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Doubly Truncated of Mixture Topp-Leone and Exponential Distribution

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Abstract
In this paper, the double truncated distribution has been found for a combined distribution of Topp-Leon generated with exponential distribution(TLG-E), and symbolize of double distribution (DT-TLGE). obtained for doubly truncated distribution are functions CDF, PDF reliability, hazard and moment. These functions are illustrated with graphs, since double truncated was evident through graphics. also derived two methods for estimating moment method and maximum likelihood method.

Keywords: LT-TLGE, TLG-E, PDF, Moment function, Reliability, Hazard.

1. Introduction

Distribution Topp-Leone was discovered by researchers Topp C. W. & Leone F. C. (1955) and found isolated in Journal of the American Statistical Association. It was rediscovered by researchers Nadarajah, S., & Kotz, S. (2003). In a previous research, we combined it with the exponential distribution and obtained CDF and PDF functions
As shown below

$$F(\rho) = \left\{\left(1-e^{-\lambda\rho}\right)^{\theta}\left(2-\left(1-e^{-\lambda\rho}\right)^{\theta}\right)\right\}^{\beta} \quad \dots\dots\dots\dots (1)$$

Can be writing as

$$F(\rho) = \sum_{k=0}^{\infty}\binom{\beta}{k}2^{\beta-k}\left(1-e^{-\lambda\rho}\right)^{\theta(\beta+k)} \quad \dots\dots\dots\dots (2)$$

Has a derivation of a relationship (1) that results a PDF

$$f(\rho) = 2\beta\,\theta\lambda e^{-\lambda\rho}\left(1-e^{-\lambda\rho}\right)^{\beta\theta-1}\left(2-\left(1-e^{-\lambda\rho}\right)^{\theta}\right)^{\beta-1}\left(1-\left(1-e^{-\lambda\rho}\right)^{\theta}\right) \quad \dots\dots\dots\dots (3)$$

Let parameters $\beta, \theta, \lambda > 0$ and random variable $\rho > 0$. β, θ are parameters location and λ is scale.

Also has a derivation of a relationship (2) that results a PDF

$$f(\rho) = \sum_{k=0}^{\infty}\binom{\beta}{k}2^{\beta-k}\,\theta\lambda(\beta+k)e^{-\lambda\rho}\left(1-e^{-\lambda\rho}\right)^{\theta(\beta+k)} \quad \dots\dots\dots\dots (4)$$

This distribution has wide applications we applied for it double truncated

2. Doubly Truncated of TLG-E distribution (DT-TLGE)

Let ρ be a random variable of doubly truncated TLGE distribution on support $\mathfrak{a} \le \rho < \mathfrak{b}$ .in this case it is truncated on the doubly sides, the CDF is the follows

$$F_{\mathcal{D}}(\rho; \mathfrak{a} \le \rho < \mathfrak{b}) = \frac{F(\rho)-F(\mathfrak{a})}{F(\mathfrak{b})-F(\mathfrak{a})} \text{ from (2)}$$

$$= \frac{\sum_{k=0}^{\infty}\left(1-e^{-\lambda\rho}\right)^{\theta(\beta+k)}-\sum_{k=0}^{\infty}\left(1-e^{-\lambda\mathfrak{a}}\right)^{\theta(\beta+k)}}{\sum_{k=0}^{\infty}\left(1-e^{-\lambda\mathfrak{b}}\right)^{\theta(\beta+k)}-\sum_{k=0}^{\infty}\left(1-e^{-\lambda\mathfrak{a}}\right)^{\theta(\beta+k)}} \quad \dots\dots\dots\dots (5)$$

And simplify it in another way

$$F_{\mathcal{D}}(\rho)=\frac{\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\left(e^{-\lambda h\rho}-e^{-\lambda h a}\right)}{\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\left(e^{-\lambda h b}-e^{-\lambda h a}\right)}=\frac{\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\left(e^{-\lambda h(\rho-a)}-1\right)}{\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\left(e^{-\lambda h(b-a)}-1\right)}=\frac{\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\left(1-e^{-\lambda h(\rho-a)}\right)}{\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\left(1-e^{-\lambda h(b-a)}\right)}$$

Since $\left(1-e^{-\lambda h(b-a)}\right)$ is a constant, suppose that $\frac{1}{\left(1-e^{-\lambda h(b-a)}\right)}=\mathcal{C}_{k,h}$

$$F_{\mathcal{D}}(\rho)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\mathcal{C}_{k,h}\left(1-e^{-\lambda h(\rho-a)}\right) \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

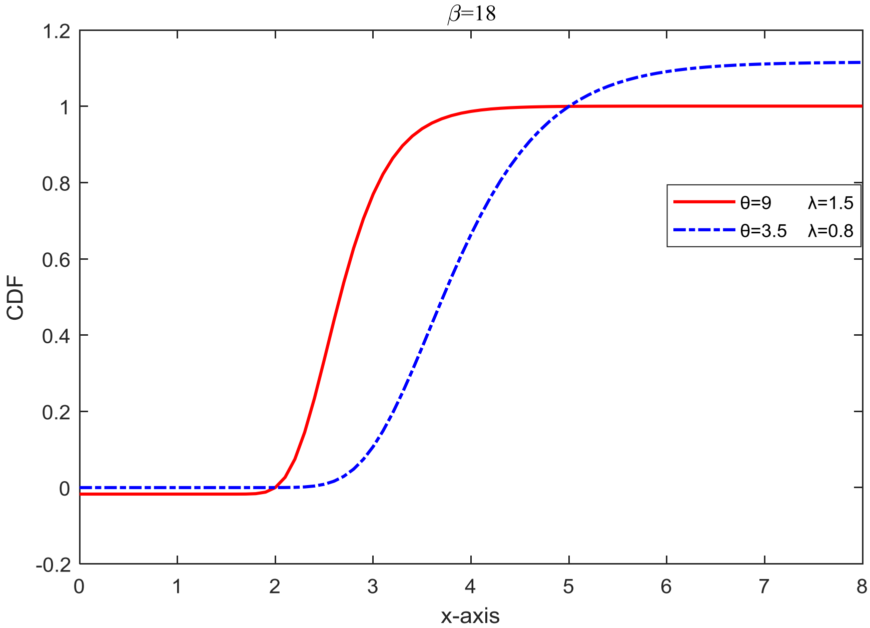

Figure (1) represents CDF of DT-TLGE, which is determined within the constrained period $2 \leq \rho < 5$ and values of parameters $\theta = 9, 3.5$, $\lambda = 1.5, 0.8$, and $\beta = 18$. It is also noticed that the curves intersection within truncated period, as well as anomalous and extreme values, to the left and right of the period.

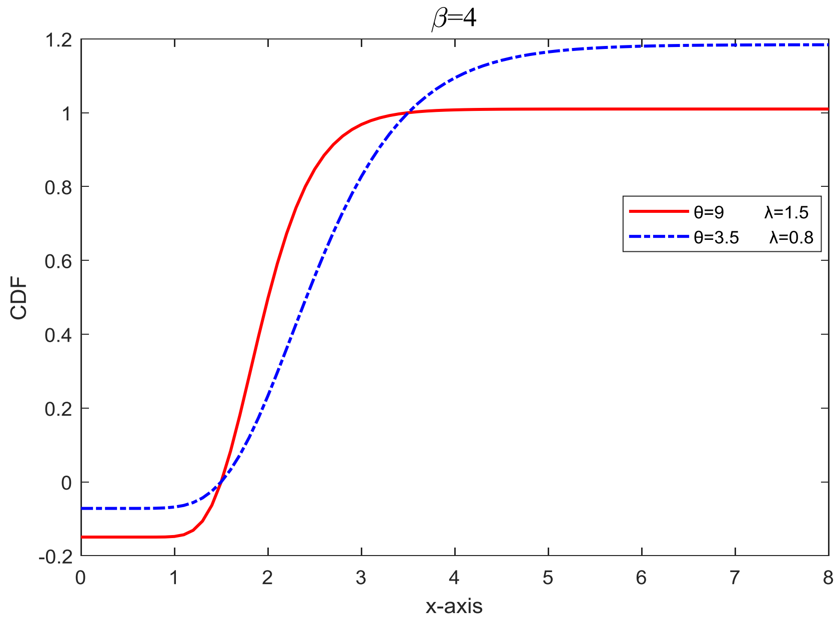

Figure (2) represents CDF of DT-TLGE, which is determined within the constrained period $1.5 \leq \rho < 3.5$ and values of parameters $\theta = 9, 3.5$, $\lambda = 1.5, 0.8$, and $\beta = 4$. It is also noticed that the curves intersection within truncated period, as well as anomalous and extreme values, to the left and right of the period.

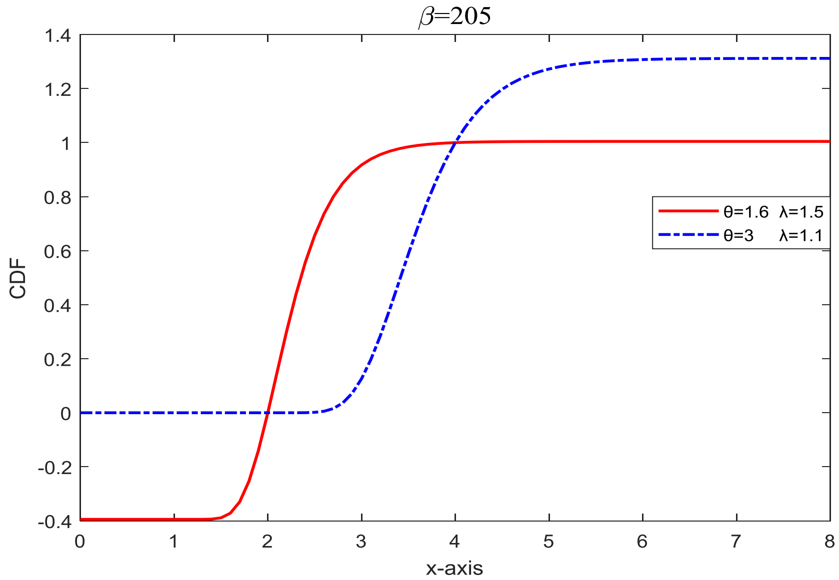

Figure (3) represents CDF of DT-TLGE, which is determined within the constrained period $2 \leq \rho < 4$ and values of parameters $\theta = 1.6, 3$, $\lambda = 1.5, 1.1$, and $\beta = 205$. It is also noticed that the curves intersection. within truncated period, as well as anomalous and extreme values, to the left and right of the period.

From (2) We find a PDF truncated doubly of TLGE and Symbolize it $\mathcal{T}_{\mathcal{D}}(\rho)$

$$\mathcal{T}_{\mathcal{D}}(\rho) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} \mathcal{C}_{k,h} \left(\lambda h e^{-\lambda h(\rho - a)}\right) \dots \dots \dots \dots \dots \dots (7)$$

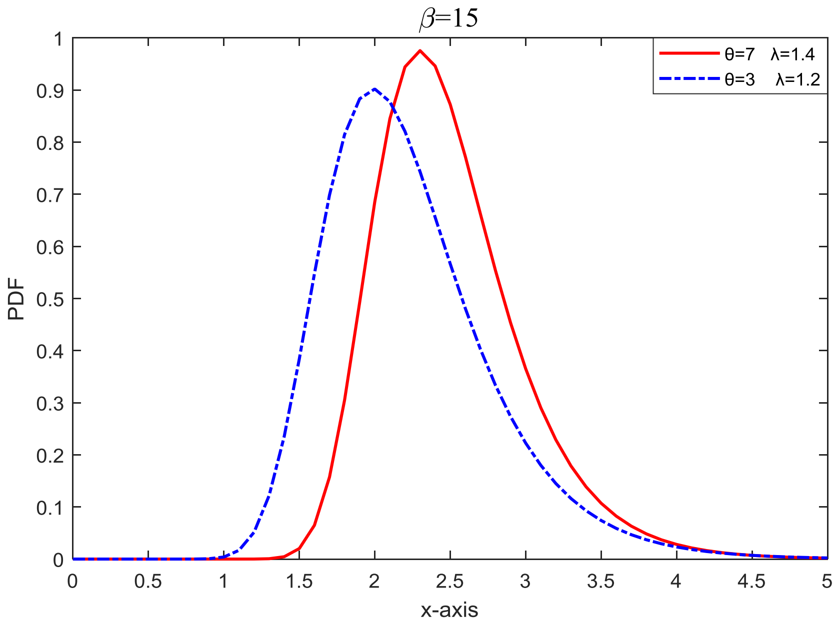

Figure (4) represents PDF of DT-TLGE, which is determined within the constrained period $1.5 \leq \rho < 3.5$ and values of parameters $\theta = 7, 3$, $\lambda = 1.4, 1.2$, and $\beta = 15$.

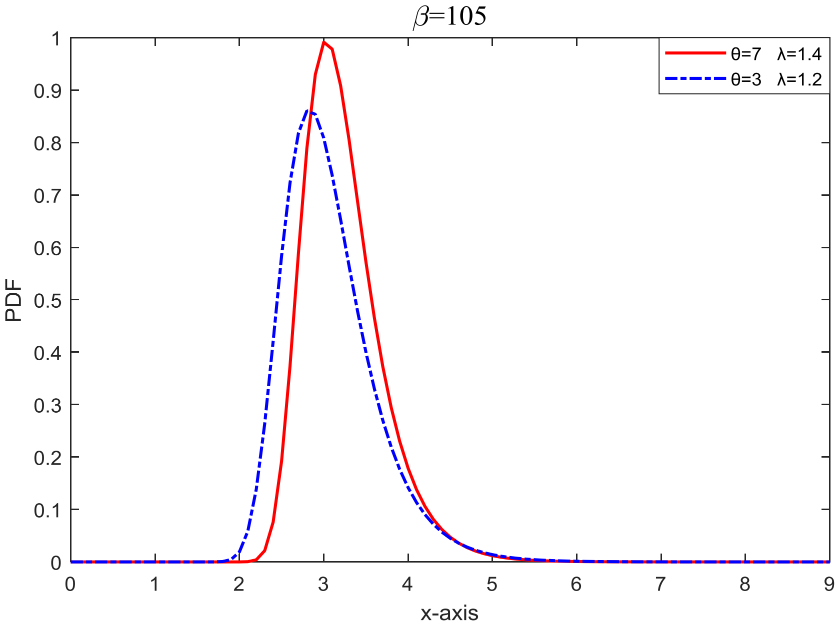

Figure (5) represents PDF of DT-TLGE, which is determined within the constrained period $2 \leq \rho < 5$ and values of parameters $\theta = 7, 3,\ \lambda = 1.4, 1.2$, and $\beta = 105$.

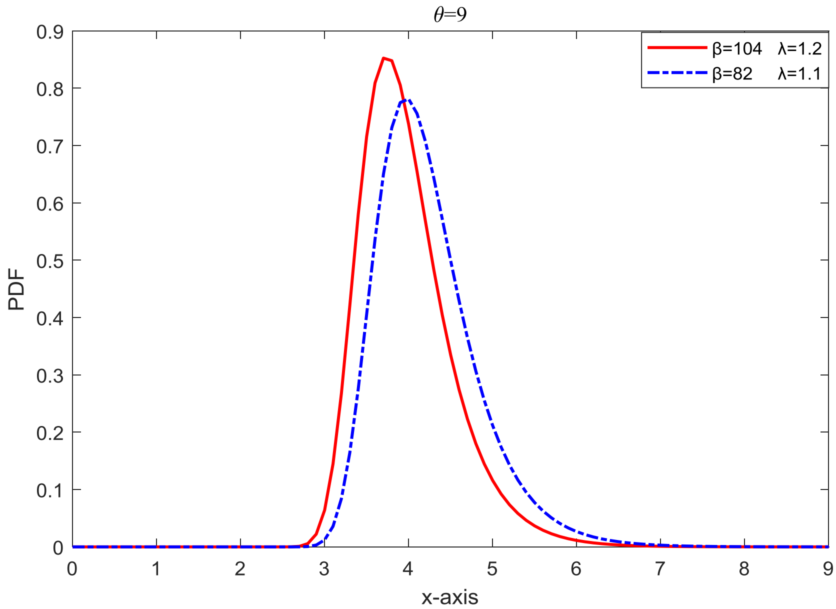

Figure (6) represents PDF of DT-TLGE, which is determined within the constrained period $3 \leq \rho < 6$ and values of parameters $\beta = 104, 82,\ \lambda = 1.2, 1.1$, and $\theta = 9$. Deduced through CDF and PDF figures

location parameters are directly proportional to the truncated distribution parameters.

3. Moment of DT-TLGE

$$\mu'_s = \int_a^b \rho^s \mathcal{T}_{\mathcal{D}}(\rho) d\rho$$

From (7) getting to

$$\mu'_s = \int_a^b \rho^s \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} \mathcal{C}_{k,h} \left(\lambda h e^{-\lambda h(\rho - a)}\right) dx$$

$$\mu'_s = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \mathcal{C}_{k,h}\, e^{-\lambda h \mathfrak{a}} \int_{\mathfrak{a}}^{\mathfrak{b}} \rho^s\, \lambda h e^{-\lambda h \rho} d\rho \qquad \dots\dots\dots\dots (8)$$

It is possible to find a solution $\int_{\mathfrak{a}}^{\mathfrak{b}} \rho^s\, \lambda h e^{-\lambda h \rho} d\rho$, by dividing the period and using one of the specified integration properties

$$\int_{\mathfrak{a}}^{\mathfrak{b}} \rho^s\, \lambda h e^{-\lambda h \rho} d\rho = \int_0^{\mathfrak{b}} \rho^s\, \lambda h e^{-\lambda h \rho} d\rho - \int_0^{\mathfrak{a}} \rho^s\, \lambda h e^{-\lambda h \rho} d\rho \quad \dots\dots\dots (9)$$

$$\int_0^{\mathfrak{b}} \rho^s\, \lambda h e^{-\lambda h \rho} d\rho = -\mathfrak{b}^s e^{-\lambda h \mathfrak{b}} + s\int_0^{\mathfrak{b}} \rho^{s-1}\, e^{-\lambda h \rho} d\rho$$

by incomplete lower gamma function then

$$\int_0^{\mathfrak{b}} \rho^s\, \lambda h e^{-\lambda h x} dx = \frac{s}{(\lambda h)^s}\gamma(s,\mathfrak{b}) - \mathfrak{b}^s e^{-\lambda h \mathfrak{b}}$$

in a similar way

$$\int_0^{\mathfrak{a}} \rho^s\, \lambda h e^{-\lambda h x} dx = \frac{s}{(\lambda h)^s}\gamma(s,\mathfrak{a}) - \mathfrak{a}^s e^{-\lambda h \mathfrak{a}}$$

Now, then(9) become

$$\int_{\mathfrak{a}}^{\mathfrak{b}} \rho^s\, \lambda h e^{-\lambda h \rho} d\rho = \frac{s}{(\lambda h)^s}\gamma(s,\mathfrak{b}) - \mathfrak{b}^s e^{-\lambda h \mathfrak{b}} - \frac{s}{(\lambda h)^s}\gamma(s,\mathfrak{a}) + \mathfrak{a}^s e^{-\lambda h \mathfrak{a}} \qquad \dots\dots\dots\dots (10)$$

Thus, (8) will be as follows

$$\mu'_s = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \mathcal{C}_{k,h}\, e^{-\lambda h \mathfrak{a}} \left[\frac{s}{(\lambda h)^s}\gamma(s,\mathfrak{b}) - \frac{s}{(\lambda h)^s}\gamma(s,\mathfrak{a}) + \mathfrak{a}^s e^{-\lambda h \mathfrak{a}} - \mathfrak{b}^s e^{-\lambda h \mathfrak{b}}\right] \quad \dots\dots\dots\dots (11)$$

Relationship (11) represents moment of distribution DT-TLGE.

4. Moment about mean of DT-TLGE

The moment generating function (m.g.f) of random variable ρ of DT-TLGE

4.1. Mean

To find the mean of (11) is when s = 1 That is $\mu'_1 = E(\rho)$

$$\mu'_1 = E(\rho) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \mathcal{C}_{k,h}\, e^{-\lambda h \mathfrak{a}} \left[\frac{1}{(\lambda h)^1}\gamma(1,\mathfrak{b}) - \frac{1}{(\lambda h)^1}\gamma(1,\mathfrak{a}) + \mathfrak{a}^1 e^{-\lambda h \mathfrak{a}} - \mathfrak{b}^1 e^{-\lambda h \mathfrak{b}}\right] \quad \dots\dots\dots\dots (12)$$

4.2. Variance

$\mu_2 = \mu'_2 - \mu'^2_1$

$$\mu_2 = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \mathcal{C}_{k,h}\, e^{-\lambda h \mathfrak{a}} \left[\frac{2}{(\lambda h)^2}\gamma(2,\mathfrak{b}) - \frac{2}{(\lambda h)^2}\gamma(2,\mathfrak{a}) + \mathfrak{a}^2 e^{-\lambda h \mathfrak{a}} - \mathfrak{b}^2 e^{-\lambda h \mathfrak{b}}\right] - \left\{\sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \mathcal{C}_{k,h}\, e^{-\lambda h \mathfrak{a}} \left[\frac{1}{(\lambda h)^1}\gamma(1,\mathfrak{b}) - \frac{1}{(\lambda h)^1}\gamma(1,\mathfrak{a}) + \mathfrak{a}^1 e^{-\lambda h \mathfrak{a}} - \mathfrak{b}^1 e^{-\lambda h \mathfrak{b}}\right]\right\}^2 \quad \dots\dots\dots\dots (13)$$

4.3. Skewness

$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2{\mu'_1}^3$

$$\mu_3 = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \mathcal{C}_{k,h}\, e^{-\lambda h \mathfrak{a}} \left[\frac{3}{(\lambda h)^3}\gamma(3,\mathfrak{b}) - \frac{3}{(\lambda h)^3}\gamma(3,\mathfrak{a}) + \mathfrak{a}^3 e^{-\lambda h \mathfrak{a}} - \mathfrak{b}^3 e^{-\lambda h \mathfrak{b}}\right] - 3\sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \mathcal{C}_{k,h}\, e^{-\lambda h \mathfrak{a}} \left[\frac{2}{(\lambda h)^2}\gamma(2,\mathfrak{b}) - \frac{2}{(\lambda h)^2}\gamma(2,\mathfrak{a}) + \mathfrak{a}^2 e^{-\lambda h \mathfrak{a}} - \mathfrak{b}^2 e^{-\lambda h \mathfrak{b}}\right] \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \mathcal{C}_{k,h}\, e^{-\lambda h \mathfrak{a}} \left[\frac{1}{(\lambda h)^1}\gamma(2,\mathfrak{b}) - \right.$$

$\frac{1}{(\lambda h)^1}\gamma(1,\mathfrak{a}) + \mathfrak{a}^1 e^{-\lambda h\mathfrak{a}} - \mathfrak{b}^1 e^{-\lambda h\mathfrak{b}}\Big] + 2\Big\{\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\mathcal{C}_{k,h}\, e^{-\lambda h\mathfrak{a}}\Big[\frac{1}{(\lambda h)^1}\gamma(1,\mathfrak{b}) - \frac{1}{(\lambda h)^1}\gamma(1,\mathfrak{a}) + \mathfrak{a}^1 e^{-\lambda h\mathfrak{a}} - \mathfrak{b}^1 e^{-\lambda h\mathfrak{b}}\Big]\Big\}^3$ ………… (14)

4.4. Moment Generated function of DT-TLGE

$$\mathcal{M}_\rho(t) = E(e^{t\rho}) = \sum_{s=0}^{\infty}\frac{t^s}{s!}\mu'_s$$

$$\mathcal{M}_\rho(t) = \sum_{s=0}^{\infty}\frac{t^s}{s!}\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\mathcal{C}_{k,h}\, e^{-\lambda h\mathfrak{a}}\left[\frac{s}{(\lambda h)^s}\gamma(s,\mathfrak{b}) - \frac{s}{(\lambda h)^s}\gamma(s,\mathfrak{a}) + \mathfrak{a}^s e^{-\lambda h\mathfrak{a}} - \mathfrak{b}^s e^{-\lambda h\mathfrak{b}}\right] \quad \text{………… (15)}$$

5. Reliability of DT-TLGE

The reliability function of doubly truncated can be represented as follows

$R_{\mathcal{D}}(t) = 1 - F_{\mathcal{D}}(t)$ from (6)

$$R_{\mathcal{D}}(t) = 1 - \sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\mathcal{C}_{k,h}\left(1 - e^{-\lambda h(\rho-\mathfrak{a})}\right) \quad \text{……………… (16)}$$

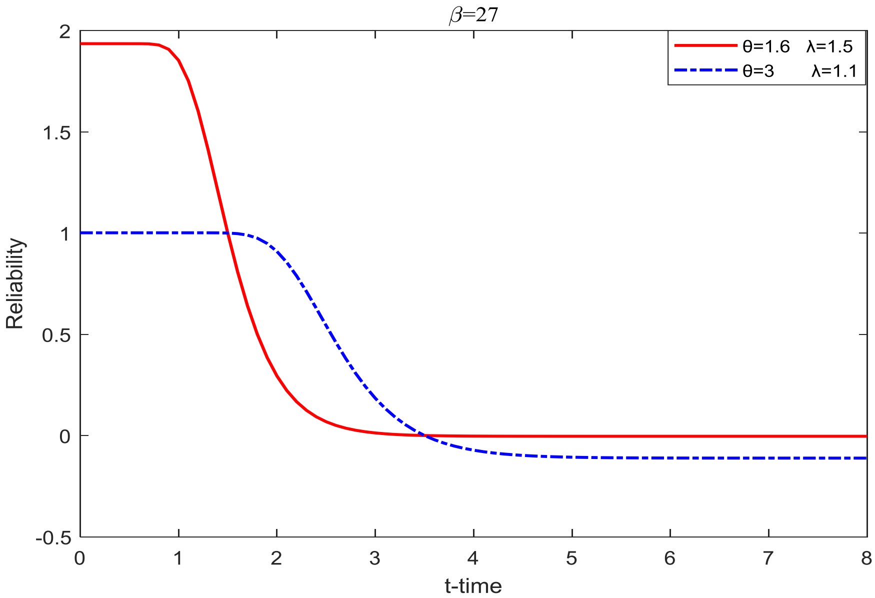

Figure (7) represents reliability of DT-TLGE, which is determined within the constrained period $1.5 \leq t < 3.5$ and values of parameters $\theta = 1.6, 3$, $\lambda = 1.5, 1.1$, and $\beta = 9$. It is noticed that curves intersection within truncated period, as well as anomalous and extreme values, to left and right of the period.

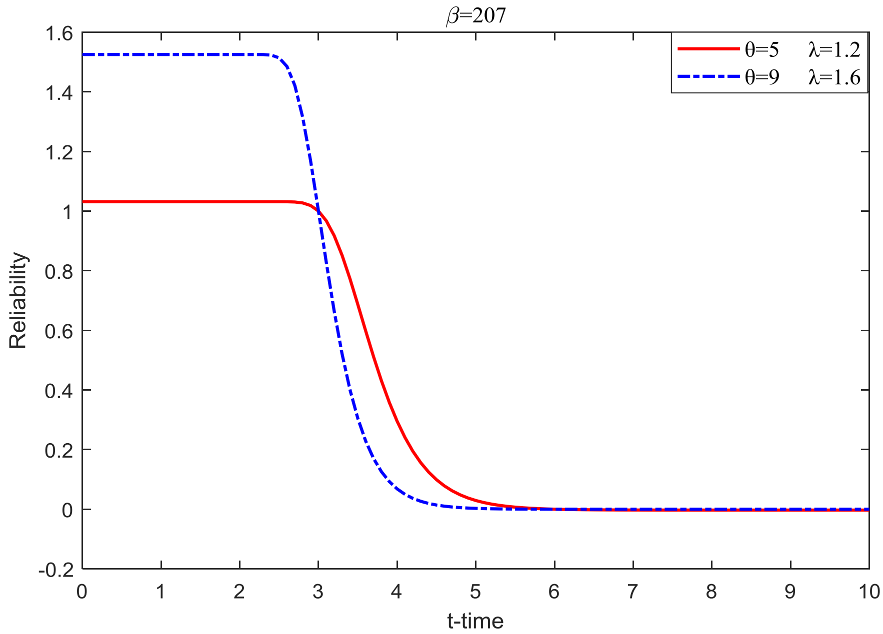

Figure (8) represents reliability function of DT-TLGE, which is determined within the constrained period $1.5 \leq \mathrm{t} < 3.5$ and values of parameters $\theta = 1.6, 3,\ \lambda = 1.5, 1.1$, and $\beta = 9$.

6. Hazard Function

$$\mathcal{H}_{\mathcal{D}}(\mathrm{t}) = \frac{\mathcal{f}_{\mathcal{D}}(\mathrm{t})}{\mathrm{R}_{\mathcal{D}}(\mathrm{t})} = \frac{\sum_{\mathrm{k}=0}^{\infty}\sum_{\mathrm{h}=0}^{\infty}\mathcal{C}_{\mathrm{k,h}}\left(\lambda \mathrm{h} e^{-\lambda \mathrm{h}(\rho-\mathfrak{a})}\right)}{1-\sum_{\mathrm{k}=0}^{\infty}\sum_{\mathrm{h}=0}^{\infty}\mathcal{C}_{\mathrm{k,h}}\left(1-e^{-\lambda \mathrm{h}(\rho-\mathfrak{a})}\right)} \quad \dots\dots\dots\dots (17)$$

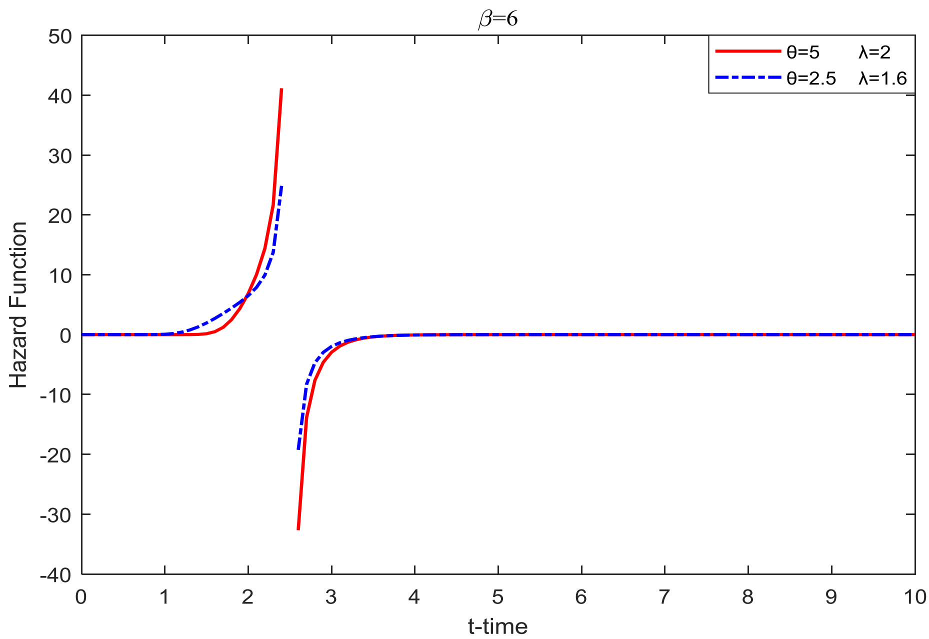

Figure (9) represents hazard function of DT-TLGE, which is determined within the constrained period $0.5 \leq \mathrm{t} < 2.5$ and values of parameters $\theta = 1.6, 3,\ \lambda = 1.5, 1.1$, and $\beta = 4$.

7. Method Moment Estimation of DT-TLGE

Let $\rho \sim$ DT-TLGE(θ, β, λ), the method of moment estimation of DT-TLGE distribution is defined as $E(\rho^s) = \sum_{j=1}^{n} \frac{1}{n} \rho_j^s$ by using (11) we getting

$$\sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \mathcal{C}_{k,h}\, e^{-\lambda h \mathfrak{a}} \left[\frac{s}{(\lambda h)^s}\gamma(s,\mathfrak{b}) - \frac{s}{(\lambda h)^s}\gamma(s,\mathfrak{a}) + \mathfrak{a}^s e^{-\lambda h \mathfrak{a}} - \mathfrak{b}^s e^{-\lambda h \mathfrak{b}}\right] = \sum_{j=1}^{n}\frac{1}{n}\rho_j^s \quad \dots\dots\dots\dots (18)$$

When $s = 1$, then

$$\sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \mathcal{C}_{k,h}\, e^{-\lambda h \mathfrak{a}} \left[\frac{1}{\lambda h}\gamma(1,\mathfrak{b}) - \frac{1}{\lambda h}\gamma(1,\mathfrak{a}) + \mathfrak{a} e^{-\lambda h \mathfrak{a}} - \mathfrak{b} e^{-\lambda h \mathfrak{b}}\right] = \sum_{j=1}^{n}\frac{1}{n}\rho_j \quad \dots\dots\dots\dots (19)$$

When $s = 2$, then

$$\sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \mathcal{C}_{k,h}\, e^{-\lambda h \mathfrak{a}} \left[\frac{2}{(\lambda h)^2}\gamma(2,\mathfrak{b}) - \frac{2}{(\lambda h)^2}\gamma(2,\mathfrak{a}) + \mathfrak{a}^2 e^{-\lambda h \mathfrak{a}} - \mathfrak{b}^2 e^{-\lambda h \mathfrak{b}}\right] = \sum_{j=1}^{n}\frac{1}{n}\rho_j^2 \quad \dots\dots\dots\dots (20)$$

When $s = 3$, then

$$\sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \mathcal{C}_{k,h}\, e^{-\lambda h \mathfrak{a}} \left[\frac{3}{(\lambda h)^3}\gamma(3,\mathfrak{b}) - \frac{3}{(\lambda h)^3}\gamma(3,\mathfrak{a}) + \mathfrak{a}^3 e^{-\lambda h \mathfrak{a}} - \mathfrak{b}^3 e^{-\lambda h \mathfrak{b}}\right] = \sum_{j=1}^{n}\frac{1}{n}\rho_j^3 \quad \dots\dots\dots\dots (21)$$

8. Maximum likelihood estimator MLE of Doubly Truncated

The (MLE) method is considered one of the most important methods used to estimate truncated distributions we denote the maximum likelihood of DT-TLGE as symbol $L_{\mathcal{D}}(\sigma_i, \rho)$. Thus it can be represented as formula:

$$L_{\mathcal{D}}(\sigma_i,\rho) = \prod_{i=1}^{n} \frac{f(\rho_i;\sigma)}{[F(\mathfrak{b}) - F(\mathfrak{a})]}$$

$$= \frac{(2\beta\theta\lambda)^n e^{-\lambda\sum_{i=1}^{n}\rho_i} \sum_{i=1}^{n}\left(1-e^{-\lambda\rho_i}\right)^{\beta\theta-1}\left[2-\left(1-e^{-\lambda\rho_i}\right)^{\theta}\right]^{\beta-1}\left(1-\left(1-e^{-\lambda\rho_i}\right)^{\theta}\right)}{[F(\mathfrak{b})-F(\mathfrak{a})]^n} \quad \dots\dots (22)$$

Taking Ln to both sides

$$\ell_{\mathcal{D}} = \ln L_{\mathcal{D}}(\sigma_i,\rho) = n\ln 2 + n\ln\theta + n\ln\beta + n\ln\lambda - \lambda\sum_{i=1}^{n}\rho_i + n(\beta\theta-1)\ln\sum_{i=1}^{n}\left(1-e^{-\lambda\rho_i}\right) + n(\beta-1)\ln\left(\sum_{i=1}^{n}\left(2-\left(1-e^{-\lambda\rho_i}\right)^{\theta}\right)\right) + n\ln\left(\sum_{i=1}^{n}\left(1-\left(1-e^{-\lambda\rho_i}\right)^{\theta}\right)\right) - n\ln\left(F(\mathfrak{b}) - F(\mathfrak{a})\right) \quad \dots\dots\dots (23)$$

Consider $\mathfrak{a}$ and $\mathfrak{b}$ constant so from (1) then

$$F(\mathfrak{a}) = \left\{\left(1-e^{-\lambda\mathfrak{a}}\right)^{\theta}\left(2-\left(1-e^{-\lambda\mathfrak{a}}\right)^{\theta}\right)\right\}^{\beta} \ \&$$

$$F(\mathfrak{b}) = \left\{\left(1-e^{-\lambda\mathfrak{b}}\right)^{\theta}\left(2-\left(1-e^{-\lambda\mathfrak{b}}\right)^{\theta}\right)\right\}^{\beta}$$ thus become (23) as following

$$\ell_{\mathcal{D}} = \ln L_{\mathcal{D}}(\sigma_i,\rho) = n\ln 2 + n\ln\theta + n\ln\beta + n\ln\lambda - \lambda\sum_{i=1}^{n}\rho_i + n(\beta\theta-1)\ln\sum_{i=1}^{n}\left(1-e^{-\lambda\rho_i}\right) + n(\beta-1)\ln\left(\sum_{i=1}^{n}\left(2-\left(1-e^{-\lambda\rho_i}\right)^{\theta}\right)\right) + n\ln\left(\sum_{i=1}^{n}\left(1-\left(1-e^{-\lambda\rho_i}\right)^{\theta}\right)\right) - n\ln\left(\left\{\left(1-e^{-\lambda\mathfrak{b}}\right)^{\theta}\left(2-\left(1-e^{-\lambda\mathfrak{b}}\right)^{\theta}\right)\right\}^{\beta} - \left\{\left(1-e^{-\lambda\mathfrak{a}}\right)^{\theta}\left(2-\left(1-e^{-\lambda\mathfrak{a}}\right)^{\theta}\right)\right\}^{\beta}\right) \quad \dots\dots\dots\dots (24)$$

Partial derivative of parameters

$$\frac{\partial\ell_{\mathcal{D}}}{\partial\beta} = \frac{n}{\beta} + n\theta\ln\left(\sum_{i=1}^{n}\left(1-e^{-\lambda\rho_i}\right)\right) + n\ln\sum_{i=1}^{n}\left(2-\left(1-e^{-\lambda\rho_i}\right)^{\theta}\right) - \frac{1}{\beta}\cdot\frac{F(\mathfrak{b})\ln(F(\mathfrak{b})) - F(\mathfrak{a})\ln F(\mathfrak{a})}{F(\mathfrak{b}) - F(\mathfrak{a})} = 0 \quad \dots\dots\dots\dots (25)$$

$$\frac{\partial\ell_{\mathcal{D}}}{\partial\theta} = \frac{n}{\theta} + n\beta\ln\sum_{i=1}^{n}\left(1-e^{-\lambda\rho_i}\right) - \frac{n(\beta-1)\sum_{i=1}^{n}\left(1-e^{-\lambda\rho_i}\right)^{\theta}\ln\sum_{i=1}^{n}\left(1-e^{-\lambda\rho_i}\right)}{\sum_{i=1}^{n}\left(2-\left(1-e^{-\lambda\rho_i}\right)^{\theta}\right)} - \frac{n\sum_{i=1}^{n}\left(1-e^{-\lambda\rho_i}\right)^{\theta}\ln\sum_{i=1}^{n}\left(1-e^{-\lambda\rho_i}\right)}{\sum_{i=1}^{n}\left(1-\left(1-e^{-\lambda\rho_i}\right)^{\theta}\right)}$$

$$\frac{\partial \ell_{\mathcal{D}}}{\partial \theta} = \frac{n}{\theta} + n\beta \ln \sum_{i=1}^{n}\left(1 - e^{-\lambda\rho_i}\right) - \frac{n(\beta-1)\sum_{i=1}^{n}\left(1-e^{-\lambda\rho_i}\right)^{\theta}\ln\sum_{i=1}^{n}\left(1-e^{-\lambda\rho_i}\right)}{\sum_{i=1}^{n}\left(2-\left(1-e^{-\lambda\rho_i}\right)^{\theta}\right)} - \frac{n\sum_{i=1}^{n}\left(1-e^{-\lambda\rho_i}\right)^{\theta}\ln\sum_{i=1}^{n}\left(1-e^{-\lambda\rho_i}\right)}{\sum_{i=1}^{n}\left(1-\left(1-e^{-\lambda\rho_i}\right)^{\theta}\right)} + \frac{2n\beta\left[\frac{e^{-\lambda b}\ln\left(1-e^{-\lambda b}\right)F(b)}{\left(2-\left(1-e^{-\lambda b}\right)^{\theta}\right)} - \frac{e^{-\lambda a}\ln\left(1-e^{-\lambda a}\right)F(a)}{\left(2-\left(1-e^{-\lambda a}\right)^{\theta}\right)}\right]}{F(b)-F(a)} = 0 \quad \dots\dots\dots\dots(26)$$

$$\frac{\partial \ell_{\mathcal{D}}}{\partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n}\rho_i + \frac{n(\beta\theta-1)\sum_{i=1}^{n}\rho_i e^{-\lambda\rho_i}}{\sum_{i=1}^{n}\left(1-e^{-\lambda\rho_i}\right)} - \frac{n\theta(\beta-1)\sum_{i=1}^{n}\left(1-e^{-\lambda\rho_i}\right)^{\theta-1}\rho_i e^{-\lambda\rho_i}}{\sum_{i=1}^{n}\left(2-\left(1-e^{-\lambda\rho_i}\right)^{\theta}\right)} - \frac{n\theta\sum_{i=1}^{n}\left(1-e^{-\lambda\rho_i}\right)^{\theta-1}\rho_i e^{-\lambda\rho_i}}{\sum_{i=1}^{n}\left(1-\left(1-e^{-\lambda\rho_i}\right)^{\theta}\right)} + \frac{2n\beta\theta\left[\frac{b e^{-\lambda b}F(b)\left(1-\left(1-e^{-\lambda b}\right)^{\theta}\right)}{\left(1-e^{-\lambda b}\right)\left(2-\left(1-e^{-\lambda b}\right)^{\theta}\right)} - \frac{a e^{-\lambda a}F(a)\left(1-\left(1-e^{-\lambda a}\right)^{\theta}\right)}{\left(1-e^{-\lambda a}\right)\left(2-\left(1-e^{-\lambda a}\right)^{\theta}\right)}\right]}{F(b)-F(a)} = 0 \quad \dots\dots\dots\dots(27)$$

9. Conclusion

It is concluded that mixture of Topp-Leone generated with exponential distribution can achieve double truncated property, as it contains two location parameters that help with truncation as shown in the functions CDF, PDF, reliability and hazard. It was evident by figures where truncation between curves was occurring at its intersection. Since DT-TLGE distribution of life, it is possible to apply an analysis of life data that is restricted within a specified period.

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