

SELECTION OF THREE STAGE CHAIN SAMPLING PLAN OF TYPE CHSP (0,1,3) WITH SINGLE SAMPLING PLAN USING QUALITY REGION

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ABSTRACT

Acceptance sampling procedures are practical tools for quality assurance applications involving product control. In this article we have developed new method for designing sampling plans based on range of quality instead of pointwise description of quality by invoking a quality regions approach. The Quality Decision Region proposes wider potential applicability in industry ensuring a higher standard of quality attainment for a product. This paper provides a selection of chain sampling plan of type (0,1,3) with Single Sampling plan indexed through quality regions. Tables are constructed by considering the various combinations of acceptable and limiting quality levels, and an example is given for illustration purpose.

KEYWORDS: Statistical quality control, Three Stage Chain sampling plan, Quality Region

Introduction: Sampling is widely used in industry and government for controlling the quality of shipment of components, supplies and final products. In general, the number marking the boundary between “a few” and “too many” defectives, (the maximum acceptable number of defectives) varies depending on the situation. This number depends on the lot size, cost of inspecting and testing a part and an assembly, cost of dismantling and repairing an assembly, loss associated with the possible failure to meet customer requirements, etc. In order to control the quality of purchased lot, two major alternatives are open to a buyer. One, complete inspection: every single item in the lot is inspected and tested. Two, partial inspection: a sample of items is taken, the sampled items are inspected and tested, and the lot as a whole is accepted or rejected depending on whether few or many defective items which are found in the sample. This type of sampling, one of many used to control the quality of manufacturing processes or lots, is known as acceptance sampling

Review of Literature: Dodge (1955) treats this problem using a procedure, called chain sampling plan (ChSP – 1). These plans make use of the cumulative inspection results from several results, from one or more samples along with the results from the current sample, in making a decision regarding acceptance or rejection of the current lot. The chain sampling plans are applicable for both small and large samples. Dodge and Stephens (1966) extended the concept of chain sampling plans and presented a set of two stage chain sampling plans based on the concept of ChSP – 1 developed by Dodge (1955). They presented expressions for OC curves of certain two – stage chain sampling plans and made comparison with single and double sampling attributes inspection plans. Cameron (1952) has developed an operating ratio based on Acceptable Quality Level (AQL) and Limiting Quality Level (LQL). A table is constructed for the selection of sampling plan under Poisson model and the ratio is a measure of discrimination. Soundararajan (1975) has indexed the single sampling plan through MAPD and $K = pT / p^*$, where MAPD is the proportion defective at the inflection point of OC curve defined and explained by Mayer (1967) and Mandelson (1962). Vedaldi (1986) has studied the SSP through Incentive and filter effects. This paper designs the parameters of the plan indexed with QDR and PQR. Suresh and Srivenkataramana (1996) have studied the selection of SSP using Producer and Consumer Quality Levels. Suresh and Saminathan (2007) have given a procedure to define multiple repetitive group sampling plans indexed with MAPD and MAAOQ. Suresh and Kaviyarasu (2008) have explained the desirability for developing Quick Switching System with Conditional RGS plan indexed through quality levels. Suresh and Divya (2009) have given the new procedure for Single Sampling Plan through Decision Region. Suresh and Sangeetha (2010) have studied the selection of Repetitive Deferred Sampling plan using Quality Regions.

Operating Procedure for Three Stage Chain Sampling Plan

Step 1: At the outset, select a random sample of n units from the lot and from each succeeding lot.

Step 2: Record the number of defectives d, in each sample and sum the number of defectives, D, in all samples from the first up to and including in the current sample.

Step 3: Accept the lot associated with each new sample during the cumulation as long as $D_i \leq c_1; 1 \leq i \leq k_1$.

Step 4: When k_1 consecutive samples have all resulted in acceptance continue to sum the defectives in the k_1 samples plus additional samples up to not more than k_2 samples.

Step 5: Accept the lot associated with each new sample during cumulation as long as $D_i \leq c_2; k_1 \leq i \leq k_2$.

Step 6: When k_2 consecutive samples have all resulted in acceptance continue to sum the defectives in the k_2 samples plus additional samples up to not more than k_3 samples.

Step 7: Accept the lot associated with each new sample during cumulation as long as $D_i \leq c_3; k_2 \leq i \leq k_3$.

Step 8: When the third stage of the restart period has been successfully completed (i.e., k_3 consecutive samples have been resulted in acceptance), start cumulation of defectives as moving total over k_3 samples by adding the current sample result while dropping from the sum, the sample result of the k_3 th preceding sample. Continue this procedure as long as $D_i \leq c_3$ and in each instance accept the lot.

Step 9: If for any sample at any stage of the above procedure, D_i is greater than the corresponding c , reject the lot.

Step 10: When a lot is rejected return to step-1 and fresh restart of the cumulation procedure.

The three-stage chain sampling plan has 7 parameters which are defined below:

n = sample size

k_1 = The maximum number of samples over which the cumulation of the defectives take place in the first stage of procedure.

k_2 = The maximum number of samples over which the cumulation of the defectives take place in the second stage of procedure.

k_3 = The maximum number of samples over which the cumulation of the defectives take place in the first of procedure.

c_1 = The allowable number of defectives in the cumulative results from k_1 or fewer sample of n . Thus, c_1 is an acceptance number for cumulative results. It is the cumulative results criterion (CRC) that must be met by cumulative sampling results during the first stage of of the restart period in order to permit acceptance of a lot.

c_2 = The allowable number of defectives in the cumulative results from $k_1 + 1$ to k_2 sample of n . Thus, c_2 is also an acceptance number for cumulative results and the CRC that must be met by cumulative sampling results during the second stage of the restart period in order to permit acceptance of a lot.

c_3 = The allowable number of defectives in the cumulative results from $k_2 + 1$ to k_3 sample of n . Thus, c_3 is also an acceptance number for cumulative results and the CRC that must be met by cumulative sampling results during the third stage of the restart period in order to permit acceptance of a lot.

Designing of Quality Interval ChSP (0,1,3) with Single sampling plan as follows

Selection of a Sampling Plan

Table 2 gives unique values of R for different values k_1, k_2, c_1 , and c_2

$$\text{Operating ratio } R = \frac{np_2 - np_1}{np_0 - np_1} = \frac{d_2}{d_0}$$

Where $d_2 = np_2 - np_1$ and $d_0 = np_0 - np_1$ is used to characterize the sampling plan.

For any given values of PQR (d_2) IQR (d_0), one can find the ratio $R = \frac{d_2}{d_0}$. Find the value in the table 2 under the column R, which is equal to or just less than specified ratio. Corresponding k_1, k_2, k_3, c_1 , and c_2 values are noted. From this ratio, one can determine the parameters for the three-stage chain sampling plan with a single sampling plan as a reference plan.

Example

Given $p_1=0.05, k_1=2, k_2=3, k_3=4, c_1=1$ and $c_2=2$. Compute the values of the IQR and PQR. Then Compute R. Select the associated values from table 2. The nearest values are IQR=0.2.1579, PQR=0.7077 and R=0.327996 with respective $k_1=2, k_2=3, k_3=4, c_1=1$ and $c_2=2$. One can obtain the values of np_1 from pa(p) table $np_1=0.1723$, which leads to $n=np_1/p_1=9.708=10$ using table 2.

Thus, the selected parameters for chain sampling plan are $n=10, k_1=2, k_2=3, k_3=4, c_1=1$ and $c_2=2$ through quality interval.

For specified PQR and IQR

Table 2 used to construct the plans when the PQR and IQR are specified. For any given values of the PQR and IQR, one can find the ratio $R=d_2/d_0$ which is a monotonic increasing function. Find the value in table 2 under the column of R, which is equal to or just less than the specified ratio. Then the corresponding values of k_1, k_2, k_3 and c are noted. From this one can determine the parameters of chain SSP plan.

Example

For given $p_1=0.05, k_1=15, k_2=18, k_3=19, c_1=1$ and $c_2=2$. Compute the values of IQR and PQR. Corresponding $k_1=15, k_2=18, k_3=19, c_1=1$ and $c_2=2$, values of $IQR=0.5604$ $PQR= 2.1297$ using table 2. $np_1=0.1299, np_0= 0.6902, np_2= 2.2598$ for specified IQR and PQR

Table 2 is used to construct the plans when IQR and PQR are specified. For any given values of IQR(d_1) and PQR (d_2), one can find the ratio $R=d_1/d_2$, which is a monotonic increasing function in k . The corresponding values in table 2, under the column R which is equal or just less than the specified ratio. Then the corresponding value of k is noted, From this, one can determine the parameters for $k_1=15, k_2=18, k_3=19, c_1=1$ and $c_2=2$ for the chain sampling plan.

Conversion of parameters

The given set of parameters can be converted into another familiar set, which will provide related information on the desired sampling plan. These parameters can be found using table 2.

Construction of tables

Indifference quality range (IQR) denoted as $d_0 = np_0 - np_1$ is derived from probability of acceptance

$$P_a(p)=$$

$$\frac{P_0 + P_1 P_0^{k_2-1} + (k_3 - k_2 - 1)P_1(P_1 * 2P_2)P_0^{k_3-2} + \binom{k_3-k_2-1}{2}P_1^3 P_0^{k_3-3} + (P_2 + P_3)P_0^{k_3-1} + P_1 P_0^{k_1} \left(\frac{1-P_0^{k_2-k_1-1}}{1-P_0}\right) + (k_2 - k_1)P_1(P_1 + P_2)P_0^{k_2-1} + P_1(P_1 + 2P_2)P_0^{k_2} \left(\frac{1-(k_3-k_2-1)P_0^{k_3-k_2-2}}{1-P_0} + \frac{P_0(1-P_0^{k_3-k_2-2})}{(1-P_0)^2}\right) + P_1^3 P_0^{k_2} \left(\sum_{j=0}^{k_3-k_2-4} \binom{j+2}{2} P_0^j + (P_2 + P_3)P_0^{k_2} \left(\frac{1-P_0^{k_3-k_2-1}}{1-P_0}\right)\right)}{1 + P_1 P_0^{k_1} \left(\frac{1-P_0^{k_2-k_1-1}}{1-P_0}\right) + (k_2 - k_1)P_1(P_1 + P_2)P_0^{k_2-1} + P_1(P_1 + 2P_2)P_0^{k_2} \left(\frac{1-(k_3-k_2-1)P_0^{k_3-k_2-2}}{1-P_0} + \frac{P_0(1-P_0^{k_3-k_2-2})}{(1-P_0)^2}\right) + P_1^3 P_0^{k_2} \left(\sum_{j=0}^{k_3-k_2-4} \binom{j+2}{2} P_0^j + (P_2 + P_3)P_0^{k_2} \left(\frac{1-P_0^{k_3-k_2-1}}{1-P_0}\right)\right)}$$

Probabilistic quality range (PQR) denoted as $d_2 = np_2 - np_1$ is derived from probability of acceptance

$$p_a (p_1 < p < p_2)$$

$$P_a(p) = \frac{p_a(1-p_c)^i - p_c p_a^i}{(1-p_c)^i}$$

where

$$p_a = \sum_{r=0}^{c_1} \frac{e^{-x} x^r}{r!}$$

and

$$p_c = \sum_{r=0}^{c_2} \frac{e^{-x} x^r}{r!} - \sum_{r=0}^{c_1} \frac{e^{-x} x^r}{r!}$$

Table 2 shows the various $d_0=nQDR$, $d_2=nPQR$ for different values of k_1, k_2, k_3, c_1, c_2 and c_3 and provides various parametric values for ChSP(0,1,3) with SSP designed through quality interval.

Conclusion: The present development would be a valuable addition to the literature and a useful device to the quality practitioners. Acceptance sampling is the technique which deals with the procedures in which decision either to accept or reject lots or process which are based on the examination of samples. This procedure reduces the cost of inspection for the producer and the consumer, gets good items. In practice it is desirable to design any sampling plan with the associated quality levels which concern to producer and consumer. Tables provided in this paper are tailor – made which are handy and readymade, which are also well considered for comparison purposes. Tables are also useful for developing and under developing countries, which have limited resources to the Industrial shop floor- situations.

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Table:1 The parametric values for three stage chain sampling plan of type ChSP(0,1,3)

Pa(p)									
K ₁	K ₂	K ₃	0.99	0.95	0.90	0.50	0.10	0.05	.001
1	2	3	0.03230	0.08012	0.14013	0.68987	2.24983	2.99014	4.19998
1	2	5	0.03011	0.08580	0.14986	0.68861	2.24733	2.99008	4.16019
1	4	5	0.03013	0.07852	0.14013	0.68798	2.30019	2.99002	4.60023
2	3	4	0.03132	0.08788	0.13012	0.68775	2.30011	2.98997	4.60015
2	5	6	0.02013	0.09012	0.14010	0.65123	2.29989	2.93414	4.59987
2	9	8	0.03012	0.08123	0.14008	0.68995	2.24531	2.98995	4.59975
3	4	5	0.03011	0.07945	0.13028	0.69023	2.24483	2.98985	4.12021
4	5	6	0.03014	0.07855	0.13025	0.69018	2.30024	2.98976	4.60022
5	6	7	0.03001	0.07982	0.13010	0.64414	2.30018	2.92957	4.60017
6	7	8	0.02980	0.08763	0.14021	0.69010	2.30021	2.99002	4.60011
7	8	9	0.02314	0.08017	0.13007	0.69005	2.24233	2.98995	4.08012
8	9	10	0.03021	0.07701	0.13002	0.68995	2.30012	2.98965	4.60010
9	10	11	0.02021	0.08001	0.12998	0.63743	2.30232	2.92500	4.60002
9	10	20	0.02897	0.08012	0.12987	0.69022	2.30422	2.98997	4.59998
10	11	12	0.02100	0.08032	0.12895	0.69019	2.23983	2.98700	4.04017
11	12	13	0.02300	0.0755	0.12021	0.69015	2.30012	2.98400	4.60021
11	12	19	0.02400	0.08012	0.35012	0.63021	2.30200	2.92043	4.60201
11	17	20	0.02827	0.08001	0.41998	0.62301	2.30400	2.91586	4.60412
12	13	14	0.02757	0.07002	0.26010	1.09025	2.53024	3.14024	4.00012
13	14	15	0.02687	0.07400	0.26012	1.12012	2.53017	3.13996	3.96010
13	14	19	0.03230	0.07253	0.24987	0.98032	2.47987	3.12018	4.64019
13	14	22	0.03011	0.09012	0.20112	0.88012	2.33023	3.00123	4.60021
13	17	22	0.03013	0.24023	0.22112	0.83998	2.31995	3.00101	4.60010
13	20	22	0.03132	0.32012	0.20210	0.82022	2.32023	2.99989	4.59994
14	15	16	0.02013	0.15987	0.18799	0.78992	2.30010	2.99015	4.56018
14	15	19	0.03012	0.17231	0.17112	0.74023	2.27012	2.97900	4.52012
14	15	22	0.03001	0.17023	0.15865	0.72012	2.26743	2.97600	4.48021
14	17	22	0.02980	0.12012	0.15121	0.70023	2.26499	2.96150	4.44001
15	16	17	0.02314	0.13998	0.15023	0.69998	2.26233	2.95700	4.40101
15	18	19	0.03021	0.12987	0.16012	0.69023	2.25983	2.99012	4.36010
15	18	22	0.02021	0.12023	0.13987	0.69017	2.25733	2.95250	4.31987
15	20	22	0.02897	0.11023	0.14013	0.69010	2.30021	2.94800	4.59878
16	17	18	0.02100	0.10112	0.12992	0.67897	2.25483	2.99031	4.28021
16	18	20	0.02300	0.09850	0.12013	0.69023	2.25233	2.99022	4.24013
16	19	22	0.02400	0.09015	0.12010	0.66521	2.29987	2.94329	4.60010
16	21	22	0.02827	0.1100	0.12005	0.65818	2.29981	2.93871	4.60005
17	18	19	0.02757	0.08860	0.12001	0.61643	2.23733	2.91129	4.00012
18	19	20	0.02687	0.08720	0.11999	0.60899	2.23483	2.90671	3.96010

Table:2 Certain characteristic values for three stage chain sampling plans through quality region

K ₁	K ₂	K ₃	np ₁	np ₀	np ₂	d ₀	d ₂	R
1	2	3	0.08012	0.68987	2.24983	0.60975	2.16971	3.55836
1	2	5	0.0858	0.68861	2.24733	0.60281	2.16153	3.585757
1	4	5	0.07852	0.68798	2.30019	0.60946	2.22167	3.645309
2	3	4	0.08788	0.68775	2.30011	0.59987	2.21223	3.687849
2	5	6	0.09012	0.65123	2.29989	0.56111	2.20977	3.938212
2	9	8	0.08123	0.68995	2.24531	0.60872	2.16408	3.555132
3	4	5	0.07945	0.69023	2.24483	0.61078	2.16538	3.54527
4	5	6	0.07855	0.69018	2.30024	0.61163	2.22169	3.632408
5	6	7	0.07982	0.64414	2.30018	0.56432	2.22036	3.934576
6	7	8	0.08763	0.6901	2.30021	0.60247	2.21258	3.672515
7	8	9	0.08017	0.69005	2.24233	0.60988	2.16216	3.545222
8	9	10	0.07701	0.68995	2.30012	0.61294	2.22311	3.626962
9	10	11	0.08001	0.63743	2.30232	0.55742	2.22231	3.986778
9	10	20	0.08012	0.69022	2.30422	0.6101	2.2241	3.645468
10	11	12	0.08032	0.69019	2.23983	0.60987	2.15951	3.540935
11	12	13	0.0755	0.69015	2.30012	0.61465	2.22462	3.619328
11	12	19	0.08012	0.63021	2.302	0.55009	2.22188	4.039121
11	17	20	0.08001	0.62301	2.304	0.543	2.22399	4.095746
12	13	14	0.07002	1.09025	2.53024	1.02023	2.46022	2.411437
13	14	15	0.074	1.12012	2.53017	1.04612	2.45617	2.347886
13	14	19	0.07253	0.98032	2.47987	0.90779	2.40734	2.651869
13	14	22	0.09012	0.88012	2.33023	0.79	2.24011	2.835582
13	17	22	0.24023	0.83998	2.31995	0.59975	2.07972	3.467645
13	20	22	0.32012	0.82022	2.32023	0.5001	2.00011	3.99942
14	15	16	0.15987	0.78992	2.3001	0.63005	2.14023	3.396921
14	15	19	0.17231	0.74023	2.27012	0.56792	2.09781	3.693848
14	15	22	0.17023	0.72012	2.26743	0.54989	2.0972	3.813854
14	17	22	0.12012	0.70023	2.26499	0.58011	2.14487	3.697351
15	16	17	0.13998	0.69998	2.26233	0.56	2.12235	3.789911
15	18	19	0.12987	0.69023	2.25983	0.56036	2.12996	3.801056
15	18	22	0.12023	0.69017	2.25733	0.56994	2.1371	3.749693
15	20	22	0.11023	0.6901	2.30021	0.57987	2.18998	3.776674
16	17	18	0.10112	0.67897	2.25483	0.57785	2.15371	3.727109
16	18	20	0.0985	0.69023	2.25233	0.59173	2.15383	3.639886
16	19	22	0.09015	0.66521	2.29987	0.57506	2.20972	3.84259
16	21	22	0.11	0.65818	2.29981	0.54818	2.18981	3.994692
17	18	19	0.0886	0.61643	2.23733	0.52783	2.14873	4.070875
18	19	20	0.0872	0.60899	2.23483	0.52179	2.14763	4.11589