JOURNAL OF CRITICAL REVIEWS

ISSN- 2394-5125 VOL 7, ISSUE 15, 2020

MULTI FUZZY GRAPH

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Received: 14 April 2020 Revised and Accepted: 8 August 2020

ABSTRACT

In this paper, the new concept multi fuzzy graph is introduced and discussed its related concepts and basic definitions with examples. Domination on multi fuzzy graph is defined and its domination number is established with example. Characterization of a minimal dominating set of a multi fuzzy graph is established.

Key Words: multi fuzzy graph, degree; order and size of a multi fuzzy graph; dominating set; domination number.

AMS Subject Classification (2010): 03E72, 03B20, 03F55, 05C72, 05C07, 05C62,

1. Introduction

Every real life problem is converted as one form of mathematical model. In such case, graph is considered as simple models of relations. A graph is a convenient way of representing information involving relationship between the objects. The objects are represented by vertices and relations by edges. When there is uncertainty involved in the description of the objects or in its relationships or in both, it is natural to design the system as a 'Fuzzy Graph Model'. The notion of fuzzy set and fuzzy relations were proposed and studied by L.A.Zadeh [10, 11] in 1965 for representing uncertainty. The concept of fuzzy graph was first introduced by Kaufmann [1] from the concept fuzzy relation introduced by L.A. Zadeh. In 1975, Rosenfeld [5] considered the fuzzy relations on fuzzy sets to develop another elaborated definition, including fuzzy vertex and fuzzy edge, and also introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. In 1998, A. Somasundaram and S. Somasundaram [8] and in 2005 A. Somasundaram [9] introduced the concepts of domination in fuzzy graphs and determined the domination number for several classes of fuzzy graphs and obtained bounds for the same. A.Nagoorgani and M. Basheer Ahamed [2] was introduced the concept of order and size in a fuzzy graph in 2003. A.Nagoorgani and K.Radha [3] was introduced the concept of regular fuzzy graph in 2008 and A.Nagoorgani and S.R. Latha [4] was introduced concept of irregular fuzzy graph in 2012.

In 2010, S.Sabu and T.V.Ramakrishnan [6,7] proposed the theory of multi-fuzzy set in terms of multi-dimensional membership functions with codomain as Lattice and investigated some properties of multilevel fuzziness. In 2013, R. Muthuraj and S. Balamurugan [2] introduced the concept of multi-fuzzy set in terms of multi-dimensional membership functions with codomain as [0, 1]. Theory of multi-fuzzy set is an extension of theory of fuzzy sets. Complete characterization of many real life problems can be done by multi-fuzzy membership functions of the objects involved in the problem.

The concept of fuzzy graph model is based on the single phenomenon and it is not enough to find the accurate solution in many of the real life problems. In this paper, the new concept of multi fuzzy graph has been introduced and defined the basic definitions related to the new concept multi fuzzy graph and established the relation among them. Regular multi fuzzy graph and various types of irregular multi fuzzy graph is defined and established the relation among them. These concepts are explained with example. The concept of multi fuzzy graph is the extension of a fuzzy graph with single phenomenon into a fuzzy graph with multi phenomenon which suits to describe the real life problems in a better manner than a fuzzy graph. The dominating set of a multi fuzzy graph is introduced and the domination number is established.

2. Multi fuzzy graph

In this section, the concept of multi fuzzy graph is introduced and discussed its related concepts.

Throughout this paper, denote the edge between two vertices u and v as uv.

2.1 Definition [5]

A fuzzy graph $G = (\sigma,\mu)$ defined on the underlying crisp graph $G^* = (V, E)$, where $E \subseteq V \times V$, is a pair of functions $\sigma: V\to [0,1]$ and $\mu:V\times V\to[0,1]$, μ is a symmetric fuzzy relation on σ such that $\mu(uv) \leq \min \{\sigma(u), \sigma(v)\}$ for all $u, v\in V$.

2.2 Definition [2, 7, 8]

Let X be a non-empty set. A multi-fuzzy set A in X is defined as a set of ordered sequences:

$A = \{(x , \mu_1(x), \mu_2(x),.....,\mu_i(x),\ldots) : x \in X\}$, where $\mu_i : X \to [0,1]$ for all i .

2.3 Remark:

i. If the sequences of the membership functions have only k-terms (finite number of terms), k is called the dimension of A.

2.4 Definition

A multi fuzzy graph (MFG) of dimension m defined on the underlying crisp graph $G^* = (V, E)$, where $E \subseteq V \times V$, is denoted as $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$, and $\sigma_i : V \to [0, 1]$ and $\mu_i : V \times V \to [0, 1]$, μ_i is a symmetric fuzzy relation on σ such that $\mu_i(uv) \leq \min \{ \sigma_i(u) , \sigma_i(v) \}$, for all $i = 1,2, 3, \ldots,m$; $u, v \in V$ and $uv \in E$.

2.5 Remark:

a. MFG with dimension 1 is the usual fuzzy graph.
b. MFG with dimension 2 is different from intuitionistic fuzzy graph.

2.6 Definition

Let $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m. Then the cardinality of a MFG G is denoted as $|G|$ and is defined as

$$|G| = \sum_{v_i\in V}\frac{1+\sigma_1(v_i)+\sigma_2(v_i)+\ldots+\sigma_m(v_i)}{m}+\sum_{v_iv_j\in E}\frac{1+\mu_1(v_iv_j)+\mu_2(v_iv_j)+\ldots+\mu_m(v_iv_j)}{m}.$$

2.7 Definition

Let $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m. Then the vertex cardinality of a MFG G or the order of a MFG is denoted as $|V|$ or O(G) or p and is defined as

$$|V| = \sum_{v_i\in V}\frac{1+\sigma_1(v_i)+\sigma_2(v_i)+\ldots+\sigma_m(v_i)}{m}.$$

2.8 Definition

Let $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m. Let $D \subseteq V$. Then the cardinality of D of G or the fuzzy cardinality of D of G is denoted as $|D|$ and is defined as

$$|D| = \sum_{v_i\in D}\frac{1+\sigma_1(v_i)+\sigma_2(v_i)+\ldots+\sigma_m(v_i)}{m}.$$

2.9 Definition

Let $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m. Then the edge cardinality of a MFG G or the size of a MFG G is denoted as $|E|$ or S(G) or q and is defined as $|E| = \sum_{uv\in E}\frac{1+\mu_1(uv)+\mu_2(uv)+\ldots+\mu_m(uv)}{m}$.

2.10 Definition

Let G = ((σ_1, $\sigma_{2, \ldots,}$ σ_m), (μ_1, $\mu_{2, \ldots,}$ μ_m)) be a MFG of dimension m. Then the degree of a vertex u $\in$ V in G is defined as $d_G(u) = \sum_{v \in V} \frac{1+\mu_1(uv)+\mu_2(uv)+\ldots+\mu_m(uv)}{m}$.

2.11 Definition

Let G = ((σ_1, $\sigma_{2, \ldots,}$ σ_m), (μ_1, $\mu_{2, \ldots,}$ μ_m)) be a MFG of dimension m. The minimum degree of G is defined as $\delta(G) = \min \{d_G(u) / u \in V\}$.

2.12 Definition

Let G = ((σ_1, $\sigma_{2, \ldots,}$ σ_m), (μ_1, $\mu_{2, \ldots,}$ μ_m)) be a MFG of dimension m. The maximum degree of G is defined as $\Delta(G) = \max \{d_G(u) / u \in V\}$.

2.13 Definition

Let G = ((σ_1, $\sigma_{2, \ldots,}$ σ_m), (μ_1, $\mu_{2, \ldots,}$ μ_m)) be a MFG of dimension m. An edge uv = e is said to be an effective edge if $\mu_i(uv) = \min \{ \sigma_i(u), \sigma_i(v) \}$, for all i = 1,2, 3, …,m.

2.14 Definition

Let G = ((σ_1, $\sigma_{2, \ldots,}$ σ_m), (μ_1, $\mu_{2, \ldots,}$ μ_m)) be a MFG of dimension m. Then the effective degree of a vertex u $\in$ V in G is defined as $d_E(u) = \sum_{v \in V} \frac{1+\mu_1(uv)+\mu_2(uv)+\ldots+\mu_m(uv)}{m}$; uv is an effective edge .

2.15 Definition

Let G = ((σ_1, $\sigma_{2, \ldots,}$ σ_m), (μ_1, $\mu_{2, \ldots,}$ μ_m)) be a MFG of dimension m. The minimum effective degree of G is defined as $\delta_E(G) = \min \{d_E(u) / u \in V\}$.

2.16 Definition

Let G = ((σ_1, $\sigma_{2, \ldots,}$ σ_m), (μ_1, $\mu_{2, \ldots,}$ μ_m)) be a MFG of dimension m. The maximum effective degree of G is defined as $\Delta_E(G) = \max \{d_E(u) / u \in V\}$.

2.17 Definition

Let G = ((σ_1, $\sigma_{2, \ldots,}$ σ_m), (μ_1, $\mu_{2, \ldots,}$ μ_m)) be a MFG of dimension m. The vertices u and v in V of G is said to be independent if $\mu_i(uv) < \min \{ \sigma_i(u), \sigma_i(v) \}$, for some i = 1,2, 3, …,m.

The subset S of V is said to be an independent set if for any two vertices u and v in S $\subseteq$ V of G is independent.

2.18 Definition

Let G = ((σ_1, $\sigma_{2, \ldots,}$ σ_m), (μ_1, $\mu_{2, \ldots,}$ μ_m)) be a MFG of dimension m. Let u $\in$ V of G. Then the neighbors (neighborhood) of u or an open neighbors of u $\in$ V of G is denoted by N(u) and is defined as

$N(u) = \{ v \in V / \mu_i(uv) = \min \{ \sigma_i(u), \sigma_i(v) \}$, for all i = 1,2, 3, …,m}.

The closed neighbors of u $\in$ V of G is denoted by N[u] and is defined as N[u] = N(u) $\cup$ {u}.

2.19 Definition

Let G = ((σ_1, $\sigma_{2, \ldots,}$ σ_m), (μ_1, $\mu_{2, \ldots,}$ μ_m)) be a MFG of dimension m. The vertex u $\in$ V of G is said to be isolated vertex if

$\mu_i(uv) < \min \{ \sigma_i(u), \sigma_i(v) \}$, for some i = 1,2, 3, …,m and for all v $\in$ V – {u}.

In other words, The vertex u $\in$ V of G is said to be isolated vertex if N(u) = ϕ or | N(u) | = 0.

2.20 Definition

Let G = ((σ_1, $\sigma_{2, \ldots,}$ σ_m), (μ_1, $\mu_{2, \ldots,}$ μ_m)) be a MFG of dimension m. The vertex u $\in$ V of G is said to be pendant vertex if | N(u) | = 1. An edge incident on the pendant vertex is called a pendant edge.

2.21 Definition

Let G = ((σ_1, $\sigma_{2,} \ldots, \sigma_m$), ($\mu_1$, μ_2, …., μ_m)) be a MFG of dimension m. Then the neighborhood degree of a vertex u ∈V in G is defined as $d_N(u) = \sum_{v \in N(u)} \frac{1+\sigma_1(v)+\sigma_2(v)+\ldots+\sigma_m(v)}{m}$.

2.22 Definition

Let G = ((σ_1, $\sigma_{2,} \ldots, \sigma_m$), ($\mu_1$, μ_2, …., μ_m)) be a MFG of dimension m. The minimum neighborhood degree of G is defined as $\delta_N(G) = \min\{d_N(u) / u \in V\}$.

2.23 Definition

Let G = ((σ_1, $\sigma_{2,} \ldots, \sigma_m$), ($\mu_1$, μ_2, …., μ_m)) be a MFG of dimension m. The maximum neighborhood degree of G is defined as $\Delta_N(G) = \max\{d_N(u) / u \in V\}$.

2.24 Proposition

Let G = ((σ_1, $\sigma_{2,} \ldots, \sigma_m$), ($\mu_1$, μ_2, …., μ_m)) be a MFG of dimension m. The sum of the degree of all vertices in a MFG is equal to twice the sum of the membership values of all edges. That is, $\sum_{u \in V} d_G(u) = 2|E|$.

Proof:

Let G = ((σ_1, $\sigma_{2,} \ldots, \sigma_m$), ($\mu_1$, μ_2, …., μ_m)) be a MFG of dimension m. Let u, v ∈V of G and uv ∈ E. Then, it is clear from the definition that, the edge uv contribute the value $\frac{1+\mu_1(uv)+\mu_2(uv)+\ldots+\mu_m(uv)}{m}$ twice to the sum of degree of all vertices of G. That is,

$$\sum_{u \in V} d_G(u) = 2 \times \sum_{uv \in E} \frac{1+\mu_1(uv)+\mu_2(uv)+\ldots+\mu_m(uv)}{m} = 2|E|.$$

2.25 Example

Consider G = ((σ_1, σ_2, σ_3), (μ_1, μ_2, μ_3)) be a MFG of dimension 3. Here,

V = { a, b, c, d} ; E = { ab, bc, cd, da,bd }

All edges except bd and bc are effective edges.

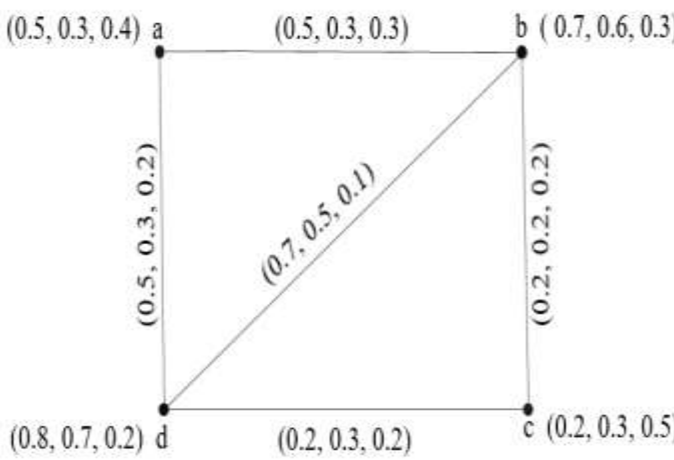

Fig. 2.1
MFG with dimension 3

In figure 2.1,

$$d_G(u) = \sum_{uv \in E} \frac{1+\mu_1(uv)+\mu_2(uv)+\ldots+\mu_m(uv)}{m}$$

$$d_E(u) = \sum_{v \in V} \frac{1+\mu_1(uv)+\mu_2(uv)+\ldots+\mu_m(uv)}{m};$$

uv is an effective edge

$d_G(a) = 0.7 + 0.667 = 1.367$ $d_E(a) = 0.7 + 0.667 = 1.367$

$d_G(b) = 0.7 + 0.533 + 0.767 = 2$ $d_E(b) = 0.7 = 0.7$

$d_G(c) = 0.533 + 0.567 = 1.1$ $d_E(c) = 0.567 = 0.567$

$d_G(d) = 0.667 + 0.567 + 0.767 = 2.001$ $d_E(d) = 0.667 + 0.567 = 1.234$

$\delta(G) = \min \{d_G(u) / u \in V\} = 1.1$ $\delta_E(G) = \min \{d_E(u) / u \in V\} = 0.567$

$\Delta(G) = \max \{d_G(u) / u \in V\} = 2.001$ $\Delta_E(G) = \max \{d_E(u) / u \in V\} = 1.367$

$N(u) = \{v \in V / \mu_i(uv) = \min\{\sigma_i(u), \sigma_i(v)\}$

$$d_N(u) = \sum_{v \in N(u)} \frac{1+\sigma_1(v)+\sigma_2(v)+\ldots+\sigma_m(v)}{m}$$

$N(a) = \{ b , d \}$ $d_N(a) = 0.867 + 0.9 = 1.767$

$N(b) = \{ a \}$ $d_N(b) = 0.733 = 0.733$

$N(c) = \{ d \}$ $d_N(c) = 0.9 = 0.9$

$N(d) = \{ a , c \}$ $d_N(d) = 0.733 + 0.667 = 1.4$

$|V| = 0.733 + 0.867 + 0.667 + 0.9 = 3.167$ $|E| = 0.7 + 0.533 + 0.567 + 0.667 + 0.767 = 3.234$

$|G| = 3.167 + 3.234 = 6.401$

$$\sum_{u \in V} d_G(u) = 1.367+2+1.1+2.001= 6.468=2\times|E|$$

The sets { a, c }, { b, c } and { b, d }are independent sets.

2.26 Observations

Let $G = ((\sigma_1, \sigma_{2, \ldots,} \sigma_m), (\mu_1, \mu_{2, \ldots,} \mu_m))$ be a MFG of dimension m. Let n be the number of vertices of G.

i. $d_E(u) \le d_G(u)$, for all $u \in V$ of G.

ii. $\delta_E(G) \le \delta(G)$.

iii. $\Delta_E(G) \le \Delta(G)$.

iv. $\Delta(G) \le S(G)$.

v. $S(G) \le O(G)$ if G has equal numbers of vertices (n) and edges and $n > 3$.

vi. $\delta(G) \le \Delta(G) \le S(G) \le O(G)$ if G has equal numbers of vertices (n) and edges and $n > 3$.

vii. $\delta_E(G) \le \Delta_E(G) \le S(G) \le O(G)$ if G has equal numbers of vertices (n) and edges and $n > 3$.

viii. $N(u) \subseteq N[u]$, for all $u \in V$ of G.

2.27 Definition

Let $G = ((\sigma_1, \sigma_{2, \ldots,} \sigma_m), (\mu_1, \mu_{2, \ldots,} \mu_m))$ be a MFG of dimension m. Then G is said to be

ISSN- 2394-5125 VOL 7, ISSUE 15, 2020

i. a strong MFG of dimension m if $\mu_i(uv) = \min\{\sigma_i(u), \sigma_i(v)\}$, for all $i = 1,2,3,\ldots,m$; $u, v \in V$ and $uv \in E$.
ii. a complete MFG of dimension m if $\mu_i(uv) = \min\{\sigma_i(u), \sigma_i(v)\}$, for all $i = 1,2,3,\ldots,m$; $u, v \in V$ and $uv \in E$.
iii. a regular MFG of dimension m if $d_G(u) = k$ for all $u \in V$. That is, if each vertex has a same degree k then G is said to be a regular MFG of degree k or k- regular MFG.
iv. an irregular MFG of dimension m if there is a vertex which is adjacent only to vertices with distinct degrees.
v. a highly irregular MFG of dimension m if every vertex of V in G is adjacent to vertices with distinct degrees.
vi. a neighborly irregular MFG of dimension m if every two adjacent vertices of G have distinct degrees.

2.28 Example

Let us discuss the examples of definition 2.15.

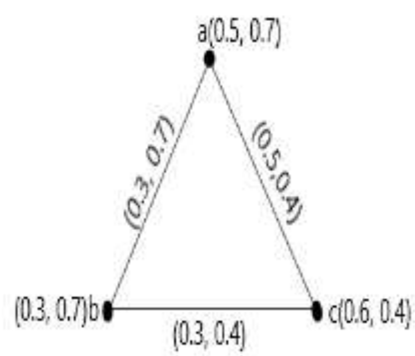

Fig. 2.2
Strong MFG of dimension 2
Complete MFG of dimension 2

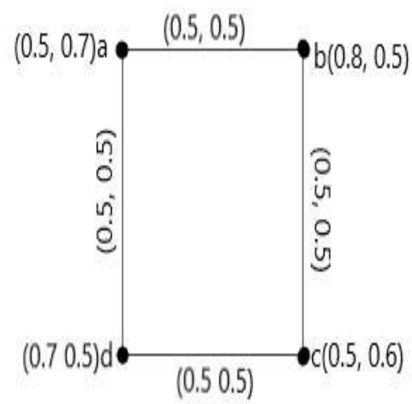

Fig. 2.3
Regular MFG of dimension 2
2– Regular MFG

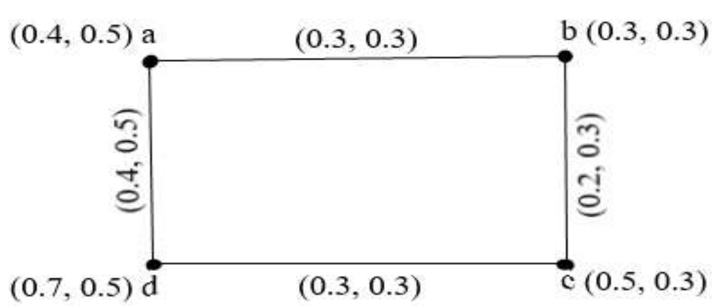

Fig. 2.4
MFG of dimension 2
Irregular, highly irregular but not a neighborly irregular

Here, $d_G(a) = 1.75$; $d_G(b) = 1.55$; $d_G(c) = 1.55$; $d_G(d) = 1.75$.

For every vertex v in V of G, the adjacent vertices have distinct degrees. Hence G is highly irregular MFG. But $d_G(a) = d_G(d) = 1.75$ and $d_G(b) = d_G(c) = 1.55$ then G is not a neighborly irregular MFG.

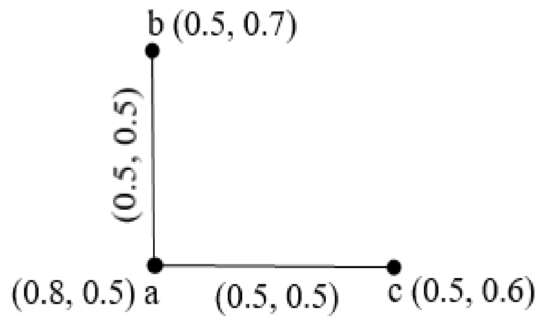

Fig. 2.5
MFG of dimension 2
Irregular, neighborly irregular but not highly irregular

Here, $d_G(a) = 2$; $d_G(b) = 1$; $d_G(c) = 1$.
No two adjacent vertices have the same degree. So G is neighborly irregular MFG.
The vertex 'a' is adjacent to the vertex 'b' and 'c', but $d_G(b) = d_G(c) = 1$.
Hence, G is not a highly irregular MFG.

2.29 Remark:

i. A highly irregular MFG need not be a neighborly irregular MFG.
ii. A neighborly irregular MFG need not be a highly irregular MFG.

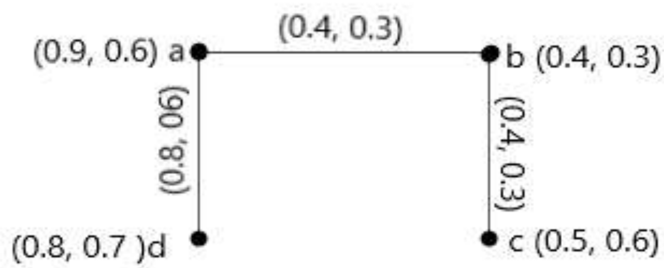

Fig. 2.6
MFG of dimension 2
Irregular, neighborly irregular and highly irregular

Here, $d_G(a) = 2.05$; $d_G(b) = 1.7$; $d_G(c) = 0.85$; $d_G(d) = 1.2$.
No two adjacent vertices have the same degree. So G is neighborly irregular MFG.
For every vertex $v \in V$ of G, the adjacent vertices have distinct degrees. Hence G is highly irregular MFG.

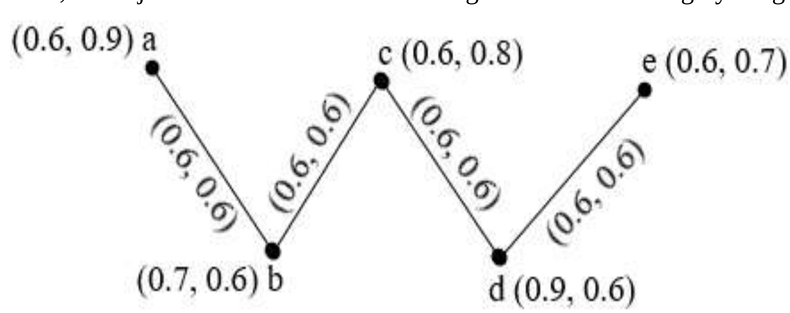

Fig. 2.7
MFG of dimension 2

Irregular but neither neighborly irregular nor highly irregular

Here, $d_G(a) = 1.1$; $d_G(b) = 2.2$; $d_G(c) = 2.2$; $d_G(d) = 2.2$; $d_G(e) = 1.1$.

The vertex 'b' and 'c' are adjacent and $d_G(b) = d_G(c) = 2.2$ and the vertex 'c' and 'd' are adjacent and $d_G(c) = d_G(d) = 2.2$. Hence G is not a neighborly irregular MFG.

The vertex 'c' is adjacent to the vertex 'b' and 'd', but $d_G(b) = d_G(d) = 2.2$.

Hence, G is not a highly irregular MFG.

3. Domination on MFG

In this section, the concept of domination on MFG is defined and its domination number is obtained for MFG with examples. Throughout this section, domination on MFG using effective edges only considered.

3.1 Definition

Let $G = ((\sigma_1, \sigma_{2, \ldots,} \sigma_m), (\mu_1, \mu_{2, \ldots,} \mu_m))$ be a MFG of dimension m. Let u, v $\in$ D. We say that u dominates v in G if $\mu_i(uv) = \min \{ \sigma_i(u), \sigma_i(v) \}$, for all i = 1,2, 3, …,m ; uv $\in$ E.

A subset D of V is said to be a dominating set in G if for every v $\in$ V – D there exist u $\in$ D such that u dominates v.

A dominating set D of V is said to be a minimal dominating set if no proper subset of D is a dominating set of G.

The minimum fuzzy cardinality of a minimal dominating set in G is called the domination number of G and is denoted by $\gamma(G)$ or simply γ.

3.2 Example

Consider the MFG with dimension 3.

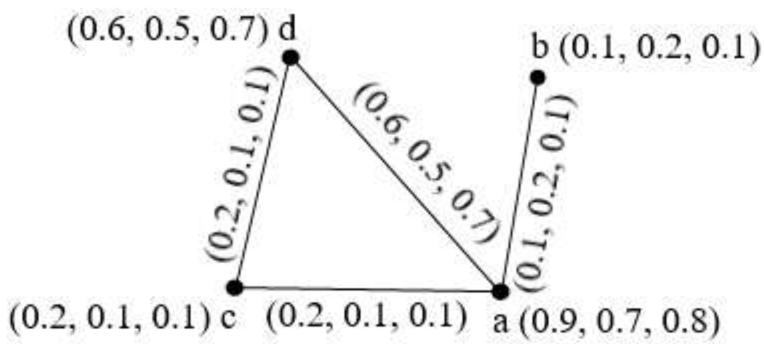

Fig. 2.8

MFG of dimension 3

Here, $D_1 = \{ a \}$; $D_2 = \{ b, c \}$; $D_3 = \{ b, d \}$ are the minimal dominating sets of G with cardinality 1.133, 0.934, 1.4 respectively and the domination number is $\gamma = 0.934$.

3.3 Remark

Let $G = ((\sigma_1, \sigma_{2, \ldots,} \sigma_m), (\mu_1, \mu_{2, \ldots,} \mu_m))$ be a MFG of dimension m.

a. For any u, v $\in$ V, if u dominates v then v dominates u and hence domination is a symmetric relation on V.
b. For any u $\in$ V, N(u) is precisely the set of all v$\in$V which are dominated by u.
c. If $\mu_i(uv) < \min \{ \sigma_i(u), \sigma_i(v) \}$ for all u, v $\in$ V, then obviously the only dominating set in G is V.
d. An isolated vertex does not dominate any other vertex in G.

The following Theorem gives a characterization of minimal dominating sets which is analogous to the result of A.Somasundaram and S. Somasundaram (1998) [7] in the case of a fuzzy graph.

3.4 Theorem

Let $G = ((\sigma_1, \sigma_{2, \ldots,} \sigma_m), (\mu_1, \mu_{2, \ldots,} \mu_m))$ be a MFG of dimension m. A dominating set D of G is a minimal dominating set if and only if for each u$\in$D, one of the following two conditions holds.

i. $N(u) \cap D = \phi$.

ii. There is a vertex $v \in V - D$ such that $N(v) \cap D = \{u\}$.

Proof

Let $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m. Let D be a minimal dominating set of G and $u \in D$. Let $D_u = D - \{u\}$. Then D_u is not a dominating set as D is a minimal dominating set. Hence there exists $v \in V - D_u$ such that v is not dominated by any element of D_u.

Case i: If v = u, then v = u is not dominated by any element of D_u and hence it is not dominated by any element of D and hence, $N(u) \cap D = \phi$.

Case ii: If $v \neq u$ then u dominates v as D is a minimal dominating set of G and hence, $N(v) \cap D = \{u\}$.

Conversely, let D be a dominating set of G and for each $u \in D$, one of the following two conditions holds.

i. $N(u) \cap D = \phi$.

ii. There is a vertex $v \in V - D$ such that $N(v) \cap D = \{u\}$.

Suppose if D is not a minimal dominating set of G then $D_1 \subset D$ is a dominating set of G.

Consider an element $u \in D$ and $u \notin D_1$. Then $u \in V - D_1$ and there exists $w \in D_1$ such that w dominates u and so $w \in N(u)$. Also $w \in D_1 \subset D$ and hence $N(u) \cap D \neq \phi$.

Given D is not a minimal dominating set, then there is a vertex $v \in V - D$ such that either v is dominated by more than one vertex of D or there exist an element $u \in D$ such that u does not dominate any v for all $v \in V - D$.

Case i: Let $u, w \in D$ dominates v and $u, w \in N(v)$. Then $N(v) \cap D = \{u, w\} \neq \{u\}$.

Case ii: Then for this $u \in D$, $N(v) \cap D \neq \{u\}$ for all $v \in V - D$.

Hence, conditions i and ii do not hold because of the assumption that D is not a minimal dominating set of G. Hence D is a minimal dominating set of G.

3.5 Theorem

Let $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m without isolated vertices. Let D be the minimal dominating set of G. Then V – D is a dominating set of G.

Proof

Let $G = ((\sigma_1, \sigma_2, \ldots, \sigma_m), (\mu_1, \mu_2, \ldots, \mu_m))$ be a MFG of dimension m without isolated vertices. Let D be the minimal dominating set of G. Let $u \in D$. Since G has no isolated vertices then $v \in N(u)$.

Case i: If $v \in V - D$, then, every element of D is dominated by some element of V – D. Hence, V – D is a dominating set of G.

Case ii: If $v \in D$ and D is a minimal dominating set, then, there exists an element $x \in V - D$ such that $x \in N(u)$.

That is, for every element $u \in D$, there exists an element $x \in V - D$ such that x dominates u. Hence V – D is a dominating set of G.

3.6 Theorem

For any MFG without isolated vertices, $\gamma \leq \frac{p}{2}$, where p is the order of MFG.

Proof

Any MFG without isolated vertices has two disjoint dominating sets and hence $\gamma \leq \frac{p}{2}$, where p is the order of MFG.

3.7 Conclusion

The authors introduced the concept of multi fuzzy graph and its based definitions and the concept of domination on multi fuzzy graph The application of multi fuzzy graph and operations and other dominations on multi fuzzy graph will be reported in forth coming papers.

Acknowledgement

The authors are more grateful for your valuable comments and suggestions in improving this paper.

References

[1] Kaufmann. A., "Introduction to the theory of Fuzzy Subsets", Academic Press, Newyork (1975).

[2] Muthuraj. R, Balamurugan. S., Multi – fuzzy group and its level subgroups, Gen. Math. Notes, Vol. 17, No. 1, July, 2013, pp. 74 – 81.

[3] Nagoor Gani. A and Basheer Ahamed. M, Order and Size in Fuzzy graph, Bulletin of pure and applied sciences, Vol.22E (No.1) 2003 P. 145 – 148.

[4] Nagoor Gani A, K. Radha, "On regular fuzzy graphs", Journal of Physical Sciences, Vol.12, 2008, pp. 33-40.

[5] Nagoor Gani A, S.R. Latha, "On Irregular fuzzy graphs", Applied mathematical sciences, Vol.6, 2012, no. 11, pp. 517-523.

[6] Rosenfeld A., Fuzzy graphs, In: L.A.Zadeh, K.S.Fu and M.Shimura, Eds, Fuzzy Sets and their Applications, Academic Press, New York, (1975), pp. 77- 95.

[7] Sabu. S and.Ramakrishnan T.V, 2010, "Multi-fuzzy sets," International Mathematical Forum, 50, pp. 2471-2476.

[8] Sabu S and.Ramakrishnan T.V, 2011, "Multi-fuzzy Topology,"International Journal of Applied Mathematics, Vol 24, No.1, pp. 117-129.

[9] Somasundaram . A and Somasundaram .S., Domination in Fuzzy Graphs, Pattern Recognition Lett., 19 (1998), pp.787-791.

[10] Somasundaram . A., Domination in Fuzzy Graphs-II, Journal of Fuzzy Mathematics, 13 (2) (2005), pp.281-288.

[11] Zadeh L.A., Fuzzy sets, Information and Control, 8 (1965), pp.338-353.

[12] Zadeh L.A., Toward a generalized theory of uncertainty (GTU)-an outline, Information Sciences, 172 (1-2) (2005), pp.1-40.