

SOLVING PROBLEM USING THE TOWER OF HANOI: A GOUNDED THEORY

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ABSTRACT:

This paper explained the process of generating the Honrales Solving Problem Theory with Puzzle. It shed light on how the students behaved and learned in solving the Tower of Hanoi puzzle. A Grounded theory approach was utilized to investigate the phenomenon of interest in which the verification used was in-depth interview in semi-structured type given to the seventeen students. Results showed that if a task is repeatedly done, patterns followed and rules are memorized, learning experiences are meaningful and useful. It can now be concluded that repetition, mastery, memorization, positive disposition and an interactive learning environment are basic skills needed in learning problem solving. It is recommended that teaching and learning process should be enhanced by including games and puzzles in the learning environment of the students.

KEYWORDS: Tower of Hanoi, learning experience, Grounded Theory, Theory Generation, Honrales Solving Problem Theory

I. INTRODUCTION

Education is considered as a driving force to change for the better (Prahmana, 2010). In order for change to take place, quality education must engage students to actively learn and directs the formation of values needed by students in their academic journey. Simmers (2011) says that mathematics is often experienced as difficult. Mathematics is one of the very important auxiliary science in everyday life as well as in supporting the advancement of science and technology (Case, 1992; Gooding, 2009; Seifi, et al, 2012). Abdurrahman (2009) suggests several reasons why mathematics should be taught to students, such as: (1) is always used in all facets of life; (2) all fields of study require appropriate math skills; (3) is a powerful means of communication, clear and concise; (4) can be used to present information in a variety of ways; (5) improve the ability to think logically, accuracy and spatial awareness; (6) to give satisfaction to the efforts to solve challenging problems. Mathematical solving problem ability is a major part of the learning objectives to be achieved in mathematics. NCTM (Pehkonen et al, 2013) suggests that the problem-solving is defined as a teaching method that can improve the quality of teaching mathematics in schools. The importance of solving the problem is also expressed by Beigie (2008) which says that through problem-solving, students can learn about deepening their understanding of mathematical concepts by working through the issues carefully selected which use the application of mathematics to real problems. The development of mathematical problem solving ability can equip students to think logical, analytical, systematic, critical, and creative. Some indicators of mathematical problem-solving ability by NCTM (in Widjajanti & Wahyudin, 2011; Novita, Zulkardi, & Hartono, 2012) are as follows: (1) identify the elements that are known, were asked, and the adequacy of the required elements; (2) formulate a mathematical problem or to develop a mathematical model; (3) implement strategies to solve the problem (and the kind of new problems) inside or outside of mathematics; (4) explain or interpret the results according the problem of origin; (5) using mathematical significantly. Besides the amount of research in the cognitive aspects in recent years began to affective aspects studied, among others, self-confidence (confidence) that are expected to increase students' mathematical problem-solving ability. McPheat (2010) concluded that confidence can proceed on the belief that a person has the ability to succeed in a task, based on the presence or not they have been able to do that task earlier. Someone with confidence to have confidence that they will be able to recover, reduce negative attitudes, and experience a positive attitude. Meanwhile, according to Hendriana (2012), there are several characteristics to assess the confidence of individuals, such as: believe in their own abilities, to act independently in making decisions, have a positive self-concept, and the courage to express opinions. Solving problem is a skill every student should master. This is one of the reasons why problem solving activities in school are introduced in order to prepare students in solving real-life problems.

How can teachers make learning Mathematics fun, exciting and enjoyable while at the same time making use of the students' mathematical muscles in a more creative way. An alternative solution could be games, puzzles, riddles, strategic games such as chess, math puzzles like Sudoku because all of these activities revolve around mathematical concepts and can be considered in learning problem solving. Tower of Hanoi is a puzzle where one can learn problem solving by using it. It is introduced as a problem solving activity in GE 3 Course (Mathematics in the Modern World). In playing the puzzle, the lived experiences of the students were uncovered which leads to a theory generation which is the focus of this study.

II. LITERATURE REVIEW

People of different ages played games and puzzles for centuries. They are interwoven throughout history and are fun, entertaining and useful. The Tower of Hanoi puzzles is one of them. This puzzle has been used for number of decades (Simons, 1975). It was invented by a French mathematician Edouard Lucas in 1883. The Tower of Hanoi is a well-known problem-solving task that has been used many times in experimental setting (Anderson, 2001). This puzzle consists of a search process within a problem space about which the problem-solver has very little specific domain knowledge (Newell, 1972). One study of the Tower of Hanoi carried out a detailed analysis of strategies employed to solve the puzzle and an in-depth study of one person solving a five disk, four times during a one and a half-hour period. It was found that each time this participant attempted the puzzle, she adopted a different strategy that was more efficient than the previous one (Anzai, 1979).

In recent years, research has provided evidence that brain exercises help stave of the dementia and Alzheimer's disease that are so common in old people. Solving puzzles tend to contribute to one's self esteem. For many people, their expertise in solving puzzles plays a role in their social interaction with other people. The mind learns by developing and storing patterns. As problem solving is done or accomplished, stored patterns of data, information, knowledge, and wisdom are used. In order to solve a problem, it is essential that an individual understands the problem. Problem solving begins with construction of a mental representation of the problem (Robertson, 2001).

III. STATEMENT OF OBJECTIVE

The aim of this study was to generate a theory on the learning experiences of the students in solving problem using the Tower of Hanoi which can shed light on: (1) how did the students learn and behave while solving the Tower of Hanoi; (2) how did they master playing the puzzle? (3) how did they feel after learning problem solving using the Tower of Hanoi?

Research Design: Grounded Theory

The Grounded theory using the Glasserian approach (1978), a qualitative research method, was utilized to investigate the phenomenon of interest in this preliminary study to generate the theory using inductive method on the learning experiences of the students in problem solving using the Tower of Hanoi. (Abadiano, 2016)

THEORY GENERATING-INDUCTIVE METHOD

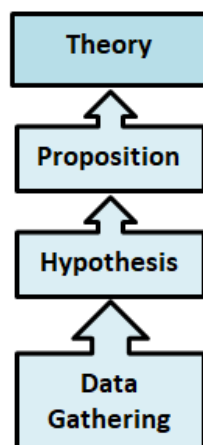


Figure 1: Research Process For The Theory Generation

Inductive reasoning moves from specific observations to broader generalizations and theories. This is sometimes called a "bottom up" approach in inductive reasoning. It begins with specific observations and measures, detect patterns and regularities, formulate some tentative hypotheses that can be explored, and then end up developing theories.

Research Environment

The study was conducted in Samar State University, Catbaogan City, Samar. The only State University in the western part of Samar. It has four campuses namely: Main Campus, Mercedes Campus, Paranas Campus and Basey Campus. The present study was conducted in the main campus.

Research Participants/Informants

Purposive sampling was used to identify the samples. The researcher, a professor of Samar State University, tapped seventeen (17) College Freshmen students from the Bachelor of Science in Information Technology (BSIT) during the School Year 2019-2020. The selection criteria were as follows: a.) The student should have taken the GE 3 Course, and, b.) should know how to solve the puzzle.

Research Sampling

Seventeen (17) participants were interviewed among the BSIT students reaching to the point of saturation. She conducted an interview schedule with an open-ended questions pre-arranged to the participants. In order to supplement the study, the researcher observed the way the students solved the Tower of Hanoi and recorded the observations in a memo pad. This was to gather data on the mathematical concepts applied by the students in completing the puzzle. She also invited other mathematics instructors to do the observation to enhance the observation previously made by the researcher.

Research Instrument

The researcher utilized a semi-structured interview schedule consisting of two parts – Part A, demographic information and Part B, semi-structured questions.

The interview guide was constructed in two versions: English and Waray version in order to be easily understood by the informants. Content validity of the constructed interview questions was assessed by the subject expert and the research expert. The applicability of interview schedule was tested through pilot study with ten participants in the university.

IV. DATA GATHERING

Data Collection: The researcher, the students’ instructor in GE 3 (Mathematics in the Modern World) conducted a semi-structured interview with the seventeen (17) BSIT College Freshmen students of the College of Arts and Sciences. Participants were contacted through snowball sampling or by word of mouth. All the participants were briefed about the study and written informed consents were solicited for their participation and the use of audio/video recorders to record the interview more accurately. Based on the answers and feedback from the students, the researcher revised and refined his guide questions.

Coding and Categorizing Data: In this stage, the researcher listened to all the transcribed interview data in the audiotapes of her cellphone several times before the transcription to be more immersed and be able to capture the meanings the participants wanted to convey. This stage, the line-by-line reading was done immediately after the initial interview with the participants. Maximum codes were constructed about concepts related to the learning experiences of the students in solving problem using the Tower of Hanoi, where some concepts were similar to others while others were not. Themes were formulated through the progress of line-by-line codings and integration of sub-categories through constant comparison, modification and analysis of concepts.

Theoretical Sampling, Constant Comparison. In this stage, the interview was guided based on developed concepts and sub-categories from the findings of preliminary data gathering. Participant sampling was directed by emerging constructs and categories. Throughout the data gathering and analysis period, emerging categories or themes were constantly compared with each other, similar concepts were integrated and contrasting categories were further explored to identify the solid concepts related to the study topic. Constant comparative analysis was the primary strategy in the integrated coding and analyzing stages of grounded theory (Duchscher and Morgan, 2004). The goal was to clarify concepts and test hypotheses derived from the data while producing precise descriptions (Jeon, 2004). The making of constant comparisons during data analysis and collection, and theoretical sampling occurred simultaneously in order to ensure that the researcher could actually construct a theory that was grounded in the data (Jeon, 2004).

Memo Writing: The researcher started also to write his memo, reflections, analysis of the initial interview and continued until the theory generation period. He jotted down his analysis on relationship, variation, links between basic concepts, codes and categories, as well as observations, the investigator’s own logic, critical analysis and reflection on categories, the process of integrating the concepts, emerging theoretical categories and core category in the memo. Memo writing is important in grounded theory where the investigator expressed his observations,

remarks, reflections, his own ideas, rationale, evaluation and reflections on the phenomena during the study period. If the investigator missed this step and jumped to theoretical writing, then it was missed out and some doubts/questions on the validity would naturally surface.

In the present study, the investigator used memos as reference to the concepts that emerged from coding and constant comparative analysis of data, selection of number of participants for theoretical sample, development of a focused interview questionnaire for the theoretical sampling, matching of sub-categories and contrast categories to develop theoretical categories and core category, and validate these categories of the study phenomena. Glaser (1978) also suggested that the writing of theoretical memos, which are written reflections of thinking, is the core stage in the process of generating theory, and that if the analyst skips this stage by going directly from coding to sorting or to writing, then he is not doing grounded theory.

Theoretical Sensitivity: It was assumed in this preliminary theory generation process that the study of the phenomenon was presented by empirical evidence through interview and observations directly from study participants. Data were analyzed, coded, categorized into emerging themes, using field notes, and memoing of students' information sharing on the learning experiences throughout the data collection process and through the review of related literature. According to Glaser (1992), theoretical sensitivity refers to the researcher's knowledge, understanding, and skill, which foster the generation of categories and properties and increase the researcher's ability to relate them into hypotheses, and to further integrate the hypotheses, according to emergent theoretical codes. Glaser believed that theoretical sensitivity is attained through immersion in the data, line by line, comparison by comparison, memo by memo, and code by code (Walker & Myrick, 2006).

Ethical Considerations: The researcher gathered the informed participants and assured the consenting participants of strict confidentiality, following the ethical principles during the study period to formulate a preliminary theory on the learning experiences of the students in solving problem using the Tower of Hanoi. The researcher likewise asked for their consent to audiotape the interview session.

Rigors of the Study: In the present study, the investigator utilized the Lomborg and Kirkevold (2003) criteria to maintain the rigors of the study which comprised the concepts of fit, work, relevance and modifiability. The Investigator selected the study participants who met the inclusion criteria and who were to participate to share their own learning experiences.

V. DATA ANALYSIS-THEMATIC APPROACH

Thematic analysis is a method for identifying, analysing, and reporting patterns within data. It minimally organises and describes your data set in detail. However, it also often goes further than this, and interprets various aspects of the research topic (Boyatzis, 1998). The range of different possible thematic analyses will further be highlighted in relation to a number of decisions regarding it as a method. Thematic analysis is widely used, but there is no clear agreement about what thematic analysis is and how you go about doing it (Attride-Stirling, 2001; Boyatzis, 1998; Tuckett, 2005, for other examples). It can be seen as a very poorly "branded" method, in that it does not appear to exist as a "named" analysis in the same way that other methods do (e.g., narrative analysis, grounded theory). In this sense, it is often not explicitly claimed as the method of analysis, when, in actuality, we argue that a lot of analysis is essentially thematic - but is either claimed as something else (such as discourse analysis, or even content analysis (e.g., Meehan, Vermeer, & Windsor, 2000)) or not identified as any particular method at all – for example, data were "subjected to qualitative analysis for commonly recurring themes" (Braun & Wilkinson, 2003: 30). If we do not know how people went about analysing their data, or what assumptions informed their analysis, it is difficult to evaluate their research, and to compare and/or synthesise it with other studies on that topic, and it can impede other researchers carrying out related projects in the future (Attride-Stirling, 2001). For these reasons alone, clarity around process and practice of method is vital. It is hoped that this paper will lead to more clarity with regard to thematic analysis.

Chapter 2

Theory Generation Process

Results and Discussion

Grounded Theory research method is an exact opposite of traditional social science research. The first step is data collection, rather than starting it with a hypothesis, through a variety of methods. From the data gathered, the key points are marked with a series of codes, which are extracted from the text. The codes are grouped into similar concepts in order to make them more workable. From these concepts, themes are formed, which are the basis for the creation of a theory (Allan, 2003).

Study for Theory Development

The results of preliminary study for theory development were presented as follows: demographic profile of participants, findings of the learning experiences of the students in solving problem using the Tower of Hanoi presented in theoretical themes or categories.

Demographic information: As shown in Table 1, the total study participants consisted of seventeen (17) College Freshmen students. There were nine males and eight females. The participants’ ages ranged from 18 to 20. They are all single and only one belongs to the Protestant. All of the participants are BSIT students who can solve the puzzle completely.

Demographic Information

Table 1: Demographic Information of Preliminary Study Participants

Participant	Age	Sex	Civil Status	Education	Religion
1	18	M	Single	Student	Roman Catholic
2	19	F	Single	Student	Roman Catholic
3	18	M	Single	Student	Roman Catholic
4	18	F	Single	Student	Protestant
5	19	M	Single	Student	Roman Catholic
6	20	M	Single	Student	Roman Catholic
7	18	F	Single	Student	Roman Catholic
8	19	F	Single	Student	Roman Catholic
9	18	M	Single	Student	Roman Catholic
10	19	F	Single	Student	Roman Catholic
11	19	F	Single	Student	Roman Catholic
12	20	M	Single	Student	Roman Catholic
13	18	M	Single	Student	Roman Catholic
14	19	F	Single	Student	Roman Catholic
15	19	M	Single	Student	Roman Catholic
16	18	F	Single	Student	Roman Catholic
17	19	M	Single	Student	Roman Catholic

Solving the Tower of Hanoi - Learning Experiences, Feelings, Views, and Reactions

The participants shared their learning experiences, feeling, views and reactions regarding the use of the Tower of Hanoi in solving problems. From the one-to-one interview, the following comments were used in formulating and categorizing codes of the study.

Participant 1 – “...at first I was confused, but when I recalled the rules, I was able to complete the puzzle.”

Participant 2 – “I only followed what you taught me at first...but as I continued to play I discovered that it is easier to solve the TOH when the position is altered.”

Participant 3 – “I followed that there should be no bigger chips over a smaller one.”

Participant 4- “I mastered playing the odd number of chips more than the even chips.”

Participant 5 – “I felt a sense of enjoyment and satisfaction because I was able to solve the puzzle”

Participant 6 – “...I think I was addicted to the puzzle that I kept on playing it again and again. It was fun playing the puzzle”

Participant 7- “...just kept on playing to master the rules and patterns.” “I continued using the pattern until the end of the game...and one move only every time”.

“The thing I considered is the color, aside from the two rules –one move at a time and no bigger disk over a smaller disk...the same colors should not be placed together, yes, that’s what I did ma’am”

Participant 8 – “...It is fun and addictive just like a computer game, I kept on solving the puzzle repeatedly until I mastered the pattern”

Participants 9 – “...I adopted the strategies of altering the position so that my hands will not cross over...I followed also the pattern in solving”

Participant 10 – “I discovered that to obtain the minimum moves when the chips of the TOH is even, it has a different sequence from the TOH with odd chips...because the movement of the smallest chips is to the center then to the last dowel and back to the first dowel and I kept on repeating this sequence until the puzzle is solved.”

Participant 11 – “...I felt proud of myself and grateful to our teacher who shared this thing to us and taught us how to solve the puzzle. I enjoyed a lot!”

Participant 12 – “...It became my stress reliever whenever I’m stressed in school and I had fun playing it”

Participant 13- “...just kept on playing to master the rules and patterns.”

Participant 16 - “...I remembered/memorize the sequence of the game and played again and again.”

Participant 17- “The number of minimum moves to solve the puzzle can be determined even without playing the puzzle but by using the rule $2^n - 1$, where n is the number of chips in the realia.”

Categorizing, Coding and Themes

Using the Husserlian Method of Data Analysis and Collaizzis’ Thematic Process, the researcher anchored the steps, and just like Collaizzis who asserted that all research occurred through dialogue and that asking the right questions was tantamount to eliciting an accurate description of the experience from study participants (Collaizzi, 1978). On the other hand, Husserl’s focus was on meanings and identifying the essence or central theme of an experience as a way of furthering knowledge.

A descriptive analysis of transcripts revealed that four (4) major themes characterized the participants’ responses the learning experiences in using the puzzle. The four major themes were labeled using direct quotations from the participants; this was done to express the original idea conveyed by the participants.

Themes, however, are not to be interpreted as independent from each other but as complementary aspects of a unified pattern. The four (4) themes as deduced from the transcripts of the participants are:

Theme 1: Active Learning

Theme 2: Procedural Learning

Theme 3: Repetitive Learning

Theme 4: Enjoyable Learning

Theme 1. Active Learning

Playing the Tower of Hanoi is considered an active learning because it is based on the premise that in order to learn, students must do more than just listen. They must engaged in solving problems. The puzzle engages students on doing things and thinking about the things they are doing.

Participant 1 – “...at first I was confused, but when I recalled the rules, I was able to complete the puzzle.”

Participant 3 – “I followed that there should be no bigger chips over a smaller one.”

Participant 7 - “I continued using the pattern until the end of the game...and one move only everytime”

Theme 2. Procedural Learning

Doing the puzzle requires a lot of procedure like following the pattern or sequence and memorizing the next move to use. It involves the acquisition of motor skills and habits, and certain types of cognitive skills. In solving the Tower of Hanoi, the task can be completed through a procedure to be demonstrated or performed by the students.

Participant 2 – “I only followed what you taught me at first...but as I continued to play I discovered that it is easier to solve the TOH when the position is altered.”

Participant 7 – “The thing I considered is the color, aside from the two rules –one move at a time and no bigger disk over a smaller disk...the same colors should not be placed together, yes, that’s what I did ma’am”

Participants 9 – “...I adopted the strategies of altering the position so that my hands will not cross over...I followed also the pattern in solving”

Participant 10 – “I discovered that to obtain the minimum moves when the chips of the TOH is even, it has a different sequence from the TOH with odd chips...because the movement of the smallest chips is to the center then to the last dowel and back to the first dowel and I kept on repeating this sequence until the puzzle is solved.”

Participant 17 – “The number of minimum moves to solve the puzzle can be determined even without playing the puzzle but by using the rule $2^n - 1$, where n is the number of chips in the realia.”

Theme 3. Repetitive Learning

Real learning does not usually occur in one-time events. It is something that is repeated, practiced, and honed. The Tower of Hanoi provides the practice that students need to master new skills. Repetition helps to improve speed, increases confidence, and strengthens the connections in the brain that help students learn. Through repetition, the students mastered playing the Tower of Hanoi.

Participant 8- “I kept on solving the puzzle repeatedly until I mastered the pattern”

Participant 4- “I mastered playing the odd number of chips more than the even chips.”

Participant 13- “...just kept on playing to master the rules and patterns.”

Participant 16 - “...I remembered/memorize the sequence of the game and played again and again.”

Participant 7- “...just kept on playing to master the rules and patterns.”

Theme 4. Enjoyable Learning

Enjoyable learning is one that captures the attention of students. Tower of Hanoi promotes meaningful, active learning because the students find it enjoyable and fun.

Participant 5 – “I felt a sense of enjoyment and satisfaction because I was able to solve the puzzle”

Participant 6 – “...I think I was addicted to the puzzle that I kept on playing it again and again. It was fun playing the puzzle”

Participant 12 – “...It became my stress reliever whenever I’m stressed in school and I had fun playing it”

Participant 11 – “...I felt proud of myself and grateful to our teacher who shared this thing to us and taught us how to solve the puzzle. I enjoyed a lot!”

Participant 8 – “...It is fun and addictive just like a computer game.

Participant 14 – “...I love to play the puzzle again and again...it is addictive”

Hypotheses Derived from the Results:

Sub-categories and the theoretical category of information sharing behavior were considered. Going into narrative interview analysis, the researcher was able to generate the following hypotheses to explore the learning experiences of the students in problem solving using the Tower of Hanoi. In grounded theory, hypothesis generation was always derived from the empirical data. The main method of forming a hypothesis in this study involved interviews with the purposive participants. This means forming questions appropriate to the study. In each hypothesis generation, the investigator asked one central interview question and a few guided questions which are described below.

Generation of hypothesis 1: The study participants were asked to share their learning experiences in using the Tower of Hanoi in solving problem activity.

Hypothesis

H1: If a thing is done, then it is retained in the mind

H2: If a task is repeatedly done, then it is mastered

H3: If a procedure is followed, then it will complete the task accurately

H4: If a task is enjoyable and fun, then it leaves a lasting and meaningful impression on the learner.

Propositions

- P1: Hands-on activities are basic skills in problem solving.
- P2: Repetition and mastery lead to a successful problem solving activity.
- P3: Following procedure is a way to solving the problem accurately.
- P4: A satisfied and contented person is a happy person.

Theory

Honrales Solving Problem Theory states that “repetition, accuracy, and with the aid of puzzle coupled with a positive disposition are basic foundation and necessary skills needed for a successful problem solving activities”.

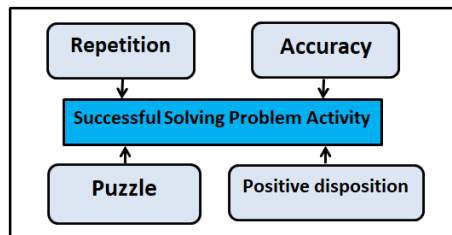


Figure 1: Conceptual Model of Honrales Solving Problem Theory

Figure 1 explains that in order to attain successful problem solving activity, one must master skills in mathematics like repetition, accuracy, with the aid of puzzle and positive disposition to make it more meaningful and long lasting. In order to attain the maximum learning skills in problem solving, repetition of knowledge-based discussion is needed. There should be a repetitive drill and mastery of the topics. Whereas, accuracy entails the students to enhance memory learning in basic knowledge level. Accuracy means the use of acquired knowledge to application and there is a correctness of the acquired and learned knowledge. The aid of puzzle make the solving problem activity more meaningful. Positive disposition also play a vital role in the successful problem solving activity wherein, the students inhibits positive attitude such as being contented, satisfied along their academic performance. Further, it emphasizes the positive outlook and motivation of the students towards their learning process. There are various studies that prove that positive attitude and motivation could help greatly in the success of the academic performance of the majority of the students.

VI. CONCLUSIONS AND RECOMMENDATION

The use puzzle is one of the basic tools to a successful solving problem activity. Other factors are repetitive tasks, accuracy in performing the task and more importantly, a positive attitude among the learners.

It is therefore highly recommended that Tower of Hanoi puzzle be part of the solving problem activities in the teaching-learning process. It should be introduced to both college and high school students as a solving problem task. Furthermore, the study may serve as a baseline for future researches like exploring the learning experiences of the students in solving the Tower of Hanoi.

Further, the findings of this study may also provide data for future intervention and policies, and an offshoot for further researches and more theory development along teaching and learning processes.

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