JOURNAL OF CRITICAL REVIEWS

ISSN- 2394-5125 VOL 7, ISSUE 13, 2020

ON CERTAIN TRANSFORMATIONS FOR MOCK THETA FUNCTIONS

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Received: April 2020 Revised and Accepted: June 2020

ABSTRACT: In this paper the various relation and transformations for mock theta function and infinite product analogues to the identities of ramanujan, established using well known identities.

KEYWORDS: mock theta function, partial mock theta function, infinite products, ramanujan identities.
A.M.S. Subject Classification: 11B65, 33D15

I. INTRODUCTION

Ramanujan listed seventeen mock theta functions and marked them as third, fifth and seventh orders without classification. Mock is a function $f(q)$ of the complex variable q, respresented by a q-series of a special type; Ramanujan calls this the Eulerian type that converges to and satisfies the following rules;

1. Exponential singularities are infinitely many roots of unity
2. For each unity root ξ there is a θ-function $\theta_\xi(q)$ so that the difference $f(q) - \theta_\xi(q)$ is radially bounded as q→ξ.
3. f is not the sum of two functions, one of which is a θ-function and the other a function bounded in all unity roots.

Watson [23], Andrews [3] introduces a study of these sums and expansions. Subsequently,Choi. [7] Defined certain q-series in the Lost Notebook and named them as Sixth, eighth and tenth order mock theta functions. Although Gordon and Mc Intosh [9] defined the order of mock theta functions, and later Bringmann and Ono [5, 6] described the order of the mock theta functions. Also, relationship involving mock theta functions and partial mock theta functions are given by srivastava [20]. Srivastava, B[21] providing relationship comparing mock theta functions and partial mock theta functions of order 3,5,6 and 10 and relationship comparing mock theta functions, partial mock theta functions of order 2,3 and 6 and Ramanujan's function $\mu(q)$.

II. DEFINITION AND NOTATION

Mock theta functions of Ramanujan have also been confusing and interesting mathematicians for centuries. Using mock Mock theta functions of order tenth, their partial sums and infinite products in the renowned Srivastava identity [12] to acquire different relationships relating mock theta functions of order 10. Ramanujan (1988,p.9)[2] described in his lost notebook four order 10 Mock theta functions , and indicated some of their relationships ,

(2.1) $\varphi_T(q) = \sum_{n=0}^{\infty} \frac{q^{\frac{n(n+1)}{2}}}{(q;q^2)_{n+1}}$

(2.2) $\psi_T(q) = \sum_{n=0}^{\infty} \frac{q^{\frac{(n+1)(n+2)}{2}}}{(q;q^2)_{n+1}}$

(2.3) $\mathrm{X}_T(q) = \sum_{n=0}^{\infty} \frac{(-1)^n q^{n^2}}{(-q;q)_{2n}}$

(2.4) $\chi_T(q) = \sum_{n=0}^{\infty} \frac{(-1)^n q^{(n+1)^2}}{(-q;q)_{2n+1}}$

The renowned identities of Rogers-Ramanujan ([14], [17], [18], pp.214-215) are

(2.5) $G(q) = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(q;q)_n} = \frac{1}{(q;q^5)_\infty (q^4;q^5)_\infty}$

(2.6) $H(q) = \sum_{n=0}^{\infty} \frac{q^{n(n+1)}}{(q;q)_n} = \frac{1}{(q^2;q^5)_\infty (q^3;q^5)_\infty}$

Where $G(q)$ and $H(q)$ are called Rogers-Ramanujan functions.

ISSN- 2394-5125 VOL 7, ISSUE 13, 2020

Two identities comparable to (1) and (2) are the identities of the so- called Gollnitz-Gordon identities [9], given by

(2.7) $S(q)=\sum_{n=0}^{\infty}\frac{(-q;q^2)_n}{(q^2;q^2)_n}q^{n^2}=\frac{1}{(q;q^8)_\infty(q^4;q^8)_\infty(q^7;q^8)_\infty}$

(2.8) $T(q)=\sum_{n=0}^{\infty}\frac{(-q;q^2)_n}{(q^2;q^2)_n}q^{n^2+2n}=\frac{1}{(q^3;q^8)_\infty(q^4;q^8)_\infty(q^5;q^8)_\infty}$

$S(q), T(q)$ are known as the Gollnitz-Gordon functions.

The following two analog Rogers – Ramanujan functions are also described in [19]

(2.9) $X(q)=\sum_{n=0}^{\infty}\frac{(-q^2;q^2)_n(1-q^{n+1})q^{n(n+2)}}{(q;q)_{2n+2}}=\frac{(q;q^{12})_\infty(q^{11};q^{12})_\infty(q^{12};q^{12})_\infty}{(q;q)_\infty}$

(2.10) $Y(q)=\sum_{n=0}^{\infty}\frac{(-q^2;q^2)_{n-1}(1+q^n)q^{n^2}}{(q;q)_{2n}}=\frac{(q^5;q^{12})_\infty(q^7;q^{12})_\infty(q^{12};q^{12})_\infty}{(q;q)_\infty}$

III. MAIN RESULTS

(3.1) $\sum_{r=0}^{\infty}\frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+2}}G_m(q)=\varphi_T(q)\frac{1}{(q;q^5)_\infty(q^4;q^5)_\infty}-\sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}\varphi_{Tm}(q)$

(3.2) $\sum_{r=0}^{\infty}\frac{q^{\frac{(r+2)(r+3)}{2}}}{(q;q^2)_{r+2}}G_m(q)=\psi_T(q)\frac{1}{(q;q^5)_\infty(q^4;q^5)_\infty}-\sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}\psi_{Tm}(q)$

(3.3) $\sum_{r=0}^{\infty}\frac{(-1)^{r+1}q^{(r+1)^2}}{(-q;q)_{2(r+1)}}G_m(q)=\mathrm{X}_T(q)\frac{1}{(q;q^5)_\infty(q^4;q^5)_\infty}-\sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}\mathrm{X}_{Tm}(q)$

(3.4) $\sum_{r=0}^{\infty}\frac{(-1)^{r+1}q^{(r+2)^2}}{(-q;q)_{2r+3}}G_m(q)=\chi_T(q)\frac{1}{(q;q^5)_\infty(q^4;q^5)_\infty}-\sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}\chi_{Tm}(q)$

(3.5) $\sum_{r=0}^{\infty}\frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+2}}H_m(q)=\varphi_T(q)\frac{1}{(q^2;q^5)_\infty(q^3;q^5)_\infty}-\sum_{m=0}^{\infty}\frac{q^{m(m+1)}}{(q;q)_m}\varphi_{Tm}(q)$

(3.6) $\sum_{r=0}^{\infty}\frac{q^{\frac{(r+2)(r+3)}{2}}}{(q;q^2)_{r+2}}H_m(q)=\psi_T(q)\frac{1}{(q^2;q^5)_\infty(q^3;q^5)_\infty}-\sum_{m=0}^{\infty}\frac{q^{m(m+1)}}{(q;q)_m}\psi_{Tm}(q)$

(3.7) $\sum_{r=0}^{\infty}\frac{(-1)^{r+1}q^{(r+1)^2}}{(-q;q)_{2(r+1)}}H_m(q)=\mathrm{X}_T(q)\frac{1}{(q^2;q^5)_\infty(q^3;q^5)_\infty}-\sum_{m=0}^{\infty}\frac{q^{m(m+1)}}{(q;q)_m}\mathrm{X}_{Tm}(q)$

(3.8) $\sum_{r=0}^{\infty}\frac{(-1)^{r+1}q^{(r+2)^2}}{(-q;q)_{2r+3}}H_m(q)=\chi_T(q)\frac{1}{(q^2;q^5)_\infty(q^3;q^5)_\infty}-\sum_{m=0}^{\infty}\frac{q^{m(m+1)}}{(q;q)_m}\chi_{Tm}(q)$

(3.9) $\sum_{r=0}^{\infty}\frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+2}}S_m(q)=\varphi_T(q)\frac{1}{(q;q^8)_\infty(q^4;q^8)_\infty(q^7;q^8)_\infty}-\sum_{m=0}^{\infty}\frac{(-q;q^2)_m}{(q^2;q^2)_m}q^{m^2}\varphi_{Tm}(q)$

(3.10) $\sum_{r=0}^{\infty}\frac{q^{\frac{(r+2)(r+3)}{2}}}{(q;q^2)_{r+2}}S_m(q)=\psi_T(q)\frac{1}{(q;q^8)_\infty(q^4;q^8)_\infty(q^7;q^8)_\infty}-\sum_{m=0}^{\infty}\frac{(-q;q^2)_m}{(q^2;q^2)_m}q^{m^2}\psi_{Tm}(q)$

(3.11) $\sum_{r=0}^{\infty}\frac{(-1)^{r+1}q^{(r+1)^2}}{(-q;q)_{2(r+1)}}S_m(q)=\mathrm{X}_T(q)\frac{1}{(q;q^8)_\infty(q^4;q^8)_\infty(q^7;q^8)_\infty}-\sum_{m=0}^{\infty}\frac{(-q;q^2)_m}{(q^2;q^2)_m}q^{m^2}\mathrm{X}_{Tm}(q)$

(3.12) $\sum_{r=0}^{\infty}\frac{(-1)^{r+1}q^{(r+2)^2}}{(-q;q)_{2r+3}}S_m(q)=\chi_T(q)\frac{1}{(q;q^8)_\infty(q^4;q^8)_\infty(q^7;q^8)_\infty}-\sum_{m=0}^{\infty}\frac{(-q;q^2)_m}{(q^2;q^2)_m}q^{m^2}\chi_{Tm}(q)$

(3.13) $\sum_{r=0}^{\infty}\frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+2}}T_m(q)=\varphi_T(q)\frac{1}{(q^3;q^8)_\infty(q^4;q^8)_\infty(q^5;q^8)_\infty}-\sum_{m=0}^{\infty}\frac{(-q;q^2)_m}{(q^2;q^2)_m}q^{m^2+2m}\varphi_{Tm}(q)$

(3.14) $\sum_{r=0}^{\infty}\frac{q^{\frac{(r+2)(r+3)}{2}}}{(q;q^2)_{r+2}}T_m(q)=\psi_T(q)\frac{1}{(q^3;q^8)_\infty(q^4;q^8)_\infty(q^5;q^8)_\infty}-\sum_{m=0}^{\infty}\frac{(-q;q^2)_m}{(q^2;q^2)_m}q^{m^2+2m}\psi_{Tm}(q)$

(3.15) $\sum_{r=0}^{\infty}\frac{(-1)^{r+1}q^{(r+1)^2}}{(-q;q)_{2(r+1)}}T_m(q)=\mathrm{X}_T(q)\frac{1}{(q^3;q^8)_\infty(q^4;q^8)_\infty(q^5;q^8)_\infty}-\sum_{m=0}^{\infty}\frac{(-q;q^2)_m}{(q^2;q^2)_m}q^{m^2+2m}\mathrm{X}_{Tm}(q)$

(3.16) $\sum_{r=0}^{\infty}\frac{(-1)^{r+1}q^{(r+2)^2}}{(-q;q)_{2r+3}}T_m(q)=\chi_T(q)\frac{1}{(q^3;q^8)_\infty(q^4;q^8)_\infty(q^5;q^8)_\infty}-\sum_{m=0}^{\infty}\frac{(-q;q^2)_m}{(q^2;q^2)_m}q^{m^2+2m}\chi_{Tm}(q)$

(3.17) $\sum_{r=0}^{\infty}\frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+2}}X_m(q)$

$$=\varphi_T(q)\frac{(q;q^{12})_\infty(q^{11};q^{12})_\infty(q^{12};q^{12})_\infty}{(q;q)_\infty}-\sum_{m=0}^{\infty}\frac{(-q^2;q^2)_m(1-q^{n+1})q^{m(m+2)}}{(q;q)_{2m+2}}\varphi_{Tm}(q)$$

(3.18) $\sum_{r=0}^{\infty}\frac{q^{\frac{(r+2)(r+3)}{2}}}{(q;q^2)_{r+2}}X_m(q)$

$$=\psi_T(q)\frac{(q;q^{12})_\infty(q^{11};q^{12})_\infty(q^{12};q^{12})_\infty}{(q;q)_\infty}-\sum_{m=0}^{\infty}\frac{(-q^2;q^2)_m(1-q^{n+1})q^{m(m+2)}}{(q;q)_{2m+2}}\psi_{Tm}(q)$$

(3.19) $\sum_{r=0}^{\infty}\frac{(-1)^{r+1}q^{(r+1)^2}}{(-q;q)_{2(r+1)}}X_m(q)$

ISSN- 2394-5125 VOL 7, ISSUE 13, 2020

$= X_T(q)\frac{(q;q^{12})_\infty(q^{11};q^{12})_\infty(q^{12};q^{12})_\infty}{(q;q)_\infty} - \sum_{m=0}^{\infty}\frac{(-q^2;q^2)_m(1-q^{n+1})q^{m(m+2)}}{(q;q)_{2m+2}}X_{Tm}(q)$

(3.20) $\sum_{r=0}^{\infty}\frac{(-1)^{r+1}q^{(r+2)^2}}{(-q;q)_{2r+3}}X_m(q)$

$= \chi_T(q)\frac{(q;q^{12})_\infty(q^{11};q^{12})_\infty(q^{12};q^{12})_\infty}{(q;q)_\infty} - \sum_{m=0}^{\infty}\frac{(-q^2;q^2)_m(1-q^{n+1})q^{m(m+2)}}{(q;q)_{2m+2}}\chi_{Tm}(q)$

(3.21) $\sum_{r=0}^{\infty}\frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+2}}Y_m(q)$

$= \varphi_T(q)\frac{(q^5;q^{12})_\infty(q^7;q^{12})_\infty(q^{12};q^{12})_\infty}{(q;q)_\infty} - \sum_{m=0}^{\infty}\frac{(-q^2;q^2)_{m-1}(1+q^m)q^{m^2}}{(q;q)_{2m}}\varphi_{Tm}(q)$

(3.22) $\sum_{r=0}^{\infty}\frac{q^{\frac{(r+2)(r+3)}{2}}}{(q;q^2)_{r+2}}Y_m(q)$

$= \psi_T(q)\frac{(q^5;q^{12})_\infty(q^7;q^{12})_\infty(q^{12};q^{12})_\infty}{(q;q)_\infty} - \sum_{m=0}^{\infty}\frac{(-q^2;q^2)_{m-1}(1+q^m)q^{m^2}}{(q;q)_{2m}}\psi_{Tm}(q)$

(3.23) $\sum_{r=0}^{\infty}\frac{(-1)^{r+1}q^{(r+1)^2}}{(-q;q)_{2(r+1)}}Y_m(q)$

$= X_T(q)\frac{(q^5;q^{12})_\infty(q^7;q^{12})_\infty(q^{12};q^{12})_\infty}{(q;q)_\infty} - \sum_{m=0}^{\infty}\frac{(-q^2;q^2)_{m-1}(1+q^m)q^{m^2}}{(q;q)_{2m}}X_{Tm}(q)$

(3.24) $\sum_{r=0}^{\infty}\frac{(-1)^{r+1}q^{(r+2)^2}}{(-q;q)_{2r+3}}Y_m(q)$

$= \chi_T(q)\frac{(q^5;q^{12})_\infty(q^7;q^{12})_\infty(q^{12};q^{12})_\infty}{(q;q)_\infty} - \sum_{m=0}^{\infty}\frac{(-q^2;q^2)_{m-1}(1+q^m)q^{m^2}}{(q;q)_{2m}}\chi_{Tm}(q)$

IV. PROOFS OF MAIN RESULTS

We will use the following renowned identity [9]

(4.1) $\sum_{m=0}^{\infty}\delta_m\sum_{r=0}^{m}\alpha_r = (\sum_{r=0}^{\infty}\alpha_r)(\sum_{m=0}^{\infty}\delta_m) - \sum_{r=0}^{\infty}\alpha_{r+1}\sum_{m=0}^{r}\delta_m$

By Taking $\delta_r = \frac{q^{r^2}}{(q;q)_r}$, $\alpha_r = \frac{q^{\frac{r(r+1)}{2}}}{(q;q^2)_{r+1}}$ in (4.1) and making use of (2.1), then after summing, we get

(4.2) $\sum_{r=0}^{\infty}\frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+2}}G_m(q) = \varphi_T(q)\frac{1}{(q;q^5)_\infty(q^4;q^5)_\infty} - \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}\varphi_{Tm}(q)$

By taking $\delta_r = \frac{q^{r^2}}{(q;q)_r}$, $\alpha_r = \frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+1}}$ in (4.1) and making use of (2.2), then after summing, we get

(4.3) $\sum_{r=0}^{\infty}\frac{q^{\frac{(r+2)(r+3)}{2}}}{(q;q^2)_{r+2}}G_m(q) = \psi_T(q)\frac{1}{(q;q^5)_\infty(q^4;q^5)_\infty} - \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}\psi_{Tm}(q)$

By taking $\delta_r = \frac{q^{r^2}}{(q;q)_r}$, $\alpha_r = \frac{(-1)^r q^{r^2}}{(-q;q)_{2r}}$ in (4.1) and making use of (2.3), then after summing, we get

(4.4) $\sum_{r=0}^{\infty}\frac{(-1)^{r+1}q^{(r+1)^2}}{(-q;q)_{2(r+1)}}G_m(q) = X_T(q)\frac{1}{(q;q^5)_\infty(q^4;q^5)_\infty} - \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}X_{Tm}(q)$

By taking $\delta_r = \frac{q^{r^2}}{(q;q)_r}$, $\alpha_r = \frac{(-1)^r q^{(r+1)^2}}{(-q;q)_{2r+1}}$ in (4.1) and making use of (2.4), then after summing, we get

(4.5) $\sum_{r=0}^{\infty}\frac{(-1)^{r+1}q^{(r+2)^2}}{(-q;q)_{2r+3}}G_m(q) = \chi_T(q)\frac{1}{(q;q^5)_\infty(q^4;q^5)_\infty} - \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}\chi_{Tm}(q)$

By taking $\delta_r = \frac{q^{r(r+1)}}{(q;q)_r}$, $\alpha_r = \frac{q^{\frac{r(r+1)}{2}}}{(q;q^2)_{r+1}}$ in (4.1) and making use of (2.1), then after summing, we get

(4.6) $\sum_{r=0}^{\infty}\frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+2}}H_m(q) = \varphi_T(q)\frac{1}{(q^2;q^5)_\infty(q^3;q^5)_\infty} - \sum_{m=0}^{\infty}\frac{q^{m(m+1)}}{(q;q)_m}\varphi_{Tm}(q)$

By taking $\delta_r = \frac{q^{r(r+1)}}{(q;q)_r}$, $\alpha_r = \frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+1}}$ in (4.1) and making use of (2.2), then after summing, we get

(4.7) $\sum_{r=0}^{\infty}\frac{q^{\frac{(r+2)(r+3)}{2}}}{(q;q^2)_{r+2}}H_m(q) = \psi_T(q)\frac{1}{(q^2;q^5)_\infty(q^3;q^5)_\infty} - \sum_{m=0}^{\infty}\frac{q^{m(m+1)}}{(q;q)_m}\psi_{Tm}(q)$

By taking$\delta_r = \frac{q^{r(r+1)}}{(q;q)_r}$, $\alpha_r = \frac{(-1)^r q^{r^2}}{(-q;q)_{2r}}$ in (4.1) and making use of (2.3), then after summing, we get

(4.8) $\sum_{r=0}^{\infty}\frac{(-1)^{r+1}q^{(r+1)^2}}{(-q;q)_{2(r+1)}}H_m(q) = X_T(q)\frac{1}{(q^2;q^5)_\infty(q^3;q^5)_\infty} - \sum_{m=0}^{\infty}\frac{q^{m(m+1)}}{(q;q)_m}X_{Tm}(q)$

By taking $\delta_r = \frac{q^{r(r+1)}}{(q;q)_r}, \alpha_r = \frac{(-1)^r q^{(r+1)^2}}{(-q;q)_{2r+1}}$ in (4.1) and making use of (2.4), then after summing, we get

(4.9) $\sum_{r=0}^{\infty} \frac{(-1)^{r+1} q^{(r+2)^2}}{(-q;q)_{2r+3}} H_m(q) = \chi_T(q) \frac{1}{(q^2;q^5)_\infty (q^3;q^5)_\infty} - \sum_{m=0}^{\infty} \frac{q^{m(m+1)}}{(q;q)_m} \chi_{Tm}(q)$

By taking $\delta_r = \frac{(-q;q^2)_r}{(q^2;q^2)_r} q^{r^2}, \alpha_r = \frac{q^{\frac{r(r+1)}{2}}}{(q;q^2)_{r+1}}$ in (4.1) and making use of (2.1), then after summing, we get

(4.10) $\sum_{r=0}^{\infty} \frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+2}} S_m(q) = \varphi_T(q) \frac{1}{(q;q^8)_\infty (q^4;q^8)_\infty (q^7;q^8)_\infty} - \sum_{m=0}^{\infty} \frac{(-q;q^2)_m}{(q^2;q^2)_m} q^{m^2} \varphi_{Tm}(q)$

By taking $\delta_r = \frac{(-q;q^2)_r}{(q^2;q^2)_r} q^{r^2}, \alpha_r = \frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+1}}$ in (4.1) and making use of (2.2), then after summing, we get

(4.11) $\sum_{r=0}^{\infty} \frac{q^{\frac{(r+2)(r+3)}{2}}}{(q;q^2)_{r+2}} S_m(q) = \psi_T(q) \frac{1}{(q;q^8)_\infty (q^4;q^8)_\infty (q^7;q^8)_\infty} - \sum_{m=0}^{\infty} \frac{(-q;q^2)_m}{(q^2;q^2)_m} q^{m^2} \psi_{Tm}(q)$

By taking $\delta_r = \frac{(-q;q^2)_r}{(q^2;q^2)_r} q^{r^2}, \alpha_r = \frac{(-1)^r q^{r^2}}{(-q;q)_{2r}}$ in (4.1) and making use of (2.3), then after summing, we get

(4.12) $\sum_{r=0}^{\infty} \frac{(-1)^{r+1} q^{(r+1)^2}}{(-q;q)_{2(r+1)}} S_m(q) = \mathrm{X}_T(q) \frac{1}{(q;q^8)_\infty (q^4;q^8)_\infty (q^7;q^8)_\infty} - \sum_{m=0}^{\infty} \frac{(-q;q^2)_m}{(q^2;q^2)_m} q^{m^2} \mathrm{X}_{Tm}(q)$

By taking $\delta_r = \frac{(-q;q^2)_r}{(q^2;q^2)_r} q^{r^2}, \alpha_r = \frac{(-1)^r q^{(r+1)^2}}{(-q;q)_{2r+1}}$ in (4.1) and making use of (2.4), then after summing, we get

(4.13) $\sum_{r=0}^{\infty} \frac{(-1)^{r+1} q^{(r+2)^2}}{(-q;q)_{2r+3}} S_m(q) = \chi_T(q) \frac{1}{(q;q^8)_\infty (q^4;q^8)_\infty (q^7;q^8)_\infty} - \sum_{m=0}^{\infty} \frac{(-q;q^2)_m}{(q^2;q^2)_m} q^{m^2} \chi_{Tm}(q)$

By taking $\delta_r = \frac{(-q;q^2)_r}{(q^2;q^2)_r} q^{r^2+2r}, \alpha_r = \frac{q^{\frac{r(r+1)}{2}}}{(q;q^2)_{r+1}}$ in (4.1) and making use of (2.1), then after summing, we get

(4.14) $\sum_{r=0}^{\infty} \frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+2}} T_m(q) = \varphi_T(q) \frac{1}{(q^3;q^8)_\infty (q^4;q^8)_\infty (q^5;q^8)_\infty} - \sum_{m=0}^{\infty} \frac{(-q;q^2)_m}{(q^2;q^2)_m} q^{m^2+2m} \varphi_{Tm}(q)$

By taking $\delta_r = \frac{(-q;q^2)_r}{(q^2;q^2)_r} q^{r^2+2r}, \alpha_r = \frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+1}}$ in (4.1) and making use of (2.2), then after summing, we get

(4.15) $\sum_{r=0}^{\infty} \frac{q^{\frac{(r+2)(r+3)}{2}}}{(q;q^2)_{r+2}} T_m(q) = \psi_T(q) \frac{1}{(q^3;q^8)_\infty (q^4;q^8)_\infty (q^5;q^8)_\infty} - \sum_{m=0}^{\infty} \frac{(-q;q^2)_m}{(q^2;q^2)_m} q^{m^2+2m} \psi_{Tm}(q)$

3) By taking $\delta_r = \frac{(-q;q^2)_r}{(q^2;q^2)_r} q^{r^2+2r}, \alpha_r = \frac{(-1)^r q^{r^2}}{(-q;q)_{2r}}$ in (4.1) and making use of (2.3), then after summing, we get

(4.16) $\sum_{r=0}^{\infty} \frac{(-1)^{r+1} q^{(r+1)^2}}{(-q;q)_{2(r+1)}} T_m(q) = \mathrm{X}_T(q) \frac{1}{(q^3;q^8)_\infty (q^4;q^8)_\infty (q^5;q^8)_\infty} - \sum_{m=0}^{\infty} \frac{(-q;q^2)_m}{(q^2;q^2)_m} q^{m^2+2m} \mathrm{X}_{Tm}(q)$

4) By taking $\delta_r = \frac{(-q;q^2)_r}{(q^2;q^2)_r} q^{r^2+2r}, \alpha_r = \frac{(-1)^r q^{(r+1)^2}}{(-q;q)_{2r+1}}$ in (4.1) and making use of (2.4), then after summing, we get

(4.17) $\sum_{r=0}^{\infty} \frac{(-1)^{r+1} q^{(r+2)^2}}{(-q;q)_{2r+3}} T_m(q) = \chi_T(q) \frac{1}{(q^3;q^8)_\infty (q^4;q^8)_\infty (q^5;q^8)_\infty} - \sum_{m=0}^{\infty} \frac{(-q;q^2)_m}{(q^2;q^2)_m} q^{m^2+2m} \chi_{Tm}(q)$

By taking $\delta_r = \frac{(-q^2;q^2)_r (1-q^{n+1}) q^{r(r+2)}}{(q;q)_{2r+2}}, \alpha_r = \frac{q^{\frac{r(r+1)}{2}}}{(q;q^2)_{r+1}}$ in (4.1) and making use of (2.1), then after summing, we get

(4.18) $\sum_{r=0}^{\infty} \frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+2}} X_m(q) = \varphi_T(q) \frac{(q;q^{12})_\infty (q^{11};q^{12})_\infty (q^{12};q^{12})_\infty}{(q;q)_\infty} - \sum_{m=0}^{\infty} \frac{(-q^2;q^2)_m (1-q^{n+1}) q^{m(m+2)}}{(q;q)_{2m+2}} \varphi_{Tm}(q)$

By taking $\delta_r = \frac{(-q^2;q^2)_r (1-q^{n+1}) q^{r(r+2)}}{(q;q)_{2r+2}}, \alpha_r = \frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+1}}$ in (4.1) and making use of (2.2), then after summing, we get

(4.19) $\sum_{r=0}^{\infty} \frac{q^{\frac{(r+2)(r+3)}{2}}}{(q;q^2)_{r+2}} X_m(q)$

$$= \psi_T(q) \frac{(q;q^{12})_\infty (q^{11};q^{12})_\infty (q^{12};q^{12})_\infty}{(q;q)_\infty} - \sum_{m=0}^{\infty} \frac{(-q^2;q^2)_m (1-q^{n+1}) q^{m(m+2)}}{(q;q)_{2m+2}} \psi_{Tm}(q)$$

By taking $\delta_r = \frac{(-q^2;q^2)_r (1-q^{n+1}) q^{r(r+2)}}{(q;q)_{2r+2}}, \alpha_r = \frac{(-1)^r q^{r^2}}{(-q;q)_{2r}}$ in (4.1) and making use of (2.3), then after summing, we get

(4.20) $\sum_{r=0}^{\infty} \frac{(-1)^{r+1} q^{(r+1)^2}}{(-q;q)_{2(r+1)}} X_m(q)$

$$= \mathrm{X}_T(q) \frac{(q;q^{12})_\infty (q^{11};q^{12})_\infty (q^{12};q^{12})_\infty}{(q;q)_\infty} - \sum_{m=0}^{\infty} \frac{(-q^2;q^2)_m (1-q^{n+1}) q^{m(m+2)}}{(q;q)_{2m+2}} \mathrm{X}_{Tm}(q)$$

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By taking $\delta_r = \frac{(-q^2;q^2)_r(1-q^{n+1})q^{r(r+2)}}{(q;q)_{2r+2}}$, $\alpha_r = \frac{(-1)^r q^{(r+1)^2}}{(-q;q)_{2r+1}}$ in (4.1) and making use of (2.4), then after summing, we get

(4.21) $\sum_{r=0}^{\infty} \frac{(-1)^{r+1} q^{(r+2)^2}}{(-q;q)_{2r+3}} X_m(q)$

$$= \chi_T(q) \frac{(q;q^{12})_\infty (q^{11};q^{12})_\infty (q^{12};q^{12})_\infty}{(q;q)_\infty} - \sum_{m=0}^{\infty} \frac{(-q^2;q^2)_m (1-q^{n+1}) q^{m(m+2)}}{(q;q)_{2m+2}} \chi_{Tm}(q)$$

By taking $\delta_r = \frac{(-q^2;q^2)_{r-1}(1+q^r)q^{r^2}}{(q;q)_{2r}}$, $\alpha_r = \frac{q^{\frac{r(r+1)}{2}}}{(q;q^2)_{r+1}}$ in (4.1) and making use of (2.1), then after summing, we get

(4.22) $\sum_{r=0}^{\infty} \frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+2}} Y_m(q) = \varphi_T(q) \frac{(q^5;q^{12})_\infty (q^7;q^{12})_\infty (q^{12};q^{12})_\infty}{(q;q)_\infty} - \sum_{m=0}^{\infty} \frac{(-q^2;q^2)_{m-1}(1+q^m)q^{m^2}}{(q;q)_{2m}} \varphi_{Tm}(q)$

By taking $\delta_r = \frac{(-q^2;q^2)_{r-1}(1+q^r)q^{r^2}}{(q;q)_{2r}}$, $\alpha_r = \frac{q^{\frac{(r+1)(r+2)}{2}}}{(q;q^2)_{r+1}}$ in (4.1) and making use of (2.2), then after summing, we get

(4.23) $\sum_{r=0}^{\infty} \frac{q^{\frac{(r+2)(r+3)}{2}}}{(q;q^2)_{r+2}} Y_m(q) = \psi_T(q) \frac{(q^5;q^{12})_\infty (q^7;q^{12})_\infty (q^{12};q^{12})_\infty}{(q;q)_\infty} - \sum_{m=0}^{\infty} \frac{(-q^2;q^2)_{m-1}(1+q^m)q^{m^2}}{(q;q)_{2m}} \psi_{Tm}(q)$

By taking $\delta_r = \frac{(-q^2;q^2)_{r-1}(1+q^r)q^{r^2}}{(q;q)_{2r}}$, $\alpha_r = \frac{(-1)^r q^{r^2}}{(-q;q)_{2r}}$ in (4.1) and making use of (2.3), then after summing, we get

(4.24) $\sum_{r=0}^{\infty} \frac{(-1)^{r+1} q^{(r+1)^2}}{(-q;q)_{2(r+1)}} Y_m(q)$

$$= \mathrm{X}_T(q) \frac{(q^5;q^{12})_\infty (q^7;q^{12})_\infty (q^{12};q^{12})_\infty}{(q;q)_\infty} - \sum_{m=0}^{\infty} \frac{(-q^2;q^2)_{m-1}(1+q^m)q^{m^2}}{(q;q)_{2m}} \mathrm{X}_{Tm}(q)$$

By taking $\delta_r = \frac{(-q^2;q^2)_{r-1}(1+q^r)q^{r^2}}{(q;q)_{2r}}$, $\alpha_r = \frac{(-1)^r q^{(r+1)^2}}{(-q;q)_{2r+1}}$ in (4.1) and making use of (2.4), then after summing, we get

(4.25) $\sum_{r=0}^{\infty} \frac{(-1)^{r+1} q^{(r+2)^2}}{(-q;q)_{2r+3}} Y_m(q)$

$$= \chi_T(q) \frac{(q^5;q^{12})_\infty (q^7;q^{12})_\infty (q^{12};q^{12})_\infty}{(q;q)_\infty} - \sum_{m=0}^{\infty} \frac{(-q^2;q^2)_{m-1}(1+q^m)q^{m^2}}{(q;q)_{2m}} \chi_{Tm}(q)$$

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