

TYPE-3 FUZZY SHORTEST CYCLE ROUTE PROBLEM BY OAM

¹S.Krishna Prabha, ²K.Jeyalakshmi

¹Assistant Professor, Department of Mathematics, PSNA College of Engineering and Technology, Dindigul, Tamilnadu.

²Professor, Department of Physics, PSNA College of Engineering and Technology, Dindigul, Tamilnadu

Email:¹ jvprbh1@gmail.com

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ABSTRACT: In the conjecture of decisiveness the concern which is confer about after Transportation and Assignment problem is the Travelling sales man problem. Shortest Cycle Route Problem alludes in minimizing the trail in a network. In this exertion we conveyed a new tactics for unraveling a Shortest Cyclic Route Problem with crisp, fuzzy and Intuitionistic fuzzy numbers as cost coefficients. A Type-3 Fuzzy Shortest Cyclic Route Problem (T-3FSCRP) is unraveled by exerting Reduced Matrix Method (RMM). This pattern is easy to understand and offer an effective tool for handling all types of FSCRP.

KEYWORDS: Type-3 Fuzzy Shortest Cyclic Route Problem, Triangular intuitionistic fuzzy numbers, and mixed constraints

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I. INTRODUCTION

The quest of locating a shortest possible route that visits each city precisely once for the prearranged catalog of cities and their pair wise distance is called the SCRP. W.R Hamilton in 19th century was the foremost to instigate SCRP. The SCRP has assorted applications even in its purest formulation, such as planning, logistics, and the manufacture of microchips. If a slight abatement is made in the predicament, it emerges as a sub-problem in many areas, such as genome sequencing. Dantzig and Fulkerson [6] have reinforced the elucidation of a large-scale SCRP. A novel technique branded as zero suffix method to unravel SCRP's has been inaugurated by Sudhakar and Navaneetha Kumar [18]. Hadi Basirzadeh [7] has launched the Ones Assignment Method for untangling Traveling Salesman Problem. Variations in measurement, deviation in accuracy, errors in computation leads to uncertainty and in exactness. In order to dole out with this uncertainty we employ fuzzy assignment problems rather than classical assignment problems. If it is not viable to clarify an imprecise notion by manipulating the conventional fuzzy set, the Intuitionistic fuzzy set can be used. The membership of an element to a fuzzy set is a single value with the degree of acceptance between zero and one. Optimization in intuitionistic fuzzy environment was anticipated by Angelov[3]. The scheme of intuitionistic fuzzy set (IFS) instigated by Atanassov [4,5] is the generalization of Zadeh's [19] fuzzy set.

Assignment and Travelling Salesman Problems with Coefficients as *LR* Fuzzy Parameters have been examined by Amit Kumar and Anila Gupta [1]. Travelling Salesman Problem (SCRP) exerting Fuzzy Quantifier is evolved by Nirmala and Anju [10]. Neighboring Optimal Solution for Fuzzy Travelling Salesman Problem is posited by Stephen Dingar and Thiripura Sundari[17]. Srinivasan and Geetharamani [16] have utilized discrete fuzzy numbers and elucidated a new algorithim for decipher a Fuzzy SCRP. New Algorithm for solving Type-3 fuzzy Assignment Problem have been perceived by Senthil Kumar and Jahir Hussain[13]. Krishna Prabha and Vimala [8] have elucidated three different methods of finding the elucidation for Intuitionistic fuzzy Assignment Problem. Shweta Rana [15] applied Genetic Algorithm to solve Travelling Salesman problem. Pavithraa [11] proposed a simple method for solving multi objective travelling salesman problem. Amit Kumar Rana [2] used Haar Hungarian algorithm approach to solve a Fuzzy Travelling Salesman Problem. Mukherjee[9] introduced some constrained covering solid travelling salesman problems.

This paper has been written as follows. In Section 2, some elementary interpretation on IFS, TrIFN'S, have been narrated and the mathematical replica of SCRP and ranking methods for converting TrIFN'S and triangular fuzzy numbers to crisp numbers is explained. In Section 3, the algorithm for solving Type-3 Fuzzy Shortest Cyclic Route Problem has been elucidated. In section 4 the above algorithm is interpreted with a numerical example. Section 5 concludes the paper.

II. PRELIMINARIES

In this module, some rudimentary connotation and arithmetic operations are reviewed

2.1 Intuitionistic fuzzy number:

Assorted precisions for Intuitionistic fuzzy sets were delineated by Atanassov [4, 5], Sagaya Roseline , Henry Amirtharaj [12] and Senthil kumar[13,14].

Definition 2.1.1: An IF Set Γ in X is given by $\Gamma = \{x, m_\Gamma(x), n_\Gamma(x) / x \in X\}$ where the functions, $m_\Gamma, n_\Gamma : X \rightarrow [0, 1]$ are functions such that $0 \leq m_\Gamma(x), n_\Gamma(x) \leq 1, \forall x \in X$. For each, the numbers $m_\Gamma(x)$ and $n_\Gamma(x)$ portray the degree of membership and degree of non-membership of the element $x \in X$.

Definition 2.1.2:

A TrIFN $\tilde{\Gamma}^I$ is an intuitionistic fuzzy set in R with the subsequent membership function $m_\Gamma(x)$ and non membership function $n_\Gamma(x)$

$$m_\Gamma(x) = \begin{cases} 0 & \text{if } x \leq \alpha \\ \frac{x-\alpha_1}{\alpha_2-\alpha_1} & \alpha_1 \leq x \leq \alpha_2 \\ 1 & x = \alpha_2 \\ \frac{\alpha_3-x}{\alpha_3-\alpha_2} & \alpha_2 \leq x \leq \alpha_3 \\ 0 & x > \alpha_3 \end{cases} \quad \text{and } n_\Gamma(x) = \begin{cases} \frac{1}{\alpha_2-x} & x < \alpha' \\ \frac{\alpha_2-x}{\alpha_2-\alpha'_1} & \alpha'_1 \leq x \leq \alpha_2 \\ 0 & x = \alpha_2 \\ \frac{x-\alpha_2}{\alpha'_3-\alpha_2} & \alpha_2 \leq x \leq \alpha'_3 \\ 1 & x \geq \alpha'_3 \end{cases}$$

Where $\alpha'_1 \leq \alpha_1 \leq \alpha_2 \leq \alpha_3 \leq \alpha'_3$ and $m_\Gamma(x), n_\Gamma(x) \leq 0.5$ for $m_\Gamma(x) = n_\Gamma(x), \forall x \in R$.

This TrIFN is represented by $\tilde{\Gamma}^I = (\alpha_1, \alpha_2, \alpha_3) (\alpha'_1, \alpha_2, \alpha'_3)$

Definition 2.1.3: The SCRP is supposed to be Type-3 F SCRP or mixed FSCRP if all the parameters of the TSP are crisp numbers, triangular fuzzy numbers or intuitionistic triangular fuzzy numbers.

2.2 Ranking Techniques:

Defuzzification: Defuzzification is the procedure of detecting singleton value (crisp value) which delineates the average value of the TrIFNs.

2.2.1. Yager's Ranking Technique

Yager's ranking technique which gratifies recompense, linearity, additivity properties and presumes verdicts which consists of human intuition. For a convex fuzzy number \tilde{a} , the Robust's Ranking Index is interpreted by,

$$R(\tilde{a}) = \int_0^1 (0.5)(a_{\infty}^L, a_{\infty}^U) d\alpha \quad \dots \quad (1)$$

Where $(a_{\infty}^L, a_{\infty}^U) = \{(b-a)\alpha + a, c - (c-b)\alpha\}$ which is the α - level cut of the fuzzy number \tilde{a}

2.2.2 Ranking of triangular intuitionistic fuzzy numbers

The Ranking of a TrIFN $\tilde{\Gamma}^I = (\alpha_1, \alpha_2, \alpha_3) (\alpha'_1, \alpha_2, \alpha'_3)$ is expounded by

$$R(\tilde{\Gamma}^I) = \frac{1}{3} \left[\frac{(\alpha'_3 - \alpha'_1)(\alpha_3 - 2\alpha'_1 - 2\alpha'_3) + (\alpha_3 - \alpha_1)(\alpha_1 + \alpha_2 + \alpha_3) + 3(\alpha'_3 - \alpha'_1)}{\alpha'_3 - \alpha'_1 + \alpha_3 - \alpha_1} \right] \quad \dots \quad (2)$$

The ranking technique [4] is: If $R(\tilde{\Gamma}^I) \leq R(\tilde{\Gamma}^I)$, then $\tilde{\Gamma}^I \leq \tilde{\Gamma}^I$ i.e, $\min \{ \tilde{\Gamma}^I, \tilde{\Gamma}^I \} = \tilde{\Gamma}^I$

2.3 Mathematical Formulation of SCRP:

Mathematically a SCRP can be framed as given bellow:

Optimize

$$\sum_{i=1}^n \sum_{j=1}^n d_{ij} x_{ij} \quad (i)$$

Subject to

$$\sum_{j=1}^n x_{ij} = 1, i = 1, 2, 3, \dots, n.$$

$$\sum_{i=1}^n x_{ij} = 1, j = 1, 2, 3, \dots, n.$$

$$x_{ij} = 0 \text{ or } 1, i = 1, 2, 3, \dots, n, j = 1, 2, 3, \dots, n. \quad (ii)$$

Where d_{ij} is the distance from the city ' i ' to city ' j ', and x_{ij} is to be some positive integer or zero, and the only possible integer is one, so the condition of

$x_{ij} = 0 \text{ or } 1$, is automatically satisfied .

III. INTUITIONISTIC FUZZY ONES ASSIGNMENT METHOD (IFOAP)

Ones Assignment Method encompass this name since the assignments are made in requisites of ones.

Step 1. Outline the cost table from the given IFAP.

Step 2. Stumble on the prospect cost table

(a) Locate the smallest/largest component in each row of the matrix and then divide that from each component of that row. By means of this maneuver we get at least one ones in each row.

(b) Trace the smallest/largest component in each column of the truncated matrix and then divide that from each component of that column. By this operation we get at least one ones in each columns and allotment are made in terms of one for IFAP.

Step 3. Wrap all zeros in the IFA matrix by means of minimum number of horizontal and vertical lines.

Assessment for Optimality.

i) If the number of lines is precisely equal to n , then the entire allotment is acquired.

ii) If the number of lines plotted is less than n , then the entire allotments is not probable and proceed to step 4.

Step 4

Verify the smallest/largest element (say d_{ij}) not enclosed by any line. Then divide by d_{ij} each element of the revealed rows or columns, which d_{ij} lies on it. This operation fabricates some new ones to this row or column.

If still an intact optimal allotment is not reached in this novel matrix, then ply step 4 and 3 iteratively. By reiterating the same procedure the optimal allotment will be achieved. Precedence plays a significant task in this technique, when we desire to consign the ones.

IV. NUMERICAL EXAMPLE:

The proposed scheme has been explained through the example given below,

$$\begin{array}{ccccc} & A & B & C & \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \left(\begin{array}{ccccc} - & (10,30,50) & (40,60,80)(20,60,100) & - & \\ 30 & - & (30,50,70) & - & \\ (40,60,80) & (30,50,70)(20,50,90) & - & (40,60,80)(20,60,100) & \\ 20 & 20 & (40,60,80) & - & (40,60,80)(20,60,100) \\ (10,30,50) & 30 & (20,40,60)(10,40,70) & (40,60,80) & - \end{array} \right) & D & E & (10,30,50) \end{array}$$

By using (1) and (2) the triangular and intuitionistic triangular fuzzy numbers are rehabilitated into crisp numbers.

After defuzzification the cost matrix of the given T-3FSCRP is given by

$$\begin{pmatrix} \infty & 30 & 60 & 20 & 30 \\ 30 & \infty & 50 & 20 & 30 \\ 60 & 50 & \infty & 60 & 40 \\ 20 & 20 & 60 & \infty & 60 \\ 30 & 30 & 40 & 60 & \infty \end{pmatrix}$$

Locate the minimum element of each row in the assignment matrix (say a_i), and mark it on RHS. Then divide each element of i^{th} row of the matrix by a_i . Thus it creates ones to each row, and the matrix reduces to following matrix.

$$\begin{pmatrix} \infty & 1.5 & 3 & 1 & 1.5 \\ 1.5 & \infty & 2.5 & 1 & 1.5 \\ 1.5 & 1.25 & \infty & 1.5 & 1 \\ 1 & 1 & 3 & \infty & 3 \\ 1 & 1 & 1.3 & 2 & \infty \end{pmatrix}$$

Locate the minimum element of each column in assignment matrix (say b_j), and mark it beneath that column. Then divide each element of j^{th} column of the matrix by b_j .

$$\begin{pmatrix} \infty & 1.5 & 2.3 & 1 & 1.5 \\ 1.5 & \infty & 1.9 & 1 & 1.5 \\ 1.5 & 1.25 & \infty & 1.5 & 1 \\ 1 & 1 & 2.3 & \infty & 3 \\ 1 & 1 & 1 & 2 & \infty \end{pmatrix}$$

Now we shall make the assignments in rows and columns having single ones .We get

$$\begin{pmatrix} \infty & 1.5 & 2.3 & (1) & 1.5 \\ 1.5 & \infty & 1.9 & \times & 1.5 \\ 1.5 & 1.25 & \infty & 1.5 & (1) \\ \times & (1) & 2.3 & \infty & 3 \\ \times & \times & (1) & 2 & \infty \end{pmatrix}$$

Since some rows and columns are devoid of assignment, the current elucidation is not optimal.Cover all the zeros by drawing minimum number of straight lines.

∞	1.5	2.3	(1)	1.5
1.5	∞	1.9	\times	1.5
1.5	1.25	∞	1.5	(1)
\times	(1)	2.3	∞	3
\times	\times	(1)	2	∞

The minimum number of lines requisite to pass through all ones is 4, and the minimum element of the uncovered rows is 1.5, so divide each uncovered element of the matrix by 1.5.

$$\begin{pmatrix} \infty & 1 & 1.5 & 1 & 1 \\ 1 & \infty & 1.26 & \times & 1 \\ 1.5 & 1.25 & \infty & 1.5 & 1 \\ \times & 1 & 2.3 & \infty & 3 \\ \times & \times & 1 & 2 & \infty \end{pmatrix}$$

Now we shall make the assignments in rows and columns having single ones.

$$\begin{pmatrix} \infty & \times & 1.5 & (1) & \times \\ (1) & \infty & 1.26 & \times & \times \\ 1.5 & 1.25 & \infty & 1.5 & (1) \\ \times & (1) & 2.3 & \infty & 3 \\ \times & \times & (1) & 2 & \infty \end{pmatrix}$$

Since each row and each column contains exactly one encircled ones, the current assignment is optimal.
City A assigns to City D → 20 Km,City B assigns to City A → 30 Km,City C assigns to City E → 40 Km,City D assigns to City B → 20 Km,City E assigns to City C → 40Km

The optimum schedule is given by A→D, B→A, C→E, D→B, and E→C
ie, A→D→B→A,C→E→C

And the resultant distance = (20+30+40+20+40) =150 Km.

But this assignment schedule does not offer the resolution of the SCRP, since it does not satisfy the route condition. Trace the next paramount resolution which satisfies the route condition also. The next minimum (non zero) cost element in the cost matrix is 1.25.so we attempt to bring 1.25 into this resolution.

We make assignment at (3, 2) as an alternative of zero assignment at (3, 5).The ensuing result is given by

$$\begin{pmatrix} \infty & \times & 1.5 & \times & (1) \\ 1 & \infty & 1.26 & (1) & \times \\ 1.5 & (1.25) & \infty & 1.5 & \times \\ 1 & \times & 2.3 & \infty & 3 \\ \times & \times & (1) & 2 & \infty \end{pmatrix}$$

The optimum schedule is given by A→E, B→D, C→B, D→A, E→C.

A→E→C→B→D→A

The optimum distance to be traversed is 160 Km.

V. CONCLUSION:

A MIFSCRP in the midst of crisp, fuzzy and Intuitionistic fuzzy is explored in this exertion. The fallouts disclose that the proposed strategies can effectively unravel the MIFSCRP. This scheme is effortless to recognize and it gives a systematic procedure for unraveling SCRP .These routines can be implemented in postal deliveries, inspection, television relays, assembly lines, job scheduling, to assign fleets of aircrafts, or assigning school buses to routes, or networking computers etc.

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