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A Study Of Ghost Dark Energy Fluid

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Abstract: The present work deals with a spatially homogeneous and anisotropic Kantowski-Sachs universe with anisotropic ghost dark energy fluid . The exact solutions of Einstein's field equations are obtained by considering the hybrid expansion law (HEL). It is seen that the anisotropic parameter of the universe and the skewness parameter of the dark energy tends to zero at later times.

Keywords: Kantowski-Sachs universe, Anisotropic ghost dark energy, Skewness parameter.

1. Introduction

Our universe is currently accelerating which is established by recent observations (Permutter et al. 1998; Riess et al. 1998; Riess et al. 1999; Spergel et al. 2003; Spergel et al. 2004; Tegmark et al. 2004). Dark energy is the most accepted hypothesis to the recent observations (Copeland et al. 2006; Frieman et al.). The supporting observations of dark energy includes Supernovae observations, CMB, BAO, LSS, the age of the universe compared to oldest stars (Amendola, Tsujikawa 2010). The astronomical observations of the present universe provide evidence for the existence of a kind of energy known as dark energy which governs the expansion of the universe. The present day acceleration is due to the negative pressure, positive energy density which violets SEC. The fluid, violating SEC is known as dark energy fluid. The violation of SEC gives a antigravitational effect. This results the universe to experience a sudden jerking and is responsible for making the universe accelerated (Caldwell et al. 2006). According to the observations cosmological constant is the simplest candidate of dark energy (Carroll 2001). There are some other candidates of dark energy, namely quintessence $(\omega > -1)$ (Steinhardt et al. 1999), phantom $(\omega < -1)$ (Caldwell 2002) etc. Some authors (Koivisto et al. 2008) have discussed that the models with anisotropic and viscous dark energy to explain the anomalous cosmological observation in CMB at the largest angles. Many authors have studied the effects of anisotropy in the early universe based on the recent observations (Battye et al. 2009; Campanelli et al. 2010; Appleby et al. 2011). Several authors (Akarsu et al. 2010; Kumar et al. 2011; Amirhashchi et al. 2011; Yadav et al. 2011) have also studied anisotropic dark energy models with constant deceleration parameter.

Recently, it is very interesting to study another type of dark energy called ghost dark energy (GDE). A new model of dark energy called Veneziano ghost dark energy has been recently proposed (Urban and Zhitnitsky, 2010; Ohta, N 2011; Cai, R. G 2011). The U(1) problem in low-energy effective theory can be explained by Veneziano ghost field. The ghost field has no contribution to the vacuum energy density in Minkowski space-time, but in a curved space-time, it contributes to the vacuum energy density proportional to $\Lambda^3_{QCD}H$ (Zhitnitsky 2010; Holdom 2011; Zhitnitsky 2011) where Λ_{QCD} is QCD mass scale and H is the Hubble parameter. The newly constructed ghost dark energy model is free from the fine tuning and cosmic coincidence problems. For the ghost dark energy the energy density is given by the relation $\rho_G = \tau H$ (Ohta 2011), where τ is a constant with dimension $[energy]^3$. Later a generalized model (Cai et al. 2012) has been introduced as $\rho_G = \tau H + \eta H^2$ with η as a constant parameter. Khurshudyan and Khurshudyan (2014) have investigated the interacting varying ghost dark energy models.

To investigate the expansion of the universe from the beginning, the geometric parameters such as Hubble parameter H and deceleration parameter q are chosen. But these fail to distinguish the cosmological models uniquely. Thus to investigate the character of the DE cosmological models Sahni et. al. (2003), Alam et. at. (2003) have introduced the pair $\{r, s\}$ the cosmological diagnosis and named it as Statefinder parameters. The pair $\{r, s\}$ is equal to $\{1, 0\}$ in the s-r plane for spatially flat Λ CDM model.

A large number worker (Ravishankar et al. 2013; Sarkar 2014; Mishra 2003) have investigated the Kantowski-Sachs model in different aspects. In this investigation our motive is to find the effect of magnetic field in the behavior of the dark energy. We have also investigated the statefinder parameter for this model. Here we have discussed the nature of EOS parameter and it is observed that the EOS parameter shows two different behaviours in two cases.

2. Metric and Field Equations

The standard representation of Kantowski-Sachs space-time is given by

$$ds^2 = dt^2 - A^2 dr^2 - B^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \tag{1}$$

where $A(t)$ and $B(t)$ are the directional scale factors.

We assume that the universe is filled with matter and anisotropic ghost dark energy fluid. The Einstein's field equation is represented by

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\left(T_{\alpha\beta} + \overline{T}_{\alpha\beta}\right). \tag{2}$$

where $R_{\alpha\beta}$ is the Ricci tensor, R is the Ricci scalar, $T_{\alpha\beta}$ and $\overline{T}_{\alpha\beta}$ are the energy momentum tensor for matter and anisotropic ghost dark energy.

The energy momentum tensor for matter is

$$T_{\alpha\beta} = diag\left[\rho_m, 0, 0, 0\right] \tag{3}$$

where ρ_m is the energy density of matter.

The energy momentum tensor for anisotropic ghost dark energy is taken as

$$\bar{T}_{\alpha\beta} = diag\left[\rho_G, -p_{G_r}, -p_{G_{\theta y}}, -p_{G_\phi}\right] \tag{4}$$

where ρ_G is the energy density for the ghost dark energy, p_{G_r}, p_{G_θ} and p_{G_ϕ} are the pressures in the directions of r, θ and ϕ respectively.

Introducing anisotropy in the EoS parameter we have

$$\begin{aligned}\bar{T}_{\alpha\beta} &= diag\left[1, -\omega_r, -\omega_\theta, -\omega_\phi\right]\rho_G \\ &= diag\left[1, -\omega_G, -(\omega_G + \delta), -(\omega_G + \delta)\right]\rho_G .\end{aligned} \tag{5}$$

where $\omega_r = \omega_G$, $\omega_\theta = \omega_G + \delta$, $\omega_\phi = \omega_G + \delta$ are the directional EoS parameters of the ghost dark energy on r, θ and ϕ axes respectively. The skewness parameter δ is the deviations from ω_G in the directions of θ and ϕ. Here ω_G and δ need not be constants and can be functions of cosmic time t.

The field equation for the line element (1) takes the form

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} = \rho_m + \rho_G , \tag{5}$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} = -\omega_G \rho_G , \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -(\omega_G + \delta)\rho_G . \tag{7}$$

where the overhead $(\bullet)$ denote derivative w. r. t. the cosmic time t.

3. Isotropization and the Solution

The directional Hubble parameters in the direction of r, θ and ϕ respectively are

$$H_r = \frac{\dot{A}}{A} \text{ and } H_\theta = H_\phi = \frac{\dot{B}}{B}. \tag{8}$$

The mean Hubble parameter is given as

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right). \tag{9}$$

where $V = AB^2$ is the spatial volume of the universe.

The anisotropy of the expansion can be parameterized after defining the directional Hubble parameters and the mean Hubble parameter of the expansion.

The anisotropic parameter of the expansion is defined as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 , \tag{10}$$

where $H_i\,(i = 1, 2, 3)$ represent the directional Hubble parameters in the direction of r, θ and ϕ respectively.

After little manipulation of equation (10) and using $H_\theta = H_\phi$, we get

$$\Delta = \frac{2}{9H^2}\left(H_r - H_\phi\right)^2 . \tag{11}$$

From equations (6) and (7) we get

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = H_r - H_\phi = \frac{c_1}{V} e^{-\int \frac{\delta\rho_G + \frac{1}{B^2}}{\frac{\dot{A}}{A} - \frac{\dot{B}}{B}} dt} . \tag{12}$$

Here the difference $H_r - H_\phi$ between the expansion rates on r and ϕ axes.

In order to solve the above equation, let us assume that

$$\delta\rho_G + \frac{1}{B^2} = \frac{\dot{A}}{A} - \frac{\dot{B}}{B} . \tag{14}$$

Using (12), equation (11) takes the form

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{c_1}{V} e^{-t} . \tag{15}$$

Using these results, the expression for the anisotropic parameter $\overline{A}_m$ reduces to

$$\overline{A}_m = \frac{2}{9H^2}\frac{c_1^2}{V^2} e^{-2t} . \tag{16}$$

The field equations (5)-(7), using (14) takes the form

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}}{A} + \frac{\dot{B}^2}{B^2} - \frac{\dot{B}}{B} = \rho_m + (1+\delta)\rho_G \tag{17}$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\dot{A}}{A} - \frac{\dot{B}}{B} = -(\omega_G - \delta)\rho_G \tag{18}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=-(\omega_G+\delta)\rho_G \tag{19}$$

The continuity equation can be written as

$$\dot{\rho}_m+\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)\rho_m+(1-\delta)\dot{\rho}_G-\dot{\delta}\rho_G+\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)(1+\omega_G)\rho_G=0 \tag{20}$$

Since the two fluids considered are non-interacting, the continuity equation (20) can be separately taken for matter as well as for ghost dark energy.

The continuity equation for matter is

$$\dot{\rho}_m+\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)\rho_m=0, \tag{21}$$

And the continuity equation for the ghost dark energy is

$$(1-\delta)\dot{\rho}_G-\dot{\delta}\rho_G+\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)(1+\omega_G)\rho_G=0. \tag{22}$$

The energy density of generalized ghost dark energy defined by Cai (Cai et al. 2012) is

$$\rho_G=\tau H+\eta H^2. \tag{23}$$

where τ and η are constants. The constant τ is of order Λ^3_{QCD} where Λ is QCD mass-scale. In the GGDE, η is a free parameter and can be adjusted for better agreement with observations.

From (24) we can write

$$\omega_G=-1+\frac{\dot{\delta}}{3H}-(1-\delta)\frac{\dot{\rho}_G}{3H\rho_G}. \tag{24}$$

In order to solve the system completely, we need one extra condition. Following Akarsu et al. (2014), we consider

$$a(t)=a_0\left(\frac{t}{t_0}\right)^{\alpha}e^{\beta\left(\frac{t}{t_0}-1\right)}. \tag{25}$$

where α and β are non-negative constants and a_0 and t_0 represent the present value of the scale factor and age of the universe respectively. The relation (25) is known as the Hybrid Expansion Law (HEL) which is the combination of power law and the exponential law. From the relation it is observed that when $\alpha=0$ it gives the exponential law and on the other hand $\beta=0$ gives the power law cosmology.

Also, the scale factor given by the relation (25) gives a time dependent deceleration parameter

$$a=V^{1/3}=a_0\left(\frac{t}{t_0}\right)^{\alpha}e^{\beta\left(\frac{t}{t_0}-1\right)}. \tag{26}$$

Using (26) in (13), the values of the scale factors are as follows

$$A = c_2^{2/3} a_0 \left(\frac{t}{t_0}\right)^{\alpha} e^{\beta\left(\frac{t}{t_0}-1\right)} \exp\left[\frac{2c_1 a_0^{-3}}{3}\int\left(\frac{t}{t_0}\right)^{-3\alpha} e^{-3\beta\left(\frac{t}{t_0}-1\right)} e^{-t} dt\right], \tag{27}$$

$$B = c_2^{-1/3} a_0 \left(\frac{t}{t_0}\right)^{\alpha} e^{\beta\left(\frac{t}{t_0}-1\right)} \exp\left[-\frac{c_1 a_0^{-3}}{3}\int\left(\frac{t}{t_0}\right)^{-3\alpha} e^{-3\beta\left(\frac{t}{t_0}-1\right)} e^{-t} dt\right]. \tag{28}$$

where c_2 is the integration constant.

The values of the directional Hubble parameters, the Hubble parameter and the anisotropic parameter are as follows

$$H_r = \frac{\alpha}{t} + \frac{\beta}{t_0} + \frac{2c_1 a_0^{-3}}{3}\left(\frac{t}{t_0}\right)^{-3\alpha} e^{-3\beta\left(\frac{t}{t_0}-1\right)}, \tag{29}$$

$$H_\theta = H_\phi = \frac{\alpha}{t} + \frac{\beta}{t_0} - \frac{c_1 a_0^{-3}}{3}\left(\frac{t}{t_0}\right)^{-3\alpha} e^{-3\beta\left(\frac{t}{t_0}-1\right)}, \tag{30}$$

$$H = \frac{\alpha}{t} + \frac{\beta}{t_0}. \tag{31}$$

$$\overline{A}_m = \frac{2}{9}\left(\frac{\alpha}{t} + \frac{\beta}{t_0}\right)^{-2} c_1 a_0^{-6} \left(\frac{t}{t_0}\right)^{-6\alpha} e^{-6\beta\left(\frac{t}{t_0}-1\right)} e^{-2t}. \tag{32}$$

The expression for the deceleration parameter is as follows

$$q = -1 + \alpha\left(\alpha + \frac{\beta t}{t_0}\right)^{-2}. \tag{33}$$

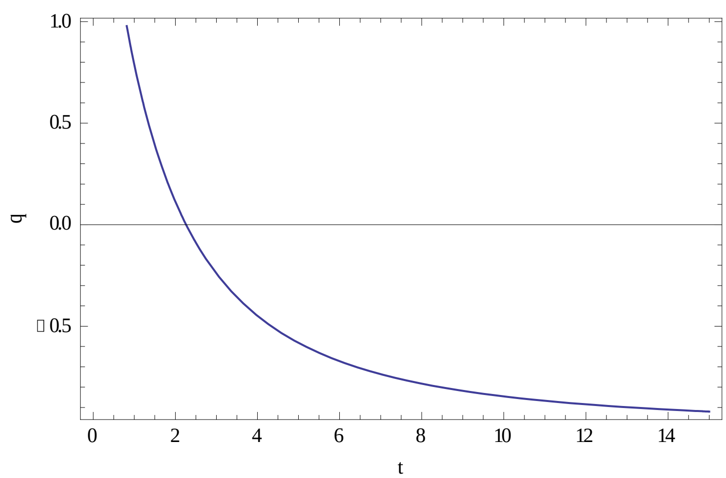

Figure 1. Deceleration parameter q vs. cosmic time t

Figure 1, shows that at the beginning the value of the deceleration parameter is positive but at later times it becomes negative. Thus our universe is in the accelerating phase at later times.
Using the results from equations (27)-(31) in (21), (23), (24) and (14), the values of the energy density of matter ρ_m, the energy density of ghost dark energy ρ_G, the skewness parameter δ and the EoS parameter of ghost dark energy ω_G becomes

$$\rho_m = c_3 t^{-3\alpha} e^{-3\beta t/t_0}, \tag{34}$$

$$\rho_G = \varsigma\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)+\eta\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^2, \tag{35}$$

$$\delta = \frac{H_r - H_\theta - \dfrac{1}{B^2}}{\varsigma H + \eta H^2}, \tag{36}$$

$$\omega_G = -1 + \frac{3\dot{H} - 3\dot{H}_\theta + 2H_\theta B^{-2} - \varsigma\dot{H} - 2\eta H\dot{H}}{3H\left(\varsigma H + \eta H^2\right)}. \tag{37}$$

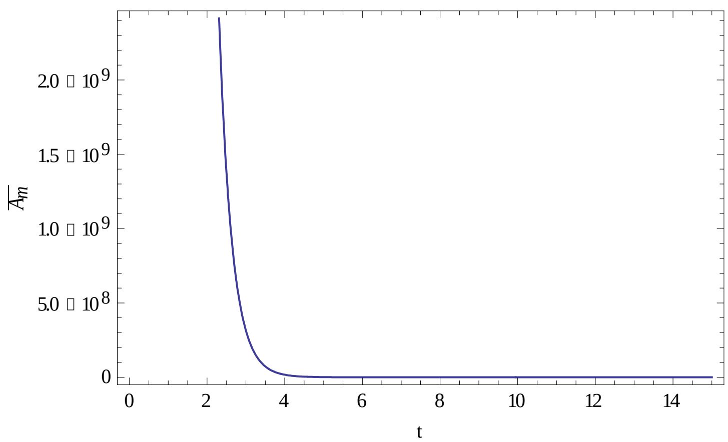

Figure 2. The anisotropic parameter $\overline{A}_m$ vs. cosmic time t

From Figure 2, it is clear that at late times the universe becomes isotropic.

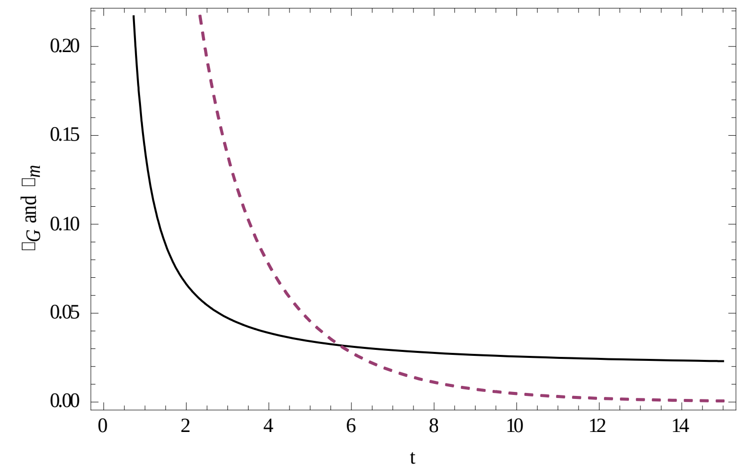

Figure 3. the matter energy density ρ_m and ghost dark energy density ρ_G vs. cosmic t, here the black line represents the ghost dark energy density and the dashed line matter energy density.

From Figure 3, it is observed that at late times the ghost dark energy density tends to some constant value but the matter energy density becomes zero.

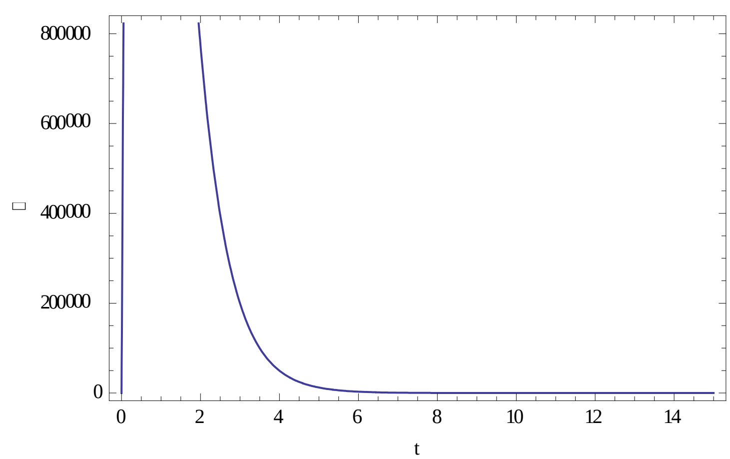

Fig4. The skewness parameter δ vs. cosmic time t

From Figure 4, it is seen that the skewness parameter $\delta \to 0$ at later times. It means that at later age of the universe the anisotropy of the ghost dark energy becomes isotropic.

The expressions for matter energy density parameter Ω_m and dark energy density parameter Ω_G are

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{c_3 t^{-3\alpha} e^{-3\beta t/t_0}}{3\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^2} \text{ and } \Omega_G = \frac{\rho_G}{3H^2} = \frac{\varsigma\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)+\eta\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^2}{3\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^2}. \tag{38}$$

Thus the total energy density parameter takes the form

$$\Omega = \Omega_m + \Omega_G = \frac{c_3}{3} t^{-3\alpha} e^{-3\beta t/t_0}\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{-2} + \frac{\varsigma}{3}\left(\frac{\alpha}{t}+\frac{\beta}{t_0}\right)^{-2} + \frac{\eta}{3}. \tag{39}$$

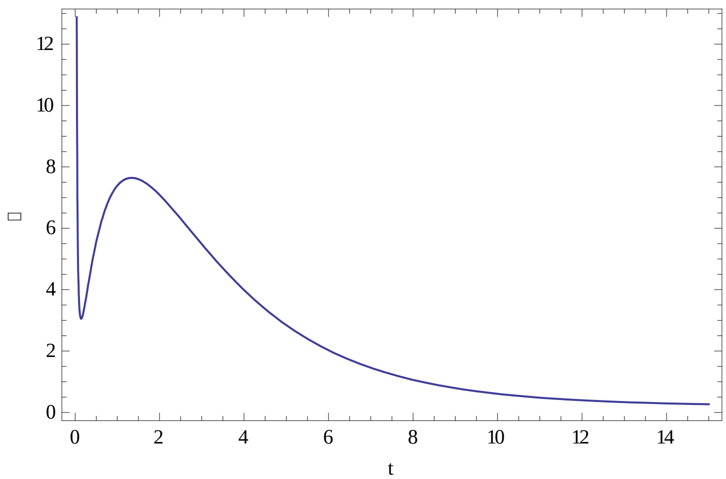

Figure.5.The total energy density Ω vs. cosmic time t

Figure.5. shows that the total energy density tends to 1. Thus at present time our universe becomes spatially homogenous, isotropic and flat.

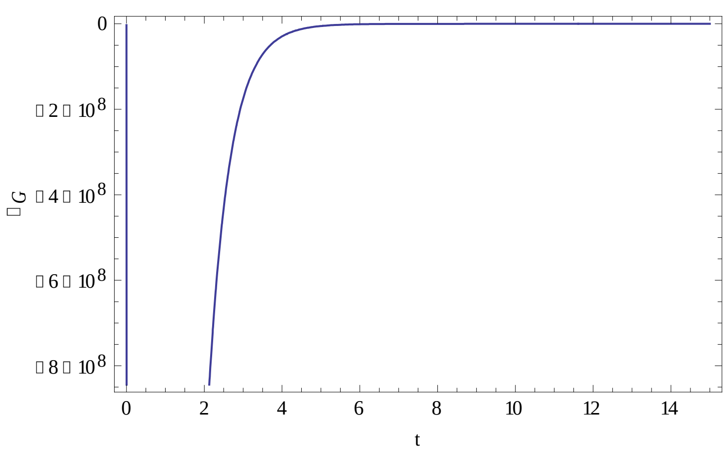

Figure.6. shows ω_G vs. cosmic time t

Figure 6, shows that the value of the EoS parameter is a negative but at late time is becomes zero.

The coincidence parameter, which is the ration between the dark energy density and the matter energy density is given by

$$\chi = \frac{\rho_G}{\rho_m} = \left\{ \varsigma\left(\frac{\alpha}{t} + \frac{\beta}{t_0}\right) + \eta\left(\frac{\alpha}{t} + \frac{\beta}{t_0}\right)^2 \right\} c^{-1}{}_3 t^{3\alpha} e^{3\beta t/t_0} . \quad (40)$$

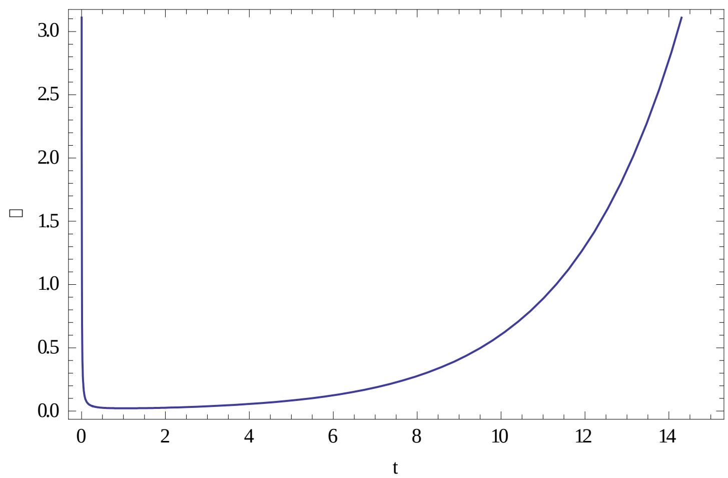

Figure 7. shows the coincidence parameter χ vs. cosmic time t

Fig.7. shows the behaviour of the coincidence parameter which is an increasing function of cosmic time. The figure gives the value of the coincidence parameter which is 2.33 at the present cosmic time $13.789(Gyr)$. So our result is consistent with the present day observations (Ade et al. 2013).

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