JOURNAL OF CRITICAL REVIEWS

ISSN- 2394-5125 VOL 7, ISSUE 15, 2020

Equitable Total Domination on Bipolar Fuzzy Graphs

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Received: 14 April 2020 Revised and Accepted: 8 August 2020

Abstract: In this paper, equitable domination in bipolar fuzzy graph, equitable total domination in bipolar fuzzy graph and its various classifications are defined. The definition of Size, Order and degree of bipolar fuzzy graph is defined with some examples. Some basic parametric conditions are introduced with suitable examples. The properties of total domination number and equitable total domination number in bipolar fuzzy graph are discussed. Some basic theorems related to the stated graphs have also been presented.

Key Words: Bipolar fuzzy graph, dominating set on bipolar fuzzy graph, strong (weak) dominating set on bipolar fuzzy graph, total dominating set on bipolar fuzzy graph, equitable dominating set on bipolar fuzzy graph, equitable total dominating set on bipolar fuzzy graph.

AMS Subject Classification: 05C69, 05C72, 05C99, 03E72, 03E55, 68R05, 68R10

1. Introduction

The concept of fuzzy graph was proposed by Kaufmann, from the fuzzy relations introduced by Zadeh [11]. In 1975, Rosenfeld introduced another elaborated concept, including fuzzy vertex and fuzzy edges and several fuzzy analogues of graph theoretic concepts such as paths, cycles, connectedness and etc. In the year 1998, the concept of domination in fuzzy graphs was investigated by A. Somasundaram, S. Somasundaram.[9] In the year 2003, A. Nagoor Gani and M. Basheer Ahamed [7] investigated Order and Size in fuzzy graph. In the year, 2004. In 2011, Muhammad Akram [5] introduced Bipolar fuzzy graph. In the year 2012, Muhammad Akram [6] was proposed regular bipolar fuzzy graph. In 2012, Muhammed Akram. Wieslaw A. Dudek [6] introduced Regular bipolar fuzzy graph. In 2014, B. Basavanagoud, V. R. Kulli and Vijay. V. Teli [1] introduced the concept of equitable total domination in graphs.

2 Preliminaries

In this section, the basic definitions needed to develop the subsequent sections definitions are discussed. Throughout this paper, the dominating set denoted as D, total dominating set as T, equitable dominating set as D_E and equitable total dominating as D_{TE}.
Throughout this paper,
The edge between the vertices u and v as uv.
G = (A, B) be a bipolar fuzzy graph, mean that G be a bipolar fuzzy graph with underlying graph G* = (V, E).

2.1 Definition [5] A fuzzy subset μ on a set X is a map $\mu: X \rightarrow [0,1]$. A map $v: X \times X \rightarrow [0,1]$ is called a fuzzy relation on X if $v(x, y) \le \min(\mu(x), \mu(y))$ for all $x, y \in X$. A fuzzy relation v is symmetric if $v(x, y) = v(y, x)$ for $x, y \in X$.

2.2 Definition [2] Let X be a non-empty set. A bipolar fuzzy set B in X is an object having the form $B = \{(x, \mu_B^P(x), \mu_B^N(x)) / x \in X\}$ where $\mu_B^P: X \rightarrow [0,1]$ and $\mu_B^N: X \rightarrow [-1,0]$ are mappings. The positive membership degree $\mu_B^P(x)$ to denote the satisfaction degree of an element x to the property corresponding to a bipolar fuzzy set B, and the negative membership degree $\mu_B^N(x)$ to denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar fuzzy set B. If $\mu_B^P(x) \neq 0$ and $\mu_B^N(x)=0$. It is the situation that is regarded as having only positive satisfaction for B. If $\mu_B^P(x)=0$ and $\mu_B^N(x) \neq 0$, it is the situation that x does not satisfy the property of B but somewhat satisfies the counter property of B. It is possible for an element x to be such that $\mu^P(x) \neq 0$ and $\mu^N(x) \neq 0$ When the membership function of the property overlaps that of its counter property over some portion of X. For the sake of simplicity, we shall use the symbol $B=(\mu_B{}^P, \mu_B{}^N)$, for the bipolar fuzzy set $B = \{(x, \mu_B{}^P(x), \mu_B{}^N(x)) / x \in X\}$.

2.3 Definition [5] Let X be a non – empty set. Then we call a mapping $A = (\mu_A^P, \mu_A^N): XxX \rightarrow [-1,1]x[-1,1]$ a bipolar fuzzy relation on X such that $\mu_A^P(x,y) \in [0,1]$ and $\mu_A^N(x,y) \in [-1,0]$.

2.4 Definition [6] A Bipolar fuzzy graph, is denoted as a pair G = (A, B), where $A=(\mu_A^P, \mu_A^N)$ and $B=(\mu_B^P, \mu_B^N)$ are bipolar fuzzy sets and $\mu_A^P: V \rightarrow [0,1]$, $\mu_A^N: V \rightarrow [-1,0]$, and $\mu_B^P: V \times V \rightarrow [0,1]$, $\mu_B^N: V \times V \rightarrow [-1,0]$ are bipolar fuzzy mappings such that $\mu_B^P(uv) \le \min\{\mu_A^P(u), \mu_A^P(v)\}$ and $\mu_B^N(uv) \ge \max\{\mu_A^N(u), \mu_A^N(v)\}$ for all $uv \in E$. A is called the bipolar fuzzy vertex set of V and B the bipolar fuzzy edge set of E respectively. Note that B is a symmetric bipolar fuzzy relation on A. That is, G = (A, B) is a bipolar fuzzy graph of the underlying crisp graph G* = (V, E), where V is a vertex set and the edge set $E \subseteq V \times V$ such that,

$$\mu_B^P(uv) \le \min\{\mu_A^P(u), \mu_A^P(v)\} \text{ and } \mu_B^N(uv) \ge \max\{\mu_A^N(u), \mu_A^N(v)\} \text{ for all } uv \in E.$$

2.5 Definition Let G = (A, B) be a bipolar fuzzy graph. Then the cardinality of V or the order of G is defined by

$$p = |V| = \sum_{u \in V} \frac{1 + \mu_A^P(u) + \mu_A^N(u)}{2}.$$

2.6 Definition Let G = (A, B) be the bipolar fuzzy graph. Then cardinality of E or the size of G is defined as

$$q = |E| = \sum_{uv \in E} \frac{1 + \mu_B^P(uv) + \mu_B^N(uv)}{2}.$$

2.7 Definition Let G = (A, B) be a bipolar fuzzy graph, then the degree of the vertex is denoted by deg(u) and it is defined as $\deg(u)=\sum_{v\in V}\frac{1+\mu_B^P(uv)+\mu_B^N(uv)}{2}$.

2.8 Definition Let G = (A, B) be a bipolar fuzzy graph. The maximum degree of a bipolar fuzzy graph is denoted by $\Delta(G) = \max\{\deg(u) / u \in V\}$.

2.9 Definition Let G = (A, B) be a bipolar fuzzy graph. The minimum degree of a bipolar fuzzy graph is denoted by $\delta(G) = \min\{\deg(u) / u \in V\}$.

2.10 Definition Let G = (A, B) be a bipolar fuzzy graph. The degree of an edge $uv\in E$ is denoted as deg(uv) and it is defined as, $\deg(uv)=\sum_{uv\in E}\frac{1+\mu_B^P(uv)+\mu_B^N(uv)}{2}$.

2.11 Definition Let G = (A, B) be a bipolar fuzzy graph. Then the neighbors (neighborhood) of u or an open neighbor of u $\in$ V of G is denoted by N(u) and is defined a $N(u)=\{v\in V/\ \mu_B^P(uv)=\min\{\mu_A^P(u),\mu_A^P(v)\}\text{ and }\mu_B^N(uv)=\max\{\mu_A^N(u),\mu_A^N(v)\}\text{ and }uv\in E\}$.

The closed neighbors of u $\in$ V of G is denoted by N[u] and is defined as $N[u] = N(u) \cup \{u\}$.

2.12 Definition Let G = (A, B) be a bipolar fuzzy graph. Then the neighbourhood degree of $u\in V$ is denoted as $\deg_N(u)$ and is defined as, $\deg_N(u)=\sum_{v\in N(u)}\frac{1+\mu_A^P(v)+\mu_A^N(v)}{2}$.

2.13 Definition Let G = (A, B) be a bipolar fuzzy graph. The maximum neighbourhood degree of a bipolar fuzzy graph is denoted by $\Delta_N(G) = \max\{\deg_N(u) / u \in V\}$.

2.14 Definition Let G = (A, B) be a bipolar fuzzy graph. The minimum neighbourhood degree of a bipolar fuzzy graph is denoted by $\delta_N(G) = \min\{\deg_N(u) / u \in V\}$.

2.15 Definition Let G = (A, B) be a bipolar fuzzy graph. An edge of G is said to be an effective edge if $\mu_B^P(uv)=\min\{\mu_A^P(u),\mu_A^P(v)\}\text{ and }\mu_B^N(uv)=\max\{\mu_A^N(u),\mu_A^N(v)\}\text{ for all }uv\in E.$

2.16 Definition Let G = (A, B) be a bipolar fuzzy graph. Then the effective degree of a vertex $u\in V$ in G is defined as $\deg_E(u)=\sum_{v\in V}\frac{1+\mu_B^P(uv)+\mu_B^N(uv)}{2};\ uv\text{ is an effective edge}$

2.17 Definition Let G = (A, B) be a bipolar fuzzy graph. The maximum effective degree of a bipolar fuzzy graph is denoted by $\Delta_E(G) = \max\{\deg_E(u) / u \in V\}$.

2.18 Definition Let G = (A, B) be a bipolar fuzzy graph. The minimum effective degree of a bipolar fuzzy graph is denoted by $\delta_E(G) = \min\{\deg_E(u) / u \in V\}$.

2.19 Example

ISSN- 2394-5125 VOL 7, ISSUE 15, 2020

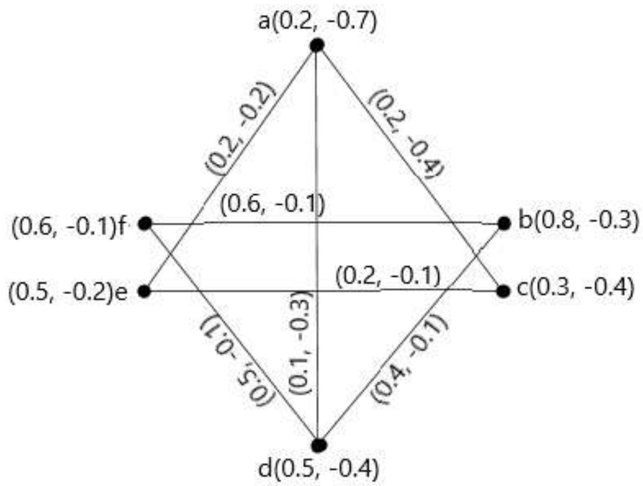

Fig. 2.1
Bipolar fuzzy graph

In figure 2.1, the edges ac, ae, df, and bf are effective edges.

$$\deg(u) = \sum_{v \in V} \frac{1 + \mu_B^P(uv) + \mu_B^N(uv)}{2}.$$

$$\deg(uv) = \sum_{uv \in E} \frac{1 + \mu_B^P(uv) + \mu_B^N(uv)}{2}.$$

deg(a) = 0.5 + 0.4 + 0.4 = 1.3

deg(b) = 0.75 + 0.65 = 1.4

deg(c) = 0.4 + 0.55 = 0.95

deg(d) = 0.7 + 0.65 + 0.4 = 1.75
deg(e) = 0.55 + 0.5 = 1.05
deg(f) = 0.75 + 0.7 = 1.45

deg(ac) = 0.4

deg(ae) = 0.5

deg(ad) = 0.4

deg(bf) = 0.75
deg(ce) = 0.55
deg(bd) = 0.65
deg(df) = 0.7

$\delta(G) = \min\{\deg(u) / u \in V\} = 0.95$

$\Delta(G) = \max\{\deg(u) / u \in V\} = 1.75$

$\delta_{\deg(uv)}(G) = \min\{\deg(uv) / uv \in E\} = 0.4$

$\Delta_{\deg(uv)}(G) = \max\{\deg(uv) / uv \in V\} = 0.75.$

$$\deg_E(u) = \sum_{v \in V} \frac{1 + \mu_B^P(uv) + \mu_B^N(uv)}{2}; \text{uv is an effective edge}$$

$\deg_E(a) = 0.5 + 0.4 = 0.9$

$\deg_E(b) = 0.75 = 0.75$

$\deg_E(c) = 0.4 = 0.4$

$\deg_E(d) = 0.7 = 0.7$

$\deg_E(e) = 0.5 = 0.5$

$\deg_E(f) = 0.75 + 0.7 = 1.45$

$\Delta_E(G) = \max\{\deg_E(u) / u \in V\} = 1.45$

$\delta_E(G) = \min\{\deg_E(u) / u \in V\} = 0.4$

$|V| = 0.25 + 0.75 + 0.45 + 0.55 + 0.65 + 0.75 = 3.4$

|E| = 0.5 + 0.4 + 0.65 + 0.7 + 0.75 + 0.55 + 0.4 = 3.95

|G| = 3.4 +3.95 = 7.35

$$\sum_{u\in V}\deg(u) = 1.3+1.4+0.95+1.75+1.05+1.45=7.9=2\times|E|$$

2.20 Definition [5] A bipolar fuzzy graph G = (A, B) is said to be a strong bipolar fuzzy graph if $\mu_B^P(uv)=\min\{\mu_A^P(u),\mu_A^P(v)\}$ and $\mu_B^N(uv)=\max\{\mu_A^N(u),\mu_A^N(v)\}$ for all $uv\in E$.

2.21 Definition A bipolar fuzzy graph G = (A, B) is said to be a complete bipolar fuzzy graph if
i. Every vertex of V in G is adjacent to every other vertex of V in G.
ii. $\mu_B^P(uv)=\min\{\mu_A^P(u),\mu_A^P(v)\}$ and $\mu_B^N(uv)=\max\{\mu_A^N(u),\mu_A^N(v)\}$ for all $uv\in E$.

2.22 Definition [5] Let G = (A, B) be a bipolar fuzzy graph. Let u, v $\in$ V. The vertex u is said to be dominates v in G if $\mu_B^P(uv)=\min\{\mu_A^P(u),\mu_A^P(v)\}$ and $\mu_B^N(uv)=\max\{\mu_A^N(u),\mu_A^N(v)\}$ and $uv\in E$.

A subset D of V is said to be a dominating set in G if for every v $\in$ V – D there exist u $\in$ D such that u dominates v.
A dominating set D of V is said to be a minimal dominating set if no proper subset of D is a dominating set of G.
The minimum fuzzy cardinality of a minimal dominating set in G is called the domination number of G and is denoted by γ(G) or simply γ.

2.23 Definition Let G = (A, B) be a bipolar fuzzy graph. Then a dominating set T of V is said to be a total dominating set T of G if the induced subgraph of T has no isolated vertex.

2.24 Example

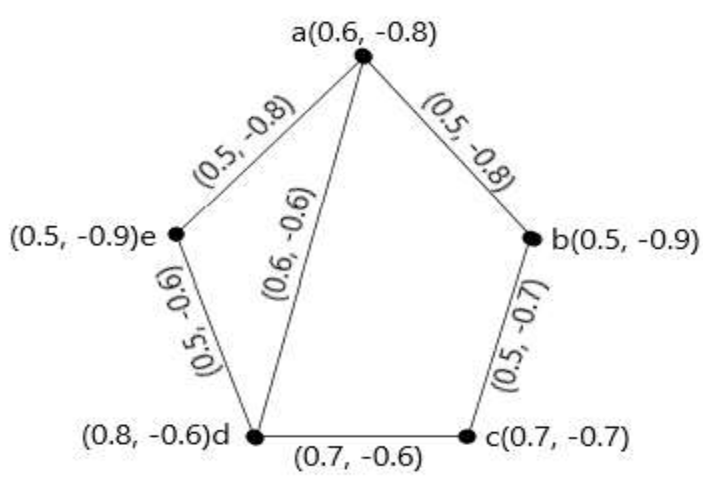

Fig. 2.2

In the Fig. 2.2, The dominating set D = {b, e}, V – D = {a, c, e}.

The domination number of D = $\gamma(G)=0.6$ = 2.1

2.25 Definition Let G = (A, B) be a bipolar fuzzy graph. Let $T \subseteq V$ be a total dominating set of G. Then T is said to be a regular total dominating set in a bipolar fuzzy graph G if all the vertices of T has same degree.

2.26 Example

The given bipolar fuzzy graph G is an example for Total dominating set in a bipolar fuzzy graph and n- regular total dominating set in a bipolar fuzzy graph.

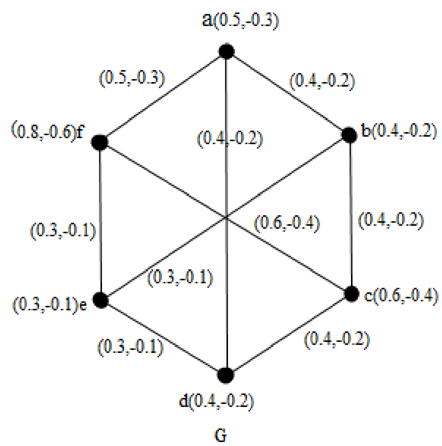

Fig 2.3

In the above graph, the total strong (weak) dominating set T = {b, e} and

V – T = {a, c, d, f}.

The domination number of a Total strong (weak) dominating set of G is $\gamma_T(G)$ = 1.2.

3. Equitable Domination on bipolar fuzzy graph.

In this section, the related concepts of equitable domination on bipolar fuzzy graph and its properties are discussed.

3.1 Definition A subset D_E of a vertex set V of a bipolar fuzzy graph G is called an equitable dominating set if for every $v \in V\text{-}D_E$ there exists a vertex $u \in D_E, \ni : uv \in E$ and $|\deg(u)\text{-}\deg(v)| \leq 1$, where deg(u) and deg(v) are denoted as the degree of a vertex u and v respectively.

The equitable domination number of a bipolar fuzzy graph $\gamma_E(G)$ is the minimum cardinality of an equitable dominating set of a bipolar fuzzy graph.

3.2 Example

Consider the following bipolar fuzzy graph G = (A, B).

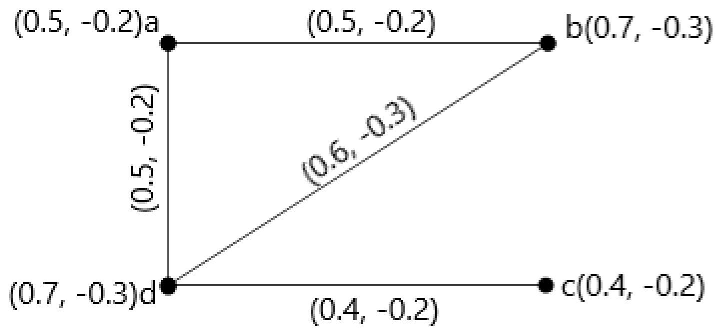

Fig. 3.1

Here, D_1 = { d }; D_2 = { a, d }; D_3 = { b, d } are the minimal equitable dominating sets of G with cardinality 0.7, 1.35, 1.4 respectively and the equitable domination number is

$\gamma_E(G)$ = 0.7

Remark:1 For the star graph (BFG), an equitable dominating set does not exist.

Remark:2 Any Friendship bipolar fuzzy graph, equitable domination does not exist.

3.3 Observation For any bipolar fuzzy graph G = (A, B) with no isolated vertices.

Then $\gamma(G) \leq \gamma_E(G)$

3.4 Theorem: Let G = (A, B) be a bipolar fuzzy graph with no isolated vertices and let D_E be the equitable dominating set. Then $\gamma(G) + \gamma_E(G)) \leq p$.

Proof: Let G = (A, B) be a bipolar fuzzy graph. Let D be the minimum equitable dominating set and let D_e be the minimal equitable dominating set of G. p be the order of the bipolar fuzzy graph and carrying the cardinality of all the vertices which is maximum. Therefore, $\gamma_E(G) \leq p - \gamma(G)$. Hence, $\gamma(G) + \gamma_E(G) \leq p$.

3.5 Theorem: For any bipolar fuzzy graph G = (A, B), then $\gamma_E(G) < p$.

Proof: Let G = (A, B) be a bipolar fuzzy graph. Then by Theorem 3.4,

$\gamma(G) + \gamma_E(G) \leq p$.

Hence, $\gamma_E(G) < p$

3.6 Theorem: Let G be a bipolar fuzzy graph without isolated vertices. Let D_E be the minimal equitable dominating set of G, then V-D_E is an equitable dominating set of G.

Proof: Let u be any vertex in D_E. Since G has no isolated vertices, there is a vertex $v \in N(u)$ .u must be dominated by at least one vertex in D_E - {u}, that is D_E - {u} is a dominating set. Thus, every vertex in D_E dominated by at least one vertex in V-D_E and V-D_E is an equitable dominating set of G.

4. Related Concepts of Equitable Total Domination on Bipolar Fuzzy Graph

4.1 Definition Let G be a bipolar fuzzy graph. A subset D_{TE} of V is called an equitable total dominating set of G if D_{TE} is an equitable dominating set and induced subgraph of D_{TE} has no isolated vertices.

The minimum fuzzy cardinality taken over all the equitable total dominating set is the equitable total domination number and is denoted by $\gamma_{TE}(G)$.

4.2 Example

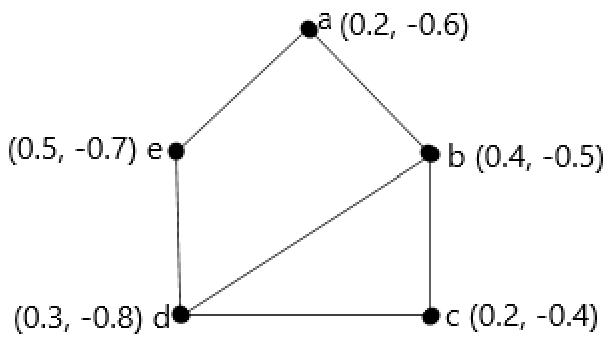

Fig. 4.1

From the above Fig.4.1,
We have the equitable total dominating set D_{TE} = {a,b}
Equitable total domination number = $\gamma_{TE}(G)$ = 0.75.

4.3 Theorem: For any bipolar fuzzy graph G without isolated vertices, $\gamma_T(G) \le \gamma_{TE}(G)$.

Proof: Every equitable total dominating set is a total dominating set, thus $\gamma_T(G) \le \gamma_{TE}(G)$.

4.4 Theorem: If G is a n - regular bipolar fuzzy graph for $n \ge 1$ then $\gamma_{TE}(G) = \gamma_T(G)$.

Proof: Suppose G is a n – regular bipolar fuzzy graph then every vertex of G is of same degree r. Let T be minimum total dominating set of G then $|T| = \gamma_T(G)$. If $v \in V\text{-}T$ then T is a total dominating se then there exist $u \in T$ also deg(u) = deg(v) = r. Therefore $|\deg(u) - \deg(v)| \le 1$. Hence T is a equitable total dominating set of bipolar fuzzy graph such that $\gamma_{TE}(G) \le |T| = \gamma_T(G)$ (1)

also we have by the previous theorem

$\gamma_T(G) \le \gamma_{TE}(G)$. (2)

From (1) and (2) we have $\gamma_{TE}(G) = \gamma_T(G)$.

4.5 Theorem: Let G = (A, B) be a bipolar fuzzy graph with at least one equitable total dominating set. Then $\gamma_T(G) + \gamma_{TE}(G) \le p$.

Proof: Let G = (A, B) be a bipolar fuzzy graph. Let T be the total dominating set and D_{TE} be the equitable total dominating set of G. Therefore, $\gamma_{TE}(G) \le p - \gamma_T(G)$.

Hence, $\gamma_T(G) + \gamma_{TE}(G) \le p$.

4.6 Theorem: For any bipolar fuzzy graph G = (A, B), then $\gamma_{TE}(G) < p$.

Proof : Let G = (A, B) be a bipolar fuzzy graph. Then by Theorem 4.5,
$\gamma_T(G) + \gamma_{TE}(G) \leq p$. Hence, $\gamma_{TE}(G) < p$.

5. Conclusion

In this paper, an equitable domination and equitable total equitable domination in a bipolar fuzzy graph was introduced and some theorems are examined. Based on these ideas, we can extend our research work to other areas of bipolar fuzzy graph.

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