)\}$ then $\mathcal{U}_\omega(x_*) \subseteq \mathcal{U}(x_*)$, $\omega \in \mathcal{U}$ so that $\cup_{\omega \in \mathcal{U}} \mathcal{U}_\omega(x_*) \subseteq \mathcal{U}(x_*)$ but $\mathcal{U}(x_*) \subseteq \cup_{\omega \in \mathcal{U}} \mathcal{U}_\omega(x_*)$ then $\mathcal{U}(x_*) = \cup_{\omega \in \mathcal{U}} \mathcal{U}_\omega(x_*)$. Now since $\mathcal{U}_\omega$ is a SBM-random open set then $\mathcal{U}_\omega(x)$ is an open set in Ω for each x thus $\cup_{\omega \in \mathcal{U}} \mathcal{U}_\omega(x_*)$ is an open set in Ω implies that $\mathcal{U}(x_*)$ is an open set in Ω from $P_r(\{x: \omega \in \mathcal{U}_\omega(x) \subseteq \mathcal{U}(x)\}) = 1$ then $\mathcal{U}(x)$ is an open set in Ω for each x. Hence $\mathcal{U}$ is a SBM-random open set in Ω.

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Definition 3.13. Let (Ω, M, P_r) be a SBM-random space and $\Omega_* \subseteq \Omega$. The union of all SBM-random open sets $\mathcal{U} \in M$ such that $P_r(\{x : \mathcal{U}(x) \subseteq \Omega_*\}) = 1$ is said to be the SBM-random interior of Ω_* and is denoted by R-$\text{int}(\Omega_*)$. The SBM-random interior of Ω_* is a SBM-random open set.

Definition 3.14. Let (Ω, M, P_r) be a SBM-random space and $\Omega_* \subseteq \Omega$. The intersection of all SBM-random closed sets $\mathcal{U} \in M$ such that $P_r(\{x : \Omega_* \subseteq \mathcal{U}(x)\}) = 1$ is said to be the SBM-random closure of Ω_* and is denoted by R-cl(Ω_*). The SBM-random closure of Ω_* is a SBM-random closed set.

Example 3.15. In the above example (3.6) a SBM-random space is (Ω, M, P_r) such that $\Omega = \{\omega_1, \omega_2, \omega_3\}$, $M = \{F_1, \dots, F_8\}$.

- **a)** Let $\Omega_* \subseteq \Omega$ such that $\Omega_* = \{\omega_2, \omega_3\}$. Then $\breve{\cup}_{\mathcal{U}\in M} \mathcal{U}$ such that $\mathcal{U}$ is a SBM-random open set and $P_r(\{x : \mathcal{U}(x) \subseteq \Omega_*\}) = 1$ is R-int$(\Omega_*) = \breve{\cup}\{F_2, F_5, F_6\} = F_2$.
- **b)** Let $\Omega_* \subseteq \Omega$ such that $\Omega_* = \{\omega_1, \omega_3\}$. Then $\breve{\cap}_{\mathcal{V}\in M} \mathcal{V}$ such that $\mathcal{V}$ is a SBM-random closed set and $P_r(\{x : \Omega_* \subseteq \mathcal{V}(x)\}) = 1$ is R-cl$(\Omega_*) = \breve{\cap}\{F_3, F_4, F_8\} = F_3$.

Theorem 3.16. Let (Ω, M, P) be a SBM-random space and $\Omega_* \subseteq \Omega$. Then $(\text{R} - \text{int}(\Omega_*))^c =$ R-cl$({\Omega_*}^c)$.

Proof: Let $\Omega_* \subseteq \Omega$ then $\text{R} - \text{int}(\Omega_*) = \breve{\cup}_{\mathcal{U}\in M} \mathcal{U}$ such that $\mathcal{U}$ is a SBM-random open set and $P_r(\{x : \mathcal{U}(x) \subseteq \Omega_*\}) = 1$ thus $\mathcal{U}(x) \subseteq \Omega_*, \forall x \in X$. $(\text{R} - \text{int}(\Omega_*))^c = (\breve{\cup}_{\mathcal{U}\in M} \mathcal{U})^c = \breve{\cap}_{\mathcal{U}\in M} \mathcal{U}^c$ such that ${\Omega_*}^c \subseteq \mathcal{U}^c(x), \forall x \in X$, $\mathcal{U}$ is a SBM-random open set thus $\mathcal{U}^c$ is a SBM-random closed set means that $(\text{R} - \text{int}(\Omega_*))^c = \breve{\cap}_{\mathcal{U}\in M} \mathcal{U}^c$ such that $\mathcal{U}^c$ is a SBM-random closed set and $P_r(\{x : {\Omega_*}^c \subseteq \mathcal{U}^c(x)\}) = 1$. Hence $(\text{R} - \text{int}(\Omega_*))^c =$ R-cl$({\Omega_*}^c)$.

IV. SBM-RANDOM SEPARATION AXIOMS

Definition 4.1. A SBM-random space (Ω, M, P_r) is said to be a SBM-random T_0-space iff for each $\omega_1, \omega_2 \in \Omega$ such that $\omega_1 \neq \omega_2$ there is a SBM-random open set $\mathcal{U} \in M$ such that $P_r(\{x: \omega_1 \in \mathcal{U}(x)\}) = 1$ and $P_r(\{x: \omega_2 \in \mathcal{U}(x)\}) \neq 1$.

Example 4.2. Let $X = \{x_1, x_2, x_3\}, \Omega = \{\omega_1, \omega_2, \omega_3\}$ such that

$\mathcal{T}_X = \{X, \Phi, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}\}$,

$\mathcal{T}_\Omega = \{\Omega, \Phi, \{\omega_2\}, \{\omega_1, \omega_2\}, \{\omega_2, \omega_3\}\}$ then $\mathcal{T}_X$ is a T_1-space and $F_{\mathcal{T}_X} = P(X)$, $F_{\mathcal{T}_\Omega} = P(\Omega)$. Consider

$F_1(x) = \{\omega_2\}, \forall x \in X$

$F_2(x_1) = \{\omega_2, \omega_3\},\ F_2(x_2) = \{\omega_1, \omega_3\}, F_2(x_3) = \{\omega_2, \omega_3\}$

$F_3(x_1) = \{\omega_2, \omega_3\}, F_3(x_2) = \{\omega_2, \omega_3\}, F_3(x_3) = \{\omega_1, \omega_3\}$

$F_4(x_1) = \Omega, F_4(x_2) = \{\omega_1, \omega_3\}, F_4(x_3) = \{\omega_1, \omega_3\}$

$F_5(x_1) = \{\omega_1, \omega_2\}, F_5(x_2) = \{\omega_1, \omega_2\}, F_3(x_3) = \Omega$

$F_6(x_1) = \{\omega_2\}, F_6(x_2) = \{\omega_1, \omega_3\}, F_6(x_3) = \{\omega_1, \omega_3\}$

$F_7(x_1) = \Omega,\ F_7(x_2) = \{\omega_3\}, F_7(x_3) = \{\omega_2, \omega_3\}$

$F_8(x_1) = \Omega, F_8(x_2) = \Omega, F_8(x_3) = \{\omega_1, \omega_2\}$

Then $M = \{F_1, \dots, F_8\}$ is a SBM-space.

Consider $P_r: F_{\mathcal{T}_X} \longrightarrow [0,1]$ is a probability function then (Ω, M, P_r) is a SBM-random space. Since $\omega_1 \neq \omega_2$, $\omega_1 \neq \omega_3$, $\omega_2 \neq \omega_3$ so that $P_r(\{x: \omega_2 \in F_1(x)\}) = 1$ and $P_r(\{x: \omega_1 \in F_1(x)\}) \neq 1$.

$P_r(\{x: \omega_1 \in F_8(x)\}) = 1$ and $P_r(\{x: \omega_3 \in F_8(x)\}) \neq 1$.

$P_r(\{x: \omega_2 \in F_1(x)\}) = 1$ and $P_r(\{x: \omega_3 \in F_1(x)\}) \neq 1$.

Such that F_1, F_8 are SBM-random open sets. Hence (Ω, M, P_r) is a SBM-random T_0-space.

Remark 4.3. A subspace of SBM-random T_0-space is not necessarily SBM-random T_0-space.

Example 4.4. In the example (4.2), a SBM-random space (Ω, M, P_r) is a T_0-space. Consider $T = \{F_1, F_2, F_3, F_4\}$ then (Ω, T, P_r) is not SBM-random T_0-space because there is no SBM-random open set F such that $P_r(\{x: \omega_1 \in F(x)\}) = 1$ and $P_r(\{x: \omega_3 \in F(x)\}) \neq 1$.

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Remark 4.5. If (Ω, M, P_r) is a SBM-random T_0-space and ω_1, ω_2 distinct points of Ω is not necessarily R-cl$(\{\omega_1\})(x) \neq$ R-cl$(\{\omega_3\})(x)$.

Example 4.6. In the example (4.2), a SBM-random space (Ω, M, P_r) is a T_0-space. Since $\breve{\cap}_{\mathcal{V}\in M} \mathcal{V}$ such that $\mathcal{V}$ is a SBM-random closed set and $P_r(\{x : \omega_1 \subseteq \mathcal{V}(x)\}) = 1$ is R-cl$(\{\omega_1\})(x) = \{F_1\}$ so that $\breve{\cap}_{\mathcal{V}\in M} \mathcal{V}$ such that $\mathcal{V}$ is a SBM-random closed set and $P_r(\{x : \omega_3 \subseteq \mathcal{V}(x)\}) = 1$ is R-cl$(\{\omega_3\})(x) = \{F_1\}$ then R-cl$(\{\omega_1\})(x) =$ R-cl$(\{\omega_3\})(x)$.

Definition 4.7. A SBM-random space (Ω, M, P_r) is said to be a SBM-random T_1-space iff for each $\omega_1, \omega_2 \in \Omega$ such that $\omega_1 \neq \omega_2$ there are SBM-random open sets $\mathcal{U}, \mathcal{V} \in M$ such that $P_r(\{x : \omega_1 \in \mathcal{U}(x)\}) = 1$, $P_r(\{x : \omega_2 \in \mathcal{U}(x)\}) = 0$ and $P_r(\{x : \omega_2 \in \mathcal{V}(x)\}) = 1$ and $P_r(\{x : \omega_1 \in \mathcal{V}(x)\}) = 0$.

Example 4.8. Let $X = \{x_1, x_2, x_3\}, \Omega = \{\omega_1, \omega_2, \omega_3\}$ such that

$\mathcal{T}_X = \{X, \Phi, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}\}$,

$\mathcal{T}_\Omega = \{\Omega, \Phi, \{\omega_1\}, \{\omega_2\}, \{\omega_3\}, \{\omega_1, \omega_2\}, \{\omega_1, \omega_3\}, \{\omega_2, \omega_3\}\}$ then $\mathcal{T}_X$ is a T_1-space and $F_{\mathcal{T}_X} = P(X)$, $F_{\mathcal{T}_\Omega} = P(\Omega)$. Consider

$F_1(x) = \{\omega_2\}, \forall x \in X$

$F_2(x_1) = \{\omega_2, \omega_3\}$, $F_2(x_2) = \{\omega_1, \omega_3\}, F_2(x_3) = \{\omega_2, \omega_3\}$

$F_3(x) = \{\omega_1\}, \forall x \in X$

$F_4(x_1) = \Omega$, $F_4(x_2) = \{\omega_1, \omega_3\}, F_4(x_3) = \{\omega_1, \omega_3\}$

$F_5(x) = \{\omega_2, \omega_3\}, \forall x \in X$

$F_6(x_1) = \{\omega_2\}$, $F_6(x_2) = \{\omega_1, \omega_3\}, F_6(x_3) = \{\omega_1, \omega_3\}$

$F_7(x_1) = \Omega$, $F_7(x_2) = \{\omega_3\}, F_7(x_3) = \{\omega_2, \omega_3\}$

$F_8(x) = \{\omega_3\}, \forall x \in X$

Then $M = \{F_1, \dots, F_8\}$ is a SBM-space.

Consider $P_r: F_{\mathcal{T}_X} \longrightarrow [0,1]$ is a probability function then (Ω, M, P_r) is a SBM-random space. Since $\omega_1 \neq \omega_2$, $\omega_1 \neq \omega_3$, $\omega_2 \neq \omega_3$ so that $P_r(\{x: \omega_1 \in F_3(x)\}) = 1$ and $P_r(\{x: \omega_2 \in F_3(x)\}) = 0$.

$P_r(\{x: \omega_2 \in F_1(x)\}) = 1$ and $P_r(\{x: \omega_1 \in F_2(x)\}) = 0$.

$P_r(\{x: \omega_1 \in F_3(x)\}) = 1$ and $P_r(\{x: \omega_3 \in F_3(x)\}) = 0$.

$P_r(\{x: \omega_3 \in F_5(x)\}) = 1$ and $P_r(\{x: \omega_1 \in F_5(x)\}) = 0$.

$P_r(\{x: \omega_2 \in F_1(x)\}) = 1$ and $P_r(\{x: \omega_3 \in F_1(x)\}) = 0$.

$P_r(\{x: \omega_3 \in F_8(x)\}) = 1$ and $P_r(\{x: \omega_2 \in F_8(x)\}) = 0$.

Such that F_1, F_3, F_5, F_8 are SBM-random open sets. Hence (Ω, M, P_r) is a SBM-random T_1-space.

Remark 4.9. A subspace of SBM-random T_1-space is not necessarily SBM-random T_1-space.

Example 4.10. In the example (4.8), a SBM-random space (Ω, M, P_r) is a T_1-space. Consider $T = \{F_1, F_2, F_3\}$ then (Ω, T, P_r) is not SBM-random T_1-space because there are no SBM-random open sets $\mathcal{U}, \mathcal{V}$ such that $P_r(\{x: \omega_3 \in \mathcal{U}(x)\}) = 1$, $P_r(\{x: \omega_2 \in \mathcal{V}(x)\}) = 0$.

Theorem 4.11. A SBM-random space (Ω, M, P_r) is a SBM-random T_1-space iff every singleton set $\{\omega\}$ is a closed set in Ω.

Proof: Let (Ω, M, P_r) be a SBM-random T_1-space. Assume that $\omega \in \Omega$ and $\omega_* \in \Omega - \{\omega\}$ then $\omega_* \neq \omega$. Consider $R(x) = \Omega - \{\omega\}, \forall x \in X$. Now (Ω, M, P_r) is a SBM-random T_1-space there are SBM-random open sets $\mathcal{U}, \mathcal{V} \in M$ such that $P_r(\{x : \omega \in \mathcal{U}(x)\}) = 1$, $P_r(\{x : \omega_* \in \mathcal{U}(x)\}) = 0$ and $P_r(\{x : \omega_* \in \mathcal{V}(x)\}) = 1$, $P_r(\{x : \omega \in \mathcal{V}(x)\}) = 0$ means that $\omega \in \mathcal{U}(x)$ and $\omega \notin \mathcal{V}(x), \forall x \in X$ so that $\mathcal{V}(x) \subseteq \Omega - \{\omega\}, \forall x \in X$ thus $P_r(\{x : \omega_* \in \mathcal{V}(x) \subseteq R(x)\}) = 1$ therefore R is a SBM-random neighborhood of each $\omega_* \in R$ implies that R is a SBM-random open set thus $R(x)$ is open set for each $x \in X$ so that $(R(x))^c = (\Omega - \{\omega\})^c = \{\omega\}$ is a closed set for each $x \in X$. Hence $\{\omega\}$ is a closed set in Ω.

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Let every singleton set $\{\omega\}$ is a closed set in Ω and $\omega_1, \omega_2 \in \Omega$ such that $\omega_1 \neq \omega_2$ then $\{\omega_1\}$, $\{\omega_2\}$ are closed sets in Ω so that $\Omega - \{\omega_1\}$, $\Omega - \{\omega_2\}$ are open sets such that $\omega_1 \in \Omega - \{\omega_2\}, \omega_2 \notin \Omega - \{\omega_2\}$ and $\omega_2 \in \Omega - \{\omega_1\}, \omega_1 \notin \Omega - \{\omega_1\}$. But $\Omega - \{\omega_1\}, \Omega - \{\omega_2\} \in F_{\mathcal{T}_\Omega}$ consider $Ɍ_1(x) = \Omega - \{\omega_1\}, \forall x \in X$ and $Ɍ_2(x) = \Omega - \{\omega_2\}, \forall x \in X$ then $Ɍ_1, Ɍ_2 \in M$ such that $P(\{x : \omega_1 \in Ɍ_2(x)\}) = 1$, $P_r(\{x : \omega_2 \in Ɍ_2(x)\}) = 0$ and $P_r(\{x : \omega_2 \in Ɍ_1(x)\}) = 1$ and $P_r(\{x : \omega_1 \in Ɍ_1(x)\}) = 0$. Hence (Ω, M, P_r) is a SBM-random T_1-space.

Theorem 4.12. Every SBM-random T_1-space is a SBM-random T_0-space.

Proof: Let (Ω, M, P_r) be a SBM-random T_1-space and $\omega_1, \omega_2 \in \Omega$ such that $\omega_1 \neq \omega_2$ since (Ω, M, P_r) is a SBM-random T_1-space there are SBM-random open sets $\mathcal{U}, \mathcal{V} \in M$ such that $P_r(\{x : \omega_1 \in \mathcal{U}(x)\}) = 1$, $P_r(\{x : \omega_2 \in \mathcal{U}(x)\}) = 0$ and $P_r(\{x : \omega_2 \in \mathcal{V}(x)\}) = 1$ and $P_r(\{x : \omega_1 \in \mathcal{V}(x)\}) = 0$ thus $P_r(\{x : \omega_1 \in \mathcal{U}(x)\}) = 1$, $P_r(\{x : \omega_2 \in \mathcal{U}(x)\}) \neq 1$. Hence (Ω, M, P_r) is a SBM-random T_0-space.

Definition 4.13. A SBM-random space (Ω, M, P_r) is said to be a SBM-random T_2-space iff for each $\omega_1, \omega_2 \in \Omega$ such that $\omega_1 \neq \omega_2$ there are SBM-random open sets $\mathcal{U}, \mathcal{V} \in M$ such that $P_r(\{x : \omega_1 \in \mathcal{U}(x)\}) = P_r(\{x : \omega_2 \in \mathcal{V}(x)\}) = 1$ and $P_r(\{x : \mathcal{U}(x) \cap \mathcal{V}(x) \neq \emptyset\}) = 0$.

Example 4.14. Let $X = \{x_1, x_2, x_3\}, \Omega = \{\omega_1, \omega_2, \omega_3\}$ such that

$\mathcal{T}_X = \{X, \Phi, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}\}$,

$\mathcal{T}_\Omega = \{\Omega, \Phi, \{\omega_1\}, \{\omega_2\}, \{\omega_3\}, \{\omega_1, \omega_2\}, \{\omega_1, \omega_3\}, \{\omega_2, \omega_3\}\}$ then $\mathcal{T}_X$ is a T_1-space and $F_{\mathcal{T}_X} = P(X)$, $F_{\mathcal{T}_\Omega} = P(\Omega)$. Consider

$Ƒ_1(x) = \{\omega_2\}, \forall x \in X$

$Ƒ_2(x_1) = \{\omega_2, \omega_3\}$, $Ƒ_2(x_2) = \{\omega_1, \omega_3\}, Ƒ_2(x_3) = \{\omega_2, \omega_3\}$

$Ƒ_3(x) = \{\omega_1\}, \forall x \in X$

$Ƒ_4(x_1) = \Omega$, $Ƒ_4(x_2) = \{\omega_1, \omega_3\}$, $Ƒ_4(x_3) = \{\omega_1, \omega_3\}$

$Ƒ_5(x) = \{\omega_2, \omega_3\}, \forall x \in X$

$Ƒ_6(x_1) = \{\omega_2\}$, $Ƒ_6(x_2) = \{\omega_1, \omega_3\}, Ƒ_6(x_3) = \{\omega_1, \omega_3\}$

$Ƒ_7(x_1) = \Omega$, $Ƒ_7(x_2) = \{\omega_3\}, Ƒ_7(x_3) = \{\omega_2, \omega_3\}$

$Ƒ_8(x) = \{\omega_3\}, \forall x \in X$

Then $M = \{Ƒ_1, \dots, Ƒ_8\}$ is a SBM-space.

Consider $P_r : F_{\mathcal{T}_X} \longrightarrow [0,1]$ is a probability function then (Ω, M, P_r) is a SBM-random space. Since $\omega_1 \neq \omega_2$, $\omega_1 \neq \omega_3$, $\omega_2 \neq \omega_3$ so that $P_r(\{x : \omega_1 \in Ƒ_3(x)\}) = 1$, $P_r(\{x : \omega_2 \in Ƒ_1(x)\}) = 0$ and $P_r(\{x : Ƒ_1(x) \cap Ƒ_3(x) \neq \emptyset\}) = 0$.

$P_r(\{x : \omega_1 \in Ƒ_3(x)\}) = 1$, $P_r(\{x : \omega_3 \in Ƒ_8(x)\}) = 0$ and $P_r(\{x : Ƒ_3(x) \cap Ƒ_8(x) \neq \emptyset\}) = 0$.

$P_r(\{x : \omega_2 \in Ƒ_1(x)\}) = 1$, $P_r(\{x : \omega_3 \in Ƒ_8(x)\}) = 0$ and $P_r(\{x : Ƒ_1(x) \cap Ƒ_8(x) \neq \emptyset\}) = 0$.

Such that $Ƒ_1, Ƒ_3, Ƒ_8$ are SBM-random open sets. Hence (Ω, M, P_r) is a SBM-random T_2-space.

Remark 4.15. A subspace of SBM-random T_2-space is not necessarily SBM-random T_2-space.

Example 4.16. In the example (4.15), a SBM-random space (Ω, M, P_r) is a T_2-space. Consider $Ţ = \{Ƒ_1, Ƒ_2, Ƒ_3\}$ then $(\Omega, Ţ, P_r)$ is not SBM-random T_2-space because there are no SBM-random open sets $\mathcal{U}, \mathcal{V}$ such that $P_r(\{x : \omega_3 \in \mathcal{U}(x)\}) = 1$, $P_r(\{x : \omega_2 \in \mathcal{V}(x)\}) = 0$ and $P_r(\{x : \mathcal{U}(x) \cap \mathcal{V}(x) \neq \emptyset\}) = 0$.

Theorem 4.17. If (Ω, M, P_r) is a SBM-random T_2-space, then every singleton set $\{\omega\}$ is a closed set in Ω.

Proof: Let (Ω, M, P_r) be a SBM-random T_2-space. Assume that $\omega \in \Omega$ and $\omega_* \in \Omega - \{\omega\}$ then $\omega_* \neq \omega$. Consider $Ɍ(x) = \Omega - \{\omega\}, \forall x \in X$. Now (Ω, M, P_r) is a SBM-random T_2-space there are SBM-random open sets $\mathcal{U}, \mathcal{V} \in M$ such that $P_r(\{x : \omega \in \mathcal{U}(x)\}) = 1$, $P_r(\{x : \omega_* \in \mathcal{V}(x)\}) = 1$ and $P_r(\{x : \mathcal{U}(x) \cap \mathcal{V}(x) \neq \emptyset\}) = 0$ means that $\omega \in \mathcal{U}(x)$ and $\mathcal{U}(x) \cap \mathcal{V}(x) = \emptyset, \forall x \in X$ thus $\omega \notin \mathcal{V}(x), \forall x \in X$ so that $\mathcal{V}(x) \subseteq \Omega - \{\omega\}, \forall x \in X$ thus $P_r(\{x : \omega_* \in \mathcal{V}(x) \subseteq Ɍ(x)\}) = 1$ therefore $Ɍ$ is a SBM-random neighborhood of each $\omega_* \in Ɍ$ implies that $Ɍ$ is a SBM-random open set thus $Ɍ(x)$ is open set for each $x \in X$ so that $\left(Ɍ(x)\right)^c = (\Omega - \{\omega\})^c = \{\omega\}$ is a closed set for each $x \in X$. Hence $\{\omega\}$ is a closed set in Ω.

Theorem 4.18. Every SBM-random T_2-space is a SBM-random T_1-space.

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Proof: Let (Ω, M, P_r) be a SBM-random T_2-space and $\omega_1, \omega_2 \in \Omega$ such that $\omega_1 \neq \omega_2$ since (Ω, M, P_r) is a SBM-random T_2-space there are SBM-random open sets $\mathcal{U}, \mathcal{V} \in M$ such that $P_r(\{x : \omega_1 \in \mathcal{U}(x)\}) = P_r(\{x : \omega_2 \in \mathcal{V}(x)\}) = 1$ and $P_r(\{x : \mathcal{U}(x) \cap \mathcal{V}(x) \neq \emptyset\}) = 0$ thus $P_r(\{x : \omega_1 \in \mathcal{U}(x)\}) = 1$, $P_r(\{x : \omega_2 \in \mathcal{U}(x)\}) = 0$ and $P_r(\{x : \omega_2 \in \mathcal{V}(x)\}) = 1$ and $P_r(\{x : \omega_1 \in \mathcal{V}(x)\})$. Hence (Ω, M, P_r) is a SBM-random T_1-space.

Definition 4.19. A SBM-random space (Ω, M, P_r) is said to be a SBM-random regular space iff for each $\omega_* \in \Omega$ and SBM-random closed set R such that $P_r(\{x : \omega_* \in R(x)\}) = 0$ there are SBM-random open sets $\mathcal{U}, \mathcal{V} \in M$ such that $P_r(\{x : \omega_* \in \mathcal{U}(x)\}) = P_r(\{x : R(x) \subseteq \mathcal{V}(x)\}) = 1$ and $P_r(\{x : \mathcal{U}(x) \cap \mathcal{V}(x) \neq \emptyset\}) = 0$.

Example 4.20. Let $X = \{x_1, x_2, x_3\}, \Omega = \{\omega_1, \omega_2, \omega_3\}$ such that

$\mathcal{T}_X = \{X, \Phi, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}\}$,

$\mathcal{T}_\Omega = \{\Omega, \Phi, \{\omega_1\}, \{\omega_2, \omega_3\}\}$ then $\mathcal{T}_X$ is a T_1-space and $F_{\mathcal{T}_X} = P(X)$, $F_{\mathcal{T}_\Omega} = P(\Omega)$. Consider

$F_1(x) = \{\omega_1\}, \forall x \in X$

$F_2(x_1) = \{\omega_2, \omega_3\}$, $F_2(x_2) = \{\Phi\}, F_2(x_3) = \{\omega_1\}$

$F_3(x) = \{\omega_2, \omega_3\}, \forall x \in X$

$F_4(x_1) = \Omega$, $F_4(x_2) = \{\omega_2, \omega_3\}, F_4(x_3) = \{\omega_2, \omega_3\}$

$F_5(x) = \Omega, \forall x \in X$

$F_6(x_1) = \{\omega_1\}$, $F_6(x_2) = \{\omega_2, \omega_3\}, F_6(x_3) = \{\omega_2, \omega_3\}$

Then $M = \{F_1, \dots, F_6\}$ is a SBM-space.

Consider $P_r : F_{\mathcal{T}_X} \longrightarrow [0,1]$ is a probability function then (Ω, M, P_r) is a SBM-random space. Since $\omega_1 \notin F_3$, $\omega_2 \notin F_1$ $\omega_3 \notin F_1$ such that F_1, F_3 are SBM-random closed sets so that

$P_r(\{x: \omega_1 \in F_1(x)\}) = P_r(\{x: F_3(x) \subseteq F_3(x)\}) = 1$ and $P_r(\{x : F_1(x) \cap F_3(x) \neq \emptyset\}) = 0$.

$P_r(\{x: \omega_2 \in F_3(x)\}) = P_r(\{x: F_1(x) \subseteq F_1(x)\}) = 1$ and $P_r(\{x : F_1(x) \cap F_3(x) \neq \emptyset\}) = 0$.

$P_r(\{x: \omega_3 \in F_3(x)\}) = P_r(\{x: F_1(x) \subseteq F_1(x)\}) = 1$ and $P_r(\{x : F_1(x) \cap F_3(x) \neq \emptyset\}) = 0$.

Such that F_1, F_3 are SBM-random open sets. Hence (Ω, M, P_r) is a SBM-random regular space.

Remark 4.21. A subspace of SBM-random regular space is not necessarily SBM-random regular space.

Example 4.22. In the example (4.21), a SBM-random space (Ω, M, P_r) is a regular space. Consider $T = \{F_1, F_2\}$ then (Ω, T, P_r) is not SBM-random regular space because there are no SBM-random open sets $\mathcal{U}, \mathcal{V}$ such that $P_r(\{x: \omega_3 \in \mathcal{U}(x)\}) = P_r(\{x: F_1(x) \subseteq \mathcal{V}(x)\}) = 1$ and $P_r(\{x : \mathcal{U}(x) \cap \mathcal{V}(x) \neq \emptyset\}) = 0$.

Theorem 4.23. A SBM-random space (Ω, M, P_r) is a SBM-random regular space. If for each $\omega_* \in \Omega$, $R \in M$ such that R is a SBM-random open set and $P_r(\{x : \omega_* \in R(x)\}) = 1$ there is a SBM-random closed set ß such that $P_r(\{x : \omega_* \in ß(x)\}) = P_r(\{x : ß(x) \subseteq R(x)\}) = 1$.

Proof: Let (Ω, M, P_r) be a SBM-random regular space and $\omega_* \in \Omega$, $R \in M$ such that R is a SBM-random open set and $P_r(\{x : \omega_* \in R(x)\}) = 1$ then R^c is a SBM-random closed set and $P_r(\{x : \omega_* \in R^c(x)\}) = 0$ since (Ω, M, P_r) is a SBM-random regular space then there are two SBM-random open sets $\mathcal{U}, \mathcal{V}$ such that $P_r(\{x : \omega_* \in \mathcal{U}(x)\}) = P_r(\{x : R^c(x) \subseteq \mathcal{V}(x)\}) = 1$ and $P_r(\{x : \mathcal{U}(x) \cap \mathcal{V}(x) \neq \emptyset\}) = 0$ means that $\omega_* \in \mathcal{U}(x), \forall x \in X$ so that $\mathcal{U}(x) \cap \mathcal{V}(x) = \emptyset, \forall x \in X$ thus $\mathcal{U}(x) \subseteq \mathcal{V}^c(x), \forall x \in X$ therefore $\omega_* \in \mathcal{V}^c(x), \forall x \in X$. Since $P_r(\{x : R^c(x) \subseteq \mathcal{V}(x)\}) = 1$ then $R^c(x) \subseteq \mathcal{V}(x), \forall x \in X$ thus $\mathcal{V}^c(x) \subseteq R(x), \forall x \in X$. Set $\mathcal{V}^c(x) = ß(x), \forall x \in X$. Hence ß is a SBM-random closed set such that $P_r(\{x : \omega_* \in ß(x)\}) = P(\{x : ß(x) \subseteq R(x)\}) = 1$.

Definition 4.24. A SBM-random space (Ω, M, P_r) is said to be a SBM-random T_3-space iff (Ω, M, P_r) is a SBM-random regular space and SBM-random T_1-space.

Remark 4.25. From remark (4.22) A subspace of SBM-random T_3-space is not necessarily SBM-random T_3-space.

Theorem 4.26. Every SBM-random T_3-space is a SBM-random T_2-space.

Proof: Let (Ω, M, P_r) be a SBM-random T_3-space and $\omega_1, \omega_2 \in \Omega$ such that $\omega_1 \neq \omega_2$ since (Ω, M, P_r) is a SBM-random T_3-space then (Ω, M, P_r) is a SBM-random regular space and SBM-random T_1-space implies that $\{\omega_2\}$ is a closed sets. Consider $R(x) = \{\omega_2\}, \forall x \in X$ then R is a SBM-random closed set such that $P_r(\{x : \omega_1 \in$

$\mathrm{R}(\mathrm{x})\}) = 0$ then there are SBM-random open sets $\mathcal{U}, \mathcal{V} \in \mathrm{M}$ such that $\mathrm{P_r}(\{\mathrm{x} : \omega_1 \in \mathcal{U}(\mathrm{x})\}) = \mathrm{P_r}(\{\mathrm{x} : \mathrm{R}(\mathrm{x}) \subseteq \mathcal{V}(\mathrm{x})\}) = 1$ and $\mathrm{P_r}(\{\mathrm{x} : \mathcal{U}(\mathrm{x}) \cap \mathcal{V}(\mathrm{x}) \neq \emptyset\}) = 0$ thus $\mathrm{P_r}(\{\mathrm{x} : \omega_1 \in \mathcal{U}(\mathrm{x})\}) = \mathrm{P}(\{\mathrm{x} : \omega_2 \subseteq \mathcal{V}(\mathrm{x})\}) = 1$ and $\mathrm{P_r}(\{\mathrm{x} : \mathcal{U}(\mathrm{x}) \cap \mathcal{V}(\mathrm{x}) \neq \emptyset\}) = 0$. Hence $(\Omega, \mathrm{M}, \mathrm{P_r})$ is a SBM-random $\mathrm{T_2}$-space.

Definition 4.27. A SBM-random space $(\Omega, \mathrm{M}, \mathrm{P_r})$ is said to be a SBM-random normal space iff for each SBM-random closed sets $\mathcal{V}_1, \mathcal{V}_2$ such that $\mathrm{P_r}(\{\mathrm{x} : \mathcal{V}_1(\mathrm{x}) \cap \mathcal{V}_2(\mathrm{x}) \neq \emptyset\}) = 0$ there are SBM-random open sets $\mathcal{U}_1, \mathcal{U}_2 \in \mathrm{M}$ such that $\mathrm{P_r}(\{\mathrm{x} : \mathcal{V}_1(\mathrm{x}) \subseteq \mathcal{U}_1(\mathrm{x})\}) = \mathrm{P_r}(\{\mathrm{x} : \mathcal{V}_2(\mathrm{x}) \subseteq \mathcal{U}_2(\mathrm{x})\}) = 1$ and $\mathrm{P_r}(\{\mathrm{x} : \mathcal{U}_1(\mathrm{x}) \cap \mathcal{U}_2(\mathrm{x}) \neq \emptyset\}) = 0$.

Example 4.28. Let $\mathrm{X} = \{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3\}, \Omega = \{\omega_1, \omega_2, \omega_3\}$ such that

$\mathcal{T}_{\mathrm{X}} = \{\mathrm{X}, \Phi, \{\mathrm{x}_1\}, \{\mathrm{x}_2\}, \{\mathrm{x}_3\}, \{\mathrm{x}_1, \mathrm{x}_2\}, \{\mathrm{x}_1, \mathrm{x}_3\}, \{\mathrm{x}_2, \mathrm{x}_3\}\}$,

$\mathcal{T}_{\Omega} = \{\Omega, \Phi, \{\omega_1\}, \{\omega_2, \omega_3\}\}$ then $\mathcal{T}_{\mathrm{X}}$ is a $\mathrm{T_1}$-space and $\mathrm{F}_{\mathcal{T}_{\mathrm{X}}} = \mathrm{P}(\mathrm{X})$, $\mathrm{F}_{\mathcal{T}_{\Omega}} = \mathrm{P}(\Omega)$. Consider

$\mathrm{F}_1(\mathrm{x}) = \{\omega_1\}, \forall\, \mathrm{x} \in \mathrm{X}$

$\mathrm{F}_2(\mathrm{x}_1) = \{\omega_2, \omega_3\},\ \mathrm{F}_2(\mathrm{x}_2) = \{\Phi\}, \mathrm{F}_2(\mathrm{x}_3) = \{\omega_1\}$

$\mathrm{F}_3(\mathrm{x}) = \{\omega_2, \omega_3\}, \forall\, \mathrm{x} \in \mathrm{X}$

$\mathrm{F}_4(\mathrm{x}_1) = \Omega,\ \mathrm{F}_4(\mathrm{x}_2) = \{\omega_2, \omega_3\}, \mathrm{F}_4(\mathrm{x}_3) = \{\omega_2, \omega_3\}$

$\mathrm{F}_5(\mathrm{x}) = \Omega, \forall\, \mathrm{x} \in \mathrm{X}$

$\mathrm{F}_6(\mathrm{x}_1) = \{\omega_1\},\ \mathrm{F}_6(\mathrm{x}_2) = \{\omega_2, \omega_3\}, \mathrm{F}_6(\mathrm{x}_3) = \{\omega_2, \omega_3\}$

Then $\mathrm{M} = \{\mathrm{F}_1, \dots, \mathrm{F}_6\}$ is a SBM-space.

Consider $\mathrm{P_r}: \mathrm{F}_{\mathcal{T}_{\mathrm{X}}} \longrightarrow [0,1]$ is a probability function then $(\Omega, \mathrm{M}, \mathrm{P_r})$ is a SBM-random space. Since $\mathrm{F}_1, \mathrm{F}_3$ are SBM-random closed sets such that $\mathrm{P_r}(\{\mathrm{x} : \mathrm{F}_1(\mathrm{x}) \cap \mathrm{F}_3(\mathrm{x}) \neq \emptyset\}) = 0$ so that

$\mathrm{P_r}(\{\mathrm{x}: \mathrm{F}_1(\mathrm{x}) \subseteq \mathrm{F}_1(\mathrm{x})\}) = \mathrm{P_r}(\{\mathrm{x}: \mathrm{F}_3(\mathrm{x}) \subseteq \mathrm{F}_3(\mathrm{x})\}) = 1$ and $\mathrm{P_r}(\{\mathrm{x} : \mathrm{F}_1(\mathrm{x}) \cap \mathrm{F}_3(\mathrm{x}) \neq \emptyset\}) = 0$.

Such that $\mathrm{F}_1, \mathrm{F}_3$ are SBM-random open sets. Hence $(\Omega, \mathrm{M}, \mathrm{P_r})$ is a SBM-random normal space.

Remark 4.29. A subspace of SBM-random normal space is not necessarily SBM-random normal space.

Example 4.30. In the example (4.15), a SBM-random space $(\Omega, \mathrm{M}, \mathrm{P_r})$ is a normal space. Consider $\mathrm{T} = \{\mathrm{F}_1, \mathrm{F}_3\}$ then $(\Omega, \mathrm{T}, \mathrm{P_r})$ is not SBM-random normal space because $\mathrm{F}_1, \mathrm{F}_3$ disjoint SBM-random closed sets but there are no SBM-random open sets $\mathcal{U}, \mathcal{V}$ such that $\mathrm{P_r}(\{\mathrm{x}: \mathrm{F}_1(\mathrm{x}) \subseteq \mathcal{U}(\mathrm{x})\}) = \mathrm{P_r}(\{\mathrm{x}: \mathrm{F}_3(\mathrm{x}) \subseteq \mathcal{V}(\mathrm{x})\}) = 1$ and $\mathrm{P_r}(\{\mathrm{x} : \mathcal{U}(\mathrm{x}) \cap \mathcal{V}(\mathrm{x}) \neq \emptyset\}) = 0$.

Theorem 4.31. A SBM-random space $(\Omega, \mathrm{M}, \mathrm{P})$ is a SBM-random normal space. If for each SBM-random closed set R_1 and SBM-random open set R_2 such that $\mathrm{P_r}(\{\mathrm{x} : \mathrm{R}_1(\mathrm{x}) \subseteq \mathrm{R}_2(\mathrm{x})\}) = 1$ there is a SBM-random closed set ß such that $\mathrm{P_r}(\{\mathrm{x} : \mathrm{R}_1(\mathrm{x}) \subseteq ß(\mathrm{x})\}) = \mathrm{P_r}(\{\mathrm{x} : ß(\mathrm{x}) \subseteq \mathrm{R}_2(\mathrm{x})\}) = 1$.

Proof: Let $(\Omega, \mathrm{M}, \mathrm{P_r})$ be a SBM-random normal space, R_1 is a SBM-random closed set and R_2 is a SBM-random open set such that $\mathrm{P_r}(\{\mathrm{x} : \mathrm{R}_1(\mathrm{x}) \subseteq \mathrm{R}_2(\mathrm{x})\}) = 1$ then $\mathrm{R}_2{}^c$ is a SBM-random closed set and $\mathrm{P_r}(\{\mathrm{x} : \mathrm{R}_1(\mathrm{x}) \cap \mathrm{R}_2{}^c(\mathrm{x}) \neq \emptyset\}) = 0$ since $(\Omega, \mathrm{M}, \mathrm{P_r})$ is a SBM-random regular space then there are two SBM-random open sets $\mathcal{U}, \mathcal{V}$ such that $\mathrm{P_r}(\{\mathrm{x} : \mathrm{R}_1(\mathrm{x}) \subseteq \mathcal{U}(\mathrm{x})\}) = \mathrm{P_r}(\{\mathrm{x} : \mathrm{R}_2{}^c(\mathrm{x}) \subseteq \mathcal{V}(\mathrm{x})\}) = 1$ and $\mathrm{P_r}(\{\mathrm{x} : \mathcal{U}(\mathrm{x}) \cap \mathcal{V}(\mathrm{x}) \neq \emptyset\}) = 0$ means that $\mathrm{R}_1(\mathrm{x}) \subseteq \mathcal{U}(\mathrm{x}), \forall\, \mathrm{x} \in \mathrm{X}$ so that $\mathcal{U}(\mathrm{x}) \cap \mathcal{V}(\mathrm{x}) = \emptyset, \forall\, \mathrm{x} \in \mathrm{X}$ thus $\mathcal{U}(\mathrm{x}) \subseteq \mathcal{V}^c(\mathrm{x}), \forall\, \mathrm{x} \in \mathrm{X}$ therefore $\mathrm{R}_1(\mathrm{x}) \subseteq \mathcal{V}^c(\mathrm{x}), \forall\, \mathrm{x} \in \mathrm{X}$. Since $\mathrm{P_r}(\{\mathrm{x} : \mathrm{R}_2{}^c(\mathrm{x}) \subseteq \mathcal{V}(\mathrm{x})\}) = 1$ then $\mathrm{R}_2{}^c(\mathrm{x}) \subseteq \mathcal{V}(\mathrm{x}), \forall\, \mathrm{x} \in \mathrm{X}$ thus $\mathcal{V}^c(\mathrm{x}) \subseteq \mathrm{R}_2(\mathrm{x}), \forall\, \mathrm{x} \in \mathrm{X}$. Set $\mathcal{V}^c(\mathrm{x}) = ß(\mathrm{x}), \forall\, \mathrm{x} \in \mathrm{X}$. Hence ß is a SBM-random closed set such that $\mathrm{P_r}(\{\mathrm{x} : \mathrm{R}_1(\mathrm{x}) \subseteq ß(\mathrm{x})\}) = \mathrm{P_r}(\{\mathrm{x} : ß(\mathrm{x}) \subseteq \mathrm{R}_2(\mathrm{x})\}) = 1$.

Definition 4.32. A SBM-random space $(\Omega, \mathrm{M}, \mathrm{P_r})$ is said to be a SBM-random $\mathrm{T_4}$-space iff $(\Omega, \mathrm{M}, \mathrm{P_r})$ is a SBM-random normal space and SBM-random $\mathrm{T_1}$-space.

Remark 4.33. From remark (4.30) A subspace of SBM-random $\mathrm{T_4}$-space is not necessarily SBM-random $\mathrm{T_4}$-space.

Theorem 4.34. Every SBM-random $\mathrm{T_4}$-space is a SBM-random $\mathrm{T_3}$-space.

Proof: Let $(\Omega, \mathrm{M}, \mathrm{P_r})$ be a SBM-random $\mathrm{T_4}$-space and $\omega_* \in \Omega$, R is a SBM-random closed set such that $\mathrm{P_r}(\{\mathrm{x} : \omega_* \in \mathrm{R}(\mathrm{x})\}) = 0$ since $(\Omega, \mathrm{M}, \mathrm{P_r})$ is a SBM-random $\mathrm{T_4}$-space then $(\Omega, \mathrm{M}, \mathrm{P_r})$ is a SBM-random normal space and SBM-random $\mathrm{T_1}$-space implies that $\{\omega_*\}$ is a closed set. Consider $ß(\mathrm{x}) = \{\omega_*\}, \forall\, \mathrm{x} \in \mathrm{X}$ then ß is a SBM-random closed set such that $\mathrm{P_r}(\{\mathrm{x} : ß(\mathrm{x}) \cap \mathrm{R}(\mathrm{x}) \neq \emptyset\}) = 0$ then there are SBM-random open sets $\mathcal{U}, \mathcal{V} \in \mathrm{M}$ such that $\mathrm{P_r}(\{\mathrm{x} : ß(\mathrm{x}) \subseteq \mathcal{U}(\mathrm{x})\}) = \mathrm{P_r}(\{\mathrm{x} : \mathrm{R}(\mathrm{x}) \subseteq \mathcal{V}(\mathrm{x})\}) = 1$ and $\mathrm{P_r}(\{\mathrm{x} : \mathcal{U}(\mathrm{x}) \cap \mathcal{V}(\mathrm{x}) \neq \emptyset\}) = 0$ thus $\mathrm{P_r}(\{\mathrm{x} : \omega_* \in$

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$\mathcal{U}(x)\}) = P_r(\{x : R(x) \subseteq \mathcal{V}(x)\}) = 1$ and $P_r(\{x : \mathcal{U}(x) \cap \mathcal{V}(x) \neq \emptyset\}) = 0$ implies that (Ω, M, P_r) is a SBM-random regular-space but (Ω, M, P_r) is a SBM-random T_1-space. Hence (Ω, M, P_r) is a SBM-random T_3-space.

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