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MILDLY α GENERALIZED BINARY CLOSED SETS IN BINARY TOPOLOGICAL SPACES

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ABSTRACT

In this paper we define and study mildly α generalized (mαg) binary closed sets in binary topological spaces. Further more binary $_{m\alpha g}\mathbf{T}_c$ space, binary $_{m\alpha g}T_\alpha$ space, $_{m\alpha g}\mathbf{T}_r$ space, $_{m\alpha g}T_{\alpha g}$ space are also introduced and their properties are investigated.

Keywords: Binary topology, binary α-closed sets, α-generalized binary closed set, mildly α generalized (mαg) binary closed set.

INTRODUCTION

The authors S. Nithyanantha jothi and P.Thangavelu **[2]** introduced the concept of binary topology and discussed some of its basic properties in 2011. Generalized closed sets in topological space are introduced by Norman Levine **[1]**. Also the authors S. Nithyanantha jothi and P.Thangavelu **[5]** studied generalized binary closed sets in binary topological space in 2014. In this paper new notion of generalized binary closed sets in binary topological spaces namely mildly α generalized binary closed sets is introduced and some of their basic properties are studied. Further, we also introduce binary $_{m\alpha g}\mathbf{T}_c$ space, binary $_{m\alpha g}T_\alpha$ space, $_{m\alpha g}\mathbf{T}_r$ space, $_{m\alpha g}T_{\alpha g}$ space and studied their relationships among them.

2. PRELIMINARIES

In this section basics of binary topological spaces are given. Throughout this paper mildly α generalized binary closed sets are denoted as mαg-binary closed sets. We recall the following definitions and results.

Definition:2.1 [5]

Let X and Y be any two nonempty sets, A binary topology from X to Y is a binary structure M⊆℘(X) ×℘(Y) that satisfies the axioms namely,

(i). (∅, ∅) and (X, Y) ∈ M,

(ii). $(A_1 \cap A_2, B_1 \cap B_2) \in M$ whenever $(A_1, B_1) \in M$ and $(A_2, B_2) \in M$

(iii).If $\{(A_\alpha, B_\alpha): \alpha\in\Delta\}$ is a family of members of M, then $(\bigcup_{\alpha\in\Delta} A\alpha, \bigcup_{\alpha\in\Delta} B\alpha) \in M$.

If M is a binary topology from X to Y then the triplet (X, Y, M) is called a binary topological space and the members of M are called the binary open subsets of the binary topological space (X,Y, M).

The elements of X×Y are called the binary points of the binary topological space (X, Y, M).

If Y=X then M is called a binary topology on X in which case we write (X, M) as a binary topological space. The examples of binary topological spaces are given in **[2].**

Definition 2.2: **[2]** Let X and Y be any two nonempty sets and let (A, B) and (C, D) ∈ ℘(X) × ℘(Y). We say that (A, B) ⊆ (C, D) if A ⊆ C and B ⊆ D, ℘(X), ℘(Y) denote the power sets of X and Y.

Definition 2.3: **[2]** Let (X, Y, M) be a binary topological space and A ⊆ X, B ⊆Y. Then (A, B) is called binary closed in (X, Y, M) if (X\A, Y\B) ∈ M.

Proposition 2.4: **[2]** Let (X, Y, M) be a binary topological space and (A,B) ⊆ (X,Y).Let $(A, B)^{1*} = \cap\{ A\alpha : (A\alpha ,B\alpha)$ is binary closed and $(A, B) \subseteq (A\alpha, B\alpha)\}$ and $(A, B)^{2*} = \cap\{ B\alpha: (A\alpha, B\alpha)$ is binary closed and $(A,B) \subseteq (A\alpha,B\alpha)\}$. Then $((A, B)^{1*}, (A, B)^{2*})$ is binary closed and $(A, B) \subseteq ((A, B)^{1*}, (A, B)^{2*})$.

Definition 2.5:[2] The ordered pair $((A, B)^{1*}, (A, B)^{2*})$ is called the binary closure of (A, B), denoted by b-cl(A, B) in the binary space (X, Y, M) where (A, B) ⊆ (X, Y).

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Definition 2.6: [2] Let X and Y be any two nonempty sets and let (A, B) and (C, D)∈℘(X) ×℘(Y). We say that (A, B) ⊄ (C, D) if one of the following holds:
(i) A⊆C and B⊄ D (ii) A ⊄ C and B ⊆D (iii) A ⊄ C and B⊄ D.

Definition 2.7: [2] Let (X, Y, M) be a binary topological space. Let (A, B) ⊆ (X ,Y). Define $M_{(A, B)}$ = {(A ∩U, B ∩V):(U,V)∈ M }. Then $M_{(A, B)}$ is a binary topology from A to B. The binary topological space (A, B, $M_{(A, B)}$) is called a binary subspace of (X, Y, M).

Definition 2.8: [3] Let (X, Y, M) be and investigated some of a binary topological space. Let (A, B) ⊆ (X, Y).Then (A, B) is called binary semi open if there exists a binary open set (U, V) such that (U, V) ⊆ (A, B) ⊆ b-cl (U, V)

Definition 2.9:[4] Let (X,Y, M) be a binary topological space. Let (A, B) ⊆ (X,Y). Then (A,B) is called binary regular open if (A,B) = b-int(b-cl(A,B)) and binary regular closed if (A,B) = b-cl(b-int(A,B)).

Definition 2.10: [5] Let (X, Y, M) be a binary topological space. Let (A, B) ∈ ℘ (X) x ℘ (Y). Then (A,B) is called generalized binary closed if b-cl(A,B) ⊆ (U,V) whenever (A,B) ⊆ (U,V) and (U,V) is binary open in (X,Y, M).

3. mαg-BINARY CLOSED SETS

In this section we introduce binary α-closed sets and α generalized binary closed sets, mildly α generalized (mαg) binary closed sets and investigated some of their properties of mαg binary closed sets.

Definition 3.1: Let (X, Y, M) be a binary topological space. Then (A, B) ⊆ (X, Y) is called binary -α closed if b-cl(b-int(b-cl(A, B)) ⊆ (A, B) and binary α-open if (A, B) ⊆ b-int(b-cl(b-int(A, B)).

Definition 3.2: Let (A, B) be a set of a binary topological space (X, Y, M), then binary-α interior and binary α-closure are defined as
b-αint(A,B)= ∪{(U,V): (U,V) is a binary α-open set in (X, Y, M) and (U,V) ⊆ (A,B)}.
b-αcl(A,B)= ∩ {(H,K): (H,K) is a binary α-closed set in (X, Y, M) and (A,B) ⊆ (H,K)}.
Also, b-αcl(A, B)=(A,B) ∪ b-cl(b-int(b-cl(A, B)).

Definition 3.3: Let (X, Y, M) be a binary topological space .Let (A, B) ⊆ (X, Y),then (A,B) is called αg-binary closed sets, if b-αcl(A,B) ⊆(U,V) whenever (A,B) ⊆ (U,V) and (U,V) is binary open.

Definition 3.4: Let (X, Y, M) be a binary topological space .Let (A, B) ⊆ (X, Y),then (A,B) is called mildly α generalized (mαg) binary closed sets, if b-cl(b-int(A, B)) ⊆ (U,V) whenever (A,B) ⊆ (U,V), and (U,V) is αg-binary open.

Proposition 3.5:
(i).Every binary closed set is binary α-closed.
(ii). Every binary regular closed set is binary α-closed .
(iii).Every binary α-closed set in binary topological space is αg-binary closed set.

Proof.
(i). Let (A, B) be binary closed in a binary topological space (X, Y, M).We shall show that (A, B) is binary α-closed. Since (A, B) is binary closed, we have (b-cl(A,B)) = (A,B). That is b-int(b-cl(A,B)= b-int(A,B) ⊆(A,B). That is b-int(b-cl(A,B)) ⊆(A,B).Therefore b-cl(b-int(b-cl(A,B)))⊆b-cl(A,B), That is b-cl(b-int(b-cl(A,B)))⊆(A,B). Thus, (A, B) is binary α-closed.

(ii). Let (A, B) be any binary regular closed set in binary topological space (X,Y, M).Suppose (U,V) be binary α-open in (X,Y, M) such that (A,B) ⊆ (U,V).Since (A, B) is a regular closed set , b-cl(b-int(A,B)) = (A,B) ⊆ (U,V). By **[4]** every regular binary closed set is binary closed, by (i) every binary closed set is binary α-closed set.(i.e) b-cl(b-int(b-cl(A,B))) ⊆ (U,V).hence (A, B) is a binary α-closed set in(X,Y, M).

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(iii). Let (X, Y, M) be a binary topological space and (A, B) is binary α-closed in (X, Y, M). Let (A,B) ⊆ (U,V), where (U,V) is binary open in (X,Y, M). Since (A, B) is binary α-closed then b-αcl(A, B) = (A, B). Hence b-αcl(A, B) ⊆ (U,V).Therefore (A, B) is αg binary closed .

Example3.6: Let X={0,1},Y={a, b, c}, M ={(φ, φ),({0},{a}),({1},{b}),(X,{a, b}),(X,Y)} is a binary topology from X to Y.

In this example {(X,{b, c})} is binary α-closed set in(X,Y, M) but not binary closed set and not regular binary closed set. Also the set ({1},{a, c}) is αg-binary closed set in(X,Y, M) but not binary α-closed .

Proposition 3.7:

(i).Every binary closed sets in a binary topological is mαg-binary closed .

(ii). Every binary regular closed set is mαg-binary closed .

(iii). Every binary α-closed set is mαg-binary closed.

(iv).Every αg-binary closed set is mαg-binary closed.

Proof:

(i). Let (X, Y, M) be a binary topological space and (A, B) is binary closed in(X, Y, M). Let (A,B) ⊆(U,V) be αg-binary open in (X,Y, M).since (A,B) is binary closed , b-cl(A,B)=(A,B)and hence b-cl(A,B) ⊆(U,V).but b-cl(b-int (A,B) ⊆ b-cl(A,B) ⊆(U,V).Therefore b-cl(b-int (A,B) ⊆ (U,V).Hence (A,B) is mαg-binary closed in (X,Y, M).

(ii). Let(A,B) be any binary regular closed set in binary topological space (X,Y, M).suppose (U,V) be αg-binary open in (X,Y, M) such that (A,B) ⊆(U,V) .since (A,B) is a regular closed set ,b-cl(b-int(A,B))=(A,B) ⊆(U,V).By **[4]** every regular binary closed set is binary closed, hence by (i) every binary closed set is mαg-binary closed set.(i.e) b-cl(b-int(A,B))⊆(U,V).hence (A,B) is a mαg-binary closed set in(X,Y, M).

(iii).Let (A, B) be any binary α-closed set in (X, Y, M).suppose (U, V) is αg-binary open in (X,Y, M), such that (A,B) ⊆(U,V). By hypothesis b-cl(b-int(b-cl(A, B)) ⊆ (A, B),Therefore b-cl(b-int(A,B))⊆b-cl(b-int(b-cl(A,B))⊆ (A,B) ⊆(U,V).Hence b-cl(b-int(A,B))⊆ U,V) Therefore (A, B) is mαg-binary closed in (X, Y, M).

(iv). Let (A,B) be any αg-binary closed set in (X,Y, M).suppose (U,V) is αg-binary open in (X, Y, M), such that (A, B) ⊆ (U,V). Since (A, B) is αg-binary closed,(i.e), b-αcl(A,B) ⊆(U,V), (i.e) b-cl(b-int(b-cl(A, B)) ⊆(U,V). Now b-cl(b-int(A,B))⊆ b-cl(b-int(b-cl(A, B)) ⊆(U,V). Hence (A, B) is mαg-binary closed in (X, Y, M).

The converses of the statements in the above theorem need not be true as shown in the following example.

Example 3.8: Let X={0,1},Y={a,b,c}, M ={(φ, φ),({0},{a}),({1},{b}),(X,{a,b}),(X,Y)} is a binary topology from X to Y.

In this example for the binary closed set ({1},{a}) in (X,Y, M), b-cl({1},{a})=({0},{b,c}) is mαg- binary closed but not binary closed set. Also the set ({0},{b,c}) is mαg-binary closed set in(X,Y, M) but not binary regular closed set and (X,{a,c}) is mαg-binary closed set in(X,Y, M) but not binary α- closed, also ({0},{a,b}) is mαg-binary closed set in (X,Y, M) but not αg-binary closed in (X,Y, M).

4. CHARACTERIZATIONS OF MILDLY α GENERALIZED BINARY CLOSED SET

Remark 4.1: Union of two mαg-binary closed set in (X,Y, M) is not mαg-binary closed.

Example 4.2: Let X= {0, 1}, Y={a, b, c}, M ={(φ, φ),({0},{a}),({1},{b}),(X,{a,b}),(X,Y)} is a binary topology from X to Y in (X,Y, M). Let (A,B) = ({0}, {a, c}) and (C,D)= ({0}, {b, c}), (A,B)∪(C,D) = ({0},{b}) ∪ ({1},{b,c}) = (X,{a,b}). Here (X,{a,b}) is not a mαg-binary closed whereas (A,B) and (C,D) are not mαg-binary closed

Remark 4.3 The intersection of two mαg-binary closed sets in (X,Y, M) is mαg-binary closed set in (X,Y, M)

Theorem 4.4: If (A, B) is a mαg binary closed set and (A, B) ⊆ (C, D) ⊆ b-cl(A,B), then (C,D) is mαg-binary closed.
Proof: Let (C, D) ⊆ (U, V) where (U, V) is αg-binary open. Now, (A, B) ⊆ (U, V). Since (A, B) is mαg-binary

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closed, b-cl(b-int(A,B))⊆(U,V). By proposition 3.7 [1] b-cl(b-int(C,D)) ⊆ b-cl(b-cl(A,B)) = b-cl(A,B) ⊆ (U,V). Consequently (C,D) is mαg-binary closed.

Theorem 4.5: Let (A,B) be mαg-binary closed set, Suppose that (C, D) is binary closed. If (A∩C, B∩D) ⊆ (U,V) where (U,V) is αg-binary open, then $((A,B)^{1*} \cap C,(A,B)^{2*} \cap D) \subseteq (U,V)$.

Proof: Since (A∩C, B∩D) ⊆ (U, V), (A, B) ⊆ (U, V) and (C, D)⊆(U, V). Since (A, B) is mαg-binary closed, b-cl(A,B)⊆(U,V) and hence $((A,B)^{1*}, (A,B)^{2*}) \subseteq (U,V)$. Consequently $((A, B)^{1*} \cap C, (A, B)^{2*} \cap D) \subseteq (U, V)$.

5. VARIOUS SPACES ASSOCIATED WITH mαg-BINARY CLOSED SET

In this section we introduce binary ${}_{m\alpha g}T_c$ space, binary ${}_{m\alpha g}T_\alpha$ space, ${}_{m\alpha g}T_r$ space, ${}_{m\alpha g}T_{\alpha g}$ space and discussed their relationships among them.

Definition 5.1: A binary topological space (X, Y, M) is said to be
(i) binary ${}_{m\alpha g}T_c$ space if every space if every mαg-binary closed set is binary closed set in it.
(ii) binary ${}_{m\alpha g}T_\alpha$ space if every mαg-binary closed subset is binary α-closed in it.
(iii) binary ${}_{m\alpha g}T_r$ space if every mαg-binary closed subset is binary regular closed in it.
(iv) binary ${}_{m\alpha g}T_{\alpha g}$ space if every mαg-binary closed subset is αg-binary closed in it.

Theorem 5.2: Every binary ${}_{m\alpha g}T_r$ space is binary ${}_{m\alpha g}T_c$ space but not conversely

Proof: Let (X, Y, M) be a binary ${}_{m\alpha g}T_r$ space, (A, B) is mαg-binary closed set in (X, Y, M). Since (X, Y, M) is a binary mαg–T_r space, By proposition 3.7 (A, B) is binary regular closed in (X, Y, M). Also by proposition 3.2 **[4]** every binary regular closed set is binary closed, (A, B) is binary closed in (X, Y, M) and hence (X, Y, M) is a binary ${}_{m\alpha g}T_c$ space.

The converse of the theorem need not be true as seen from the following example.

Example 5.3: Let X={0, 1},Y={a, b, c}, M ={(φ, φ),({0},{a}),({1},{b}),(X,{a, b}),(X,Y)} is a binary topology from X to Y. Also,{(φ, φ),({0},{a, c}),({1},{b, c}),(φ,{c}),(X,Y)} are binary closed sets in (X,Y, M). In this (X, Y, M), In this (A, B) = {({0}, {b, c})} is binary ${}_{m\alpha g}T_c$ space but not binary mαg–T_r space.

Theorem 5.4: Every binary ${}_{m\alpha g}T_c$ space is binary ${}_{m\alpha g}T_\alpha$ space but not conversely

Proof: Let (X, Y, M) be a binary ${}_{m\alpha g}T_c$ space, (A, B) is mαg-binary closed set in (X, Y, M). Since (X, Y, M) is a binary ${}_{m\alpha g}T_c$ space, By proposition 3.7 (A, B) is binary closed in (X, Y, M). Also by proposition 3.5 every binary closed set is binary α-closed, (A, B) is binary α-closed in (X, Y, M) and hence (X, Y, M) is a binary ${}_{m\alpha g}T_\alpha$ space.

The converse of the theorem need not be true as seen from the following example.

Theorem 5.5: Every binary ${}_{m\alpha g}T_r$ space is binary ${}_{m\alpha g}T_\alpha$ space but not conversely.

Proof: Let (X, Y, M) be a binary ${}_{m\alpha g}T_r$ space, Since (X, Y, M) be a binary ${}_{m\alpha g}T_r$ space , by proposition 3.7, (A, B) is binary regular closed in (X, Y, M) and by proposition 3.5 every binary regular closed set is binary α-closed set in (X, Y, M),Therefore (A, B) is binary α-closed set in
(X, Y, M).Hence (X, Y, M) is a binary ${}_{m\alpha g}T_\alpha$ space.

The converse of the theorem need not be true as seen from the following example.

Example 5.6: Let X={0,1},Y={a, b, c}, M ={(φ, φ),({0},{a}),({1},{b}),(X,{a, b}),(X,Y)} is a binary topology from X to Y. In this topological space (X, Y, M), (A, B) = {(X, {b, c})} is binary ${}_{m\alpha g}T_\alpha$ space but not binary mαg–T_r space.

Theorem 5.7: Every binary ${}_{m\alpha g}T_\alpha$ space is binary ${}_{m\alpha g}T_{\alpha g}$ space but not conversely

Proof: Let (X, Y, M) be a binary ${}_{m\alpha g}T_\alpha$ space, Since (X, Y, M) be a binary ${}_{m\alpha g}T_\alpha$ space , by proposition 3.5, (A,B) is binary α-closed in (X, Y, M) and by proposition 3.7 every binary α-closed set is binary mαg-closed set in (X, Y, M),Therefore (A,B) is mαg-binary closed set in (X, Y, M).Hence (X, Y, M) is a binary ${}_{m\alpha g}T_{\alpha g}$ space.

The converse of the theorem need not be true as seen from the following example.

Example 5.8: Let X={a,b},Y={1,2,3}, M ={(φ, φ),({a},{1}),({b},{2}),(X,{1,2}),(X,Y)} is a binary topology from X to Y. Here (A, B)=({a},{1,2}) is mαg-binary closed but not αg-binary closed set.

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