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On $\mathcal{NI}_{\widehat{g}\alpha}$-Closed Sets in Nano Ideal Topological Spaces

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ABSTRACT

The basic objective of this paper is to define and investigate a new class of sets is called $\mathcal{NI}_{\widehat{g}\alpha}$-closed sets, $\mathcal{NI}_{\widehat{g}\alpha}$-open sets, in nano ideal topological spaces. Also define a new notions like $\mathcal{NI}_{\widehat{g}\alpha}$-continuous functions and $\mathcal{NI}_{\widehat{g}\alpha}$-irresolute functions in nano ideal topological spaces, and we study the relationships between the other existing sets in nano ideal topological space. Further we have given an approprite examples to understand the abstract concept clearly.

Keywords: $\mathcal{NI}_g$-closed sets, $\mathcal{NI}_{\widehat{g}}$-closed sets, $\mathcal{NI}_{\widehat{g}\alpha}$-closed sets, $\mathcal{NI}_{\widehat{g}\alpha}$-open sets, $\mathcal{NI}_{\widehat{g}\alpha}$-continuous functions and $\mathcal{NI}_{\widehat{g}\alpha}$-irresolute functions.

1. INTRODUCTION

In 1970, Levine [15] introduced the concept of generalized closed sets in topological spaces. This concept was found to be useful to develop many results in general topology. In 1991, Balachandran et.al [2] introduced and investigated the notion of generalized continuous functions in topological spaces. In 2003, Veerakumar [19] introduced $\widehat{g}$-closed sets in topological spaces. The concept of ideal topological space was introduced by kuratowski [9]. Also he defined the local functions in ideal topological spaces. Further, Jankovic and Hamlett [8] investigated further properties of ideal topological spaces. In 2011, Ravi et.al [1] introduced $\mathcal{I}_{\widehat{g}}$-closed sets in ideal topological spaces.

The notion of nano topology was introduced by LellisThivagar [12, 13] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established and analyzed the nano forms of weakly open sets such as nano α-open sets, nano semi-open sets and nano pre-open sets. Bhuvaneswari and Mythili Gnanapriya [3], introduced and studied the concept of Nano generalized-closed sets. LellisThivagar and Sutha Devi [14] defined nano ideal topological spaces.

The structure of this manuscript is as follows. In section 2, we recall some fundamental definitions and results which are useful to prove our main results. In section 3 and 4, we define and study the notion of $\mathcal{NI}_{\widehat{g}\alpha}$-closed sets and $\mathcal{NI}_{\widehat{g}\alpha}$-open sets in nano ideal topological spaces. We also discuss the concept of $\mathcal{NI}_{\widehat{g}\alpha}$-closed sets and discussed the relationships between the other existing nano ideal sets. In section 5, we define and studied the notions of $\mathcal{NI}_{\widehat{g}\alpha}$-continuous functions and $\mathcal{NI}_{\widehat{g}\alpha}$-irresolute functions in nano ideal topological spaces. We discussed its basic properties and studied the relationships between other existing continuous functions in nano ideal topological spaces.

2. PRELIMINARIES

Definition 2.1. [12] *Let U be a non-empty finite set of objects called the universe R be an equivalence relation on U named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

(1) *The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.*
That is, $L_R(X) = \{\bigcup_{x\in U} \{R(x) : R(x) \subseteq X\}\}$, where $R(x)$ denotes the equivalence class determined by x.
(2) *The Upper approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $U_R(X)$.*
That is, $U_R(X) = \{\bigcup_{x\in U} \{R(x) : R(x) \cap X \neq \phi\}\}$
(3) *The Boundary region of X with respect to R is the set of all objects which can be classified as neither as X nor as not X with respect to R and it is denoted by$B_R(X)$.*
That is, $B_R(X) = U_R(X)$-$L_R(X)$

Definition 2.2. [12] *Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms:*

(1) *U and $\phi \in \tau_R(X)$*
(2) *The union of elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*
(3) *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$*

That is, $\tau_R(X)$ forms a topology on U is called the nano topology on U with respect to X. We call $\{U, \tau_R(X)\}$ is called the nano topological space.

Definition 2.3. [6] *An ideal $\mathcal{I}$ on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following properties*

(1) *$A \in \mathcal{I}$ and $B \subseteq A \Rightarrow B \in \mathcal{I}$.*
(2) *$A \in \mathcal{I}$ and $B \in \mathcal{I} \Rightarrow A \cup B \in \mathcal{I}$.*

An ideal topological space is a topological space (X, τ) with an ideal $\mathcal{I}$ on X and is denoted by $(X, \tau, \mathcal{I})$

Definition 2.4. *A nano topological space $\{U, \tau_R(X)\}$ with an ideal $\mathcal{I}$ on U is called a nano ideal topological space or nano ideal space and is denoted as $\{U, \tau_R(X), \mathcal{I}\}$.*

Definition 2.5. [14] *Let* $\{U, \tau_R(X), \mathcal{I}\}$ *be a nano ideal topological space. A set operator* $(A)^{\star\mathcal{N}} : P(U) \to P(U)$ *is called the nano local function of* $\mathcal{I}$ *on* U *with respect to* $\mathcal{I}$ *on* $\tau_R(X)$ *is defined as* $(A)^{\star\mathcal{N}} = \{x \in U : U \cap A \notin \mathcal{I};$ *for every* $U \in \tau_R(X)\}$ *and is denoted by* $(A)^{\star\mathcal{N}}$*, where nano closure operator is defined as* $\mathcal{N}cl^\star(A) = A \cup (A)^{\star\mathcal{N}}$.

Result 2.6. [14] *Let* $\{U, \tau_R(X), \mathcal{I}\}$ *be a nano ideal topological space and let* A *and* B *be subsets of* U*, then*

(1) $(\phi)^{\star\mathcal{N}} = \phi$.
(2) $A \subset B \to (A)^{\star\mathcal{N}} \subset (B)^{\star\mathcal{N}}$.
(3) *For another* $J \supseteq \mathcal{I}$ *on* U, $(A)^{\star\mathcal{N}}(J) \subset (A)^{\star\mathcal{N}}(\mathcal{I})$.
(4) $(A)^{\star\mathcal{N}} \subset \mathcal{N}cl^\star(A)$.
(5) $(A)^{\star\mathcal{N}}$ *is a nano closed set.*
(6) $((A)^{\star\mathcal{N}})^{\star\mathcal{N}} \subset (A)^{\star\mathcal{N}}$.
(7) $(A)^{\star\mathcal{N}} \cup (B)^{\star\mathcal{N}} = (A \cup B)^{\star\mathcal{N}}$.
(8) $(A \cap B)^{\star\mathcal{N}} = (A)^{\star\mathcal{N}} \cap (B)^{\star\mathcal{N}}$.
(9) *For every nano open set* V, $V \cap (V \cap A)^{\star\mathcal{N}} \subset (V \cap A)^{\star\mathcal{N}}$.
(10) *For* $\mathcal{I} \in \mathcal{I}$, $(A \cup \mathcal{I})^{\star\mathcal{N}} = (A)^{\star\mathcal{N}} = (A - \mathcal{I})^{\star\mathcal{N}}$.

Result 2.7. [14] *Let* $\{U, \tau_R(X), \mathcal{I}\}$ *be a nano ideal topological space and* A *be a subset of* U*, If* $A \subset (A)^{\star\mathcal{N}}$*, then* $(A)^{\star\mathcal{N}} = \mathcal{N}cl(A)^{\star\mathcal{N}} = \mathcal{N}cl(A) = \mathcal{N}cl^\star(A)$.

Definition 2.8. *Let* $(U, \tau_R(X))$ *be a nano topological space and* $A \subseteq U$*. Then* A *is said to be*

(1) $\mathcal{N}\alpha$*-open [11], if* $A \subseteq \mathcal{N}int(\mathcal{N}cl(\mathcal{N}int(A)))$.
(2) *Nano Semi-open [9], if* $A \subseteq \mathcal{N}cl(\mathcal{N}int(A))$.
(3) $\mathcal{N}\widehat{g}$*-closed* [10]*, if* $\mathcal{N}cl(A) \subseteq G$ *whenever* $A \subseteq G$ *and* G *is nano semi-open.*
(4) $\mathcal{N}g\alpha$*-closed [9], if* $\mathcal{N}\alpha cl(A) \subseteq G$ *whenever* $A \subseteq U$ *and* U *is* α*-open.*

Definition 2.9. *A function* $f : (U, \tau_R(X)) \to (V, \sigma_R(X))$ *is said to be*

(1) *nano continuous* [13]*, if the inverse image of a nano closed set in* W *is nano closed in* U.
(2) $\mathcal{N}\widehat{g}$ *- continuous* [11] *, if* $f^{-1}(W)$ *is* $\mathcal{N}\widehat{g}$ *- closed in* $(U, \tau_R(X))$ *for every nano closed set* W *in* $(V, \sigma_R(X))$.
(3) $\mathcal{N}g\alpha$ *- continuous* [4]*, if* $f^{-1}(W)$ *is* $\mathcal{N}g\alpha$ *- closed in* (U, τ_R) *for every nano closed set* W *in* $(V, \sigma_R(X))$.

Definition 2.10. *A subset* A *of a nano ideal space. Let* $(U, \tau_R(X), \mathcal{I})$ *is said to be*

(1) $\star^{\mathcal{N}}$*-closed* [16]*, if* $(A)^{\star\mathcal{N}} \subseteq A$.
(2) $\star^{\mathcal{N}}$*-dense* [16]*, if* $A \subseteq (A)^{\star\mathcal{N}}$.
(3) $\mathcal{N}\mathcal{I}_g$*-closed* [16]*, if* $(A)^{\star\mathcal{N}} \subseteq G$ *whenever* $A \subseteq G$ *and* G *is nano open.*

(4) $\mathcal{NI}_{\widehat{g}}$*-closed* [17], *if* $(A)^{\star\mathcal{N}} \subseteq G$ *whenever* $A \subseteq G$ *and* G *is nano semi-open.*

Definition 2.11. *A function* $f : (U, \tau_R(X), \mathcal{I}) \to (V, \sigma_R(X))$ *is said to be*

(1) $\mathcal{NI}_g$ *- continuous* [18], *if* $f^{-1}(W)$ *is* $\mathcal{NI}_g$ *- closed in* $(U, \tau_R(X), \mathcal{I})$ *for every nano closed set* W *in* $(V, \sigma_R(X))$,

(2) $\mathcal{NI}_{\widehat{g}}$ *- continuous* [17], *if* $f^{-1}(W)$ *is* $\mathcal{NI}_{\widehat{g}}$ *- closed in* $(U, \tau_R(X), \mathcal{I})$ *for every nano closed set* W *in* $(V, \sigma_R(X))$,

3. $\mathcal{NI}_{\widehat{g}\alpha}$-CLOSED SETS

In this section we define and study the notion of $\mathcal{NI}_{\widehat{g}\alpha}$-closed sets and $\mathcal{NI}_{\widehat{g}\alpha}$-open sets in nano ideal topological spaces.

Definition 3.1. *A subset* A *of a nano ideal space* $(U, \tau_R(X), \mathcal{I})$ *is said to be* $\mathcal{NI}_{\widehat{g}\alpha}$*-closed if* $\mathcal{N}\alpha\mathcal{I}cl(A) \subseteq G$ *whenever* $A \subseteq G$ *and* G *is* $\mathcal{NI}_{\widehat{g}}$*-open.*

Theorem 3.2. *If* $(U, \tau_R(X), \mathcal{I})$ *is any nano ideal space, then every nano closed set is a* $\mathcal{NI}_{\widehat{g}\alpha}$*-closed but not conversely.*

Proof: *Let* A *be a nano closed set and* G *be a any* $\mathcal{NI}_{\widehat{g}}$*-open set containing* A*. Then* $A \subseteq G$*. This implies that* $\mathcal{N}cl(A) \subseteq G$*. Also* $\mathcal{N}\alpha\mathcal{I}cl(A) \subseteq \mathcal{N}cl(A) \subseteq G$*. Therefore* $\mathcal{N}\alpha\mathcal{I}cl(A) \subseteq G$*. Hence* A *is* $\mathcal{NI}_{\widehat{g}\alpha}$*-closed.*

Example 3.3. *Let* $U = \{a, b, c, d\}$*, with* $U\backslash R = \{\{a\}, \{b\}, \{c, d\}\}$ *and* $X = \{a, b\}$*. Then the nano topology,* $\tau_R(X)$=$\{U, \phi, \{a, b\}\}$ *and* $\mathcal{I} = \{\phi, \{c\}, \{d, c\}\}$*. Then* $\mathcal{NI}_{\widehat{g}\alpha}$*-closed sets are* $\{U, \phi, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$ *and nano closed sets are* $\{U, \phi, \{c, d\}\}$*. It is clear that* $\{a, c\}$ *is* $\mathcal{NI}_{\widehat{g}\alpha}$*-closed set but it is not in nano closed.*

Theorem 3.4. *If* $(U, \tau_R(X), \mathcal{I})$ *is any nano ideal space, then every* $\mathcal{N}\widehat{g}$*-closed set is* $\mathcal{NI}_{\widehat{g}\alpha}$*-closed but not conversely.*

Proof: *Let* A *be a* $\mathcal{N}\widehat{g}$*-closed set and* G *be a any* $\mathcal{NI}_{\widehat{g}}$*-open set containing* A*. Then* G *is nano semi-open set containing* A*. Since* A *is* $\mathcal{N}\widehat{g}$*-closed. Therefore* $\mathcal{N}cl(A) \subseteq G$*. Since every nano closed set is* $\mathcal{N}\alpha$*-closed.* $\mathcal{N}\alpha cl(A) \subseteq G$ *and* $\mathcal{N}\alpha\mathcal{I}cl(A) \subseteq \mathcal{N}\alpha cl(A) \subseteq G$*. This implies* $\mathcal{N}\alpha\mathcal{I}cl(A) \subseteq G$*. Hence* A *is* $\mathcal{NI}_{\widehat{g}\alpha}$*-closed.*

Example 3.5. *Let* $U = \{a, b, c, d\}$*, with* $U\backslash R = \{\{a\}, \{b\}, \{c, d\}\}$ *and* $X = \{a, b\}$*. Then the nano topology,* $\tau_R(X)$=$\{U, \phi, \{a, b\}\}$ *and* $\mathcal{I} = \{\phi, \{c\}, \{d, c\}\}$*. Then* $\mathcal{NI}_{\widehat{g}\alpha}$*-closed sets are* $\{U, \phi, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$ *and* $\mathcal{N}\widehat{g}$*-closed sets are* $\{U, \phi, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$*. It is clear that* $\{a, c\}$ *is* $\mathcal{NI}_{\widehat{g}\alpha}$*-closed set but it is not in* $\mathcal{N}\widehat{g}$*-closed.*

Theorem 3.6. *If* $(U, \tau_R(X), \mathcal{I})$ *is any nano ideal space, then every* $\mathcal{N}g\alpha$*-closed set is*$\mathcal{NI}_{\widehat{g}\alpha}$*-closed but not conversely.*

Proof: *Let A be a $\mathcal{N}g\alpha$-closed and G be a $\mathcal{NI}_{\widehat{g}}$-open set containing A. Then G is α-open set containing A. Since A is $\mathcal{N}g\alpha$-closed. Therefore $\mathcal{N}cl(A) \subseteq G$. Since every nano closed set is $\mathcal{N}\alpha$-closed. $\mathcal{N}\alpha cl(A) \subseteq G$ and $\mathcal{N}\alpha\mathcal{I}cl(A) \subseteq \mathcal{N}\alpha cl(A) \subseteq G$. This implies $\mathcal{N}\alpha\mathcal{I}cl(A) \subseteq G$. Hence A is $\mathcal{NI}_{\widehat{g}\alpha}$-closed.*

Example 3.7. *Let $U = \{a, b, c, d\}$, with $U\backslash R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, b\}$. Then the nano topology, $\tau_R(X)$=$\{U, \phi, \{a, b\}\}$ and $\mathcal{I} = \{\phi, \{c\}, \{d, c\}\}$. Then $\mathcal{NI}_{\widehat{g}\alpha}$-closed sets are $\{U, \phi, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$ and $\mathcal{N}g\alpha$-closed sets are $\{U, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. It is clear that $\{a, c\}$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed set but it is not in $\mathcal{N}g\alpha$-closed.*

Theorem 3.8. *If $(U, \tau_R(X), \mathcal{I})$ is any nano ideal space, then every $\mathcal{NI}_{\widehat{g}\alpha}$-closed set is $\mathcal{NI}_g$-closed but not conversely.*

Proof: *Let A be a $\mathcal{NI}_{\widehat{g}\alpha}$-closed and G be a nano open set containing A. Then $A \subseteq G$. Since every nano open set is $\mathcal{NI}_{\widehat{g}}$-open set. This implies that $\mathcal{N}\alpha\mathcal{I}cl(A) \subseteq \mathcal{N}cl(A) \subseteq G$ and $(A)^{\star\mathcal{N}} \subseteq G$ and from $*^{\mathcal{N}}$-closed set, $(A)^{\star\mathcal{N}} \subseteq A = \mathcal{N}cl(A) \subseteq G$. Hence $(A)^{\star\mathcal{N}} \subseteq G$. Therefore A is $\mathcal{NI}_g$-closed.*

Example 3.9. *Let $U = \{a, b, c, d\}$, with $U\backslash R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, b\}$. Then the nano topology, $\tau_R(X)$=$\{U, \phi, \{a, b\}\}$ and $\mathcal{I} = \{\phi, \{c\}, \{d, c\}\}$. Then $\mathcal{NI}_{\widehat{g}\alpha}$-closed sets are $\{U, \phi, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$ and $\mathcal{NI}_g$-closed sets are $\{U, \phi, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. It is clear that $\{a, d\}$ is $\mathcal{NI}_g$-closed set but it is not in $\mathcal{NI}_{\widehat{g}\alpha}$-closed.*

Theorem 3.10. *If $(U, \tau_R(X), \mathcal{I})$ is any nano ideal space, then every $\mathcal{NI}_{\widehat{g}\alpha}$-closed set is $\mathcal{NI}_{\widehat{g}}$-closed but not conversely.*

Proof: *Let A be a $\mathcal{NI}_{\widehat{g}\alpha}$-closed and G be a nano semi-open set containing A. Then $A \subseteq G$. Since every nano semi-open set is $\mathcal{NI}_{\widehat{g}}$-open set. This implies that $\mathcal{N}\alpha\mathcal{I}cl(A) \subseteq \mathcal{N}cl(A) \subseteq G$ and $(A)^{\star\mathcal{N}} \subseteq G$ and from $*^{\mathcal{N}}$-closed set, $(A)^{\star\mathcal{N}} \subseteq A = \mathcal{N}cl(A) \subseteq G$. Hence $(A)^{\star\mathcal{N}} \subseteq G$. Therefore A is $\mathcal{NI}_{\widehat{g}}$-closed.*

Example 3.11. *Let $U = \{a, b, c, d\}$, with $U\backslash R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c, d\}$. Then the nano topology, $\tau_R(X)$=$\{U, \phi, \{a, c, d\}\}$ and $\mathcal{I} = \{\phi, \{d\}\}$. Then $\mathcal{NI}_{\widehat{g}\alpha}$-closed sets are $\{U, \phi, \{b\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}\}$ and $\mathcal{NI}_{\widehat{g}}$-closed sets are $\{U, \phi, \{b\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$. It is clear that $\{a, b\}$ is $\mathcal{NI}_{\widehat{g}}$-closed set but it is not in $\mathcal{NI}_{\widehat{g}\alpha}$-closed.*

Theorem 3.12. *Union of two $\mathcal{NI}_{\widehat{g}\alpha}$-closed sets is $\mathcal{NI}_{\widehat{g}\alpha}$- closed.*

Proof: *Let A and B be a $\mathcal{NI}_{\widehat{g}\alpha}$ closed in U. Let G be a $\mathcal{NI}_{\widehat{g}}$-open in U. Then $A \subseteq G$ and $B \subseteq G$ .Since A and B are $\mathcal{NI}_{\widehat{g}\alpha}$-closed sets, $\alpha\mathcal{NI}cl(A) \subseteq G$ and $\mathcal{N}\alpha\mathcal{I}cl(B) \subseteq G$. Hence $\mathcal{N}\alpha\mathcal{I}cl(A \cup B) = \mathcal{N}\alpha\mathcal{I}cl(A) \cup \mathcal{N}\alpha\mathcal{I}cl(B) \subseteq G$. Therefore $A \cup B$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed.*

Remark 3.13. *The intersection of any two $\mathcal{NI}_{\widehat{g}\alpha}$- closed set is $\mathcal{NI}_{\widehat{g}\alpha}$ -closed set.*

Example 3.14. *Let $U = \{a, b, c, d\}$, with $U\backslash R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, b\}$. Then the nano topology, $\tau_R(X)$=$\{U, \phi, \{a, b\}\}$ and $\mathcal{I} = \{\phi, \{c\}, \{d, c\}\}$. Then $A = \{d\}$ and $B = \{c, d\}$ are $\mathcal{NI}_{\widehat{g}\alpha}$ closed set and $A \cap B = \{d\} \cap \{c, d\} = \{d\}$ is also $\mathcal{NI}_{\widehat{g}\alpha}$ -closed set.*

Theorem 3.15. *If A is $\mathcal{NI}_{\widehat{g}\alpha}$ -closed set and $A \subseteq B \subseteq \mathcal{NI}_{\widehat{g}\alpha} - \mathcal{N}cl(A)$, then B is $\mathcal{NI}_{\widehat{g}\alpha}$ -closed.*

Proof: *Let A be $\mathcal{NI}_{\widehat{g}\alpha}$-closed and $B \subseteq G$, where B is $\mathcal{NI}_{\widehat{g}}$-open. Then $A \subseteq B$ implies $A \subseteq G$. Since A is $\mathcal{NI}_{\widehat{g}\alpha}$-closed, $\mathcal{NI}_{\widehat{g}\alpha} - \mathcal{N}cl(A) \subseteq G$ and $B \subseteq \mathcal{NI}_{\widehat{g}\alpha} - \mathcal{N}cl(A)$ implies $\mathcal{NI}_{\widehat{g}\alpha} - \mathcal{N}cl(B) \subseteq \mathcal{NI}_{\widehat{g}\alpha} - \mathcal{N}cl(A)$. Therefore $\mathcal{NI}_{\widehat{g}\alpha} - \mathcal{N}cl(B) \subseteq G$ and hence B is $\mathcal{NI}_{\widehat{g}\alpha}$-closed.*

Result 3.16. *The following table shows the relationship of $\mathcal{NI}_{\widehat{g}\alpha}$-closed sets with other existing sets. The symbol "1" in a cell means that a set implies the other and the symbol "0" means that a set does not imply the other set.*

sets	*nano closed*	$\mathcal{N}\widehat{g}$*-closed*	$\mathcal{N}g\alpha$*-closed*	$\mathcal{NI}_g$*-closed*	$\mathcal{NI}_{\widehat{g}}$*-closed*	$\mathcal{NI}_{\widehat{g}\alpha}$*-closed*
nano closed	*1*	*1*	*1*	*1*	*1*	*1*
$\mathcal{N}\widehat{g}$*-closed*	*0*	*1*	*1*	*1*	*1*	*1*
$\mathcal{N}g\alpha$*-closed*	*0*	*0*	*1*	*0*	*0*	*0*
$\mathcal{NI}_g$*-closed*	*0*	*0*	*0*	*1*	*0*	*0*
$\mathcal{NI}_{\widehat{g}}$*-closed*	*0*	*0*	*1*	*1*	*1*	*0*
$\mathcal{NI}_{\widehat{g}\alpha}$*-closed*	*0*	*0*	*0*	*1*	*1*	*1*

4. $\mathcal{NI}_{\widehat{g}\alpha}$ -OPEN SETS

Definition 4.1. *A subset A of a nano ideal space $(U, \tau_R(X), \mathcal{I})$ is said to be $\mathcal{NI}_{\widehat{g}\alpha}$-open set if $U - A$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed.*

Theorem 4.2. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal topological space. Then the following statements are hold.*

(1) *Every nano open set is $\mathcal{NI}_{\widehat{g}\alpha}$-open.*
(2) *Every $\mathcal{N}\widehat{g}$-open set is a $\mathcal{NI}_{\widehat{g}\alpha}$-open.*
(3) *Every $\mathcal{N}g\alpha$-open set is $\mathcal{NI}_{\widehat{g}\alpha}$-open.*
(4) *Every $\mathcal{NI}_{\widehat{g}\alpha}$-open set is $\mathcal{NI}_g$-open.*
(5) *Every $\mathcal{NI}_{\widehat{g}\alpha}$-open set is $\mathcal{NI}_{\widehat{g}}$-open.*

Proof: *Proof follows from the theorems 3.2, 3.4, 3.6, 3.8 and 3.10.*

Remark 4.3. $\mathcal{NI}_{\widehat{g}\alpha}$-$\mathcal{N}cl(U - A) = U$- $\mathcal{NI}_{\widehat{g}\alpha}$-$\mathcal{N}int(A)$

Theorem 4.4. *A set $A \subseteq U$ is $\mathcal{NI}_{\widehat{g}\alpha}$-open if and only if $F \subseteq \mathcal{NI}_{\widehat{g}\alpha}$-$\mathcal{N}int(A)$ whenever F is $\mathcal{NI}_{\widehat{g}}$-closed and $F \subseteq A$.*

Proof: *Necessary Part: Let A be a $\mathcal{NI}_{\widehat{g}\alpha}$-open, Let F be $\mathcal{NI}_{\widehat{g}}$-closed set and $F \subseteq A$, then $U - A \subseteq U - F$, where $U - F$ is $\mathcal{NI}_{\widehat{g}}$-open. By assumption, $\mathcal{NI}_{\widehat{g}\alpha} - \mathcal{N}cl(U - A) \subseteq U - F$. By the above remark, $U - \mathcal{NI}_{\widehat{g}\alpha} - \mathcal{N}int(A) \subseteq U - F$. Then $F \subseteq \mathcal{NI}_{\widehat{g}\alpha} - \mathcal{N}int(A)$.*

Sufficent Part: Suppose F is $\mathcal{NI}_{\widehat{g}}$-closed and $F \subseteq A$ such that $F \subseteq \mathcal{NI}_{\widehat{g}\alpha} - \mathcal{N}int(A)$. Let $U - A \subseteq G$

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where G is $\mathcal{NI}_{\widehat{g}}$-open. Then $U - G \subseteq A$, where $U - G$ is $\mathcal{NI}_{\widehat{g}}$. By hypothesis, $U - G \subseteq \mathcal{NI}_{\widehat{g}\alpha} - \mathcal{N}int(A)$ implies $U - \mathcal{NI}_{\widehat{g}\alpha} - \mathcal{N}int(A) \subseteq G$ and $\mathcal{NI}_{\widehat{g}\alpha} - \mathcal{N}cl(A) \subseteq G$. Then $U - A$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed and A is $\mathcal{NI}_{\widehat{g}\alpha}$-open.

Theorem 4.5. *If $U - \mathcal{NI}_{\widehat{g}\alpha} - \mathcal{N}int(A) \subseteq B \subseteq A$ and A is $U - \mathcal{NI}_{\widehat{g}\alpha}$-open, then B is $U - \mathcal{NI}_{\widehat{g}\alpha}$-open set.* **Proof:** *Let $\mathcal{NI}_{\widehat{g}\alpha} - \mathcal{N}int(A) \subseteq B \subseteq A$. Thus $U - A \subseteq U - B \subseteq \mathcal{NI}_{\widehat{g}\alpha} - \mathcal{N}cl(U - A)$. Since $U - A$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed. By the theorem 3.15, $U - A \subseteq U - B \subseteq \mathcal{NI}_{\widehat{g}\alpha} - \mathcal{N}cl(A)$ implies $U - B$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed. Hence B is $\mathcal{NI}_{\widehat{g}\alpha}$-open set.*

5. $\mathcal{NI}_{\widehat{g}\alpha}$ -continuous and $\mathcal{NI}_{\widehat{g}\alpha}$-irresolute Functions

In this section, we define and study the new class of nano function, namely $\mathcal{NI}_{\widehat{g}\alpha}$-continuous, $\mathcal{NI}_{\widehat{g}\alpha}$-irresolute functions, in nano ideal topological spaces. Also study some of their properties. Further we investigated the relatioships between the other existing nano continuous functions.

Definition 5.1. *A function $f : (U, \tau_R(X), \mathcal{I}) \to (V, \sigma_R(X))$ is said to be $\mathcal{NI}_{\widehat{g}\alpha}$ -continuous, if $f^{-1}(W)$ is $\mathcal{NI}_{\widehat{g}\alpha}$ -closed in $(U, \tau_R(X), \mathcal{I})$ for every nano closed set W in $(V, \sigma_R(X))$.*

Definition 5.2. *A function $f : (U, \tau_R(X), \mathcal{I}_1) \to (V, \sigma_R(X), \mathcal{I}_2)$ is said to be $\mathcal{NI}_{\widehat{g}\alpha}$ -irresolute, if $f^{-1}(W)$ is $\mathcal{NI}_{\widehat{g}\alpha}$ -closed in $(U, \tau_R(X), \mathcal{I}_1)$ for every nano closed set W in $(V, \sigma_R(X), \mathcal{I}_2)$.*

Theorem 5.3. *In a nano ideal topological space $(U, \tau_R(X), \mathcal{I})$, the following hold.*

(1) *Every nano continuous function is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous.*
(2) *Every $\mathcal{N}\widehat{g}$-continuous function is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous.*
(3) *Every $\mathcal{N}g\alpha$-continuous function is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous.*
(4) *Every $\mathcal{NI}_{\widehat{g}\alpha}$-continuous function is $\mathcal{NI}_g$-continuous.*
(5) *Every $\mathcal{NI}_{\widehat{g}\alpha}$-continuous function is $\mathcal{NI}_{\widehat{g}}$-continuous.*

Proof:

(1) *Let $f : (U, \tau_R(X), \mathcal{I}) \to (V, \sigma_R(X)$ be nano continuous function and W be any nano closed set in $(V, \sigma_R(X))$. Then $f^{-1}(W)$ is nano closed in $(U, \tau_R(X), \mathcal{I})$ as f is nano continuous. Since every nano closed set is $\mathcal{NI}_{\widehat{g}\alpha}$-closed set, $f^{-1}(W)$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed set in $(U, \tau_R(X), \mathcal{I})$. Therefore f is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous.*
(2) *Let $f : (U, \tau_R(X), \mathcal{I}) \to (V, \sigma_R(X))$ be $\mathcal{N}\widehat{g}$-continuous function and W be any nano closed set in $(V, \sigma_R(X))$. Then $f^{-1}(W)$ is $\mathcal{N}\widehat{g}$-closed in $(U, \tau_R(X), \mathcal{NI})$ as f is $\mathcal{N}\widehat{g}$-continuous. Since every $\mathcal{N}\widehat{g}$-closed set is $\mathcal{NI}_{\widehat{g}\alpha}$-closed set, $f^{-1}(W)$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed set in $(U, \tau_R(X), \mathcal{I})$. Therefore f is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous.*
(3) *Let $f : (U, \tau_R(X), \mathcal{I}) \to (V, \sigma_R(X))$ be $\mathcal{N}g\alpha$-continuous function and W be any nano closed set in $(V, \sigma_R(X))$. Then $f^{-1}(W)$ is $\mathcal{N}g\alpha$-closed in $(U, \tau_R(X), \mathcal{NI})$ as f is $\mathcal{N}g\alpha$-continuous.*

Since every $\mathcal{N}g\alpha$-closed set is $\mathcal{NI}_{\widehat{g}\alpha}$-closed set, $f^{-1}(W)$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed set in $(U, \tau_R(X), \mathcal{I})$. Therefore f is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous.

(4) *Let $f : (U, \tau_R(X), \mathcal{I}) \to (V, \sigma_R(X))$ be $\mathcal{NI}_{\widehat{g}\alpha}$-continuous function and W be any nano closed set in $(V, \sigma_R(X))$. Then $f^{-1}(W)$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed in $(U, \tau_R(X), \mathcal{I})$ as f is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous. Since every $\mathcal{NI}_{\widehat{g}\alpha}$-closed set is $\mathcal{NI}_g$-closed set, $f^{-1}(W)$ is $\mathcal{NI}_g$-closed set in $(U, \tau_R(X), \mathcal{I})$. Therefore f is $\mathcal{NI}_g$-continuous.*

(5) *Let $f : (U, \tau_R(X), \mathcal{I}) \to (V, \sigma_R(X))$ be $\mathcal{NI}_{\widehat{g}\alpha}$-continuous function and W be any nano closed set in $(V, \sigma_R(X))$. Then $f^{-1}(W)$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed in $(U, \tau_R(X), \mathcal{I})$ as f is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous. Since every $\mathcal{NI}_{\widehat{g}\alpha}$-closed set is $\mathcal{NI}_{\widehat{g}}$-closed set, $f^{-1}(W)$ is $\mathcal{NI}_{\widehat{g}}$-closed set in $(U, \tau_R(X), \mathcal{I})$. Therefore f is $\mathcal{NI}_{\widehat{g}}$-continuous.*

Example 5.4. *Let $U = \{a, b, c, d\}$, with $U \backslash R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a\}$. Then the nano topology, $\tau_R(X)$=$\{U, \phi, \{a\}\}$ and the ideal $\mathcal{I} = \{\phi, \{a\}, \{a, b\}\}$ and Let $V = \{a, b, c, d\}$, with $V \backslash R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{c\}$. Then the nano topology, $\sigma_R(X) = \{V, \phi, \{c, d\}\}$. Let $f : (U, \tau_R(X), \mathcal{I}) \to (V, \sigma_R(X))$ be the identity function. Then f is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous but not nano continuous, $\mathcal{N}\widehat{g}$-continuous and $\mathcal{N}g\alpha$-continuous*

Example 5.5. *Let $U = \{a, b, c, d\}$, with $U \backslash R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a\}$. Then the nano topology, $\tau_R(X)$=$\{U, \phi, \{a\}\}$ and the ideal $\mathcal{I} = \{\phi, \{a\}, \{a, b\}\}$ and Let $V = \{a, b, c, d\}$, with $V \backslash R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{c\}$. Then the nano topology, $\sigma_R(X) = \{V, \phi, \{c, d\}\}$ Let $f : (U, \tau_R(X), \mathcal{I}) \to (V, \sigma_R(X))$ be the identity function. Then f is $\mathcal{NI}_g$-continuous but not $\mathcal{NI}_{\widehat{g}\alpha}$-continuous.*

Example 5.6. *Let $U = \{a, b, c, d\}$, with $U \backslash R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c, d\}$. Then the nano topology, $\tau_R(X)$=$\{U, \phi, \{a, c, d\}\}$ and the ideal $\mathcal{I} = \{\phi, \{d\}\}$ and Let $V = \{a, b, c, d\}$, with $V \backslash R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{c\}$. Then the nano topology, $\sigma_R(X) = \{V, \phi, \{c, d\}\}$ Let $f : (U, \tau_R(X), \mathcal{I}) \to (V, \sigma_R(X))$ be the identity function. Then f is $\mathcal{NI}_{\widehat{g}}$-continuous but not $\mathcal{NI}_{\widehat{g}\alpha}$-continuous.*

Theorem 5.7. *Every $\mathcal{NI}_{\widehat{g}\alpha}$-irresolute function is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous function but not conversely.*

Proof: *Let W be a nano closed set in V which is $\mathcal{NI}_{\widehat{g}\alpha}$-closed set, then $f^{-1}(W)$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed in U. Hence f is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous.*

Example 5.8. *Let $U = \{a, b, c, d\}$, with $U \backslash R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{c\}$. Then the nano topology, $\tau_R(X)$=$\{U, \phi, \{c, d\}\}$ and the ideal $\mathcal{I} = \{\phi, \{b\}, \{a, b\}\}$ and Let $V = \{a, b, c, d\}$, with $V \backslash R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a\}$. Then the nano topology, $\sigma_R(X) = \{V, \phi, \{a\}\}$ and the ideal $\mathcal{I} = \{\phi, \{a\}, \{a, b\}\}$. Let $f : (U, \tau_R(X), \mathcal{I}) \to (V, \sigma_R(X))$ be the identity function. Then f is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous but not $\mathcal{NI}_{\widehat{g}}$-irresolute.*

Theorem 5.9. *A function $f : (U, \tau_R(X), \mathcal{I}) \to (V, \sigma_R(X))$ is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous if and only if the inverse image of every nano closed set is $(V, \sigma_R(X))$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed set in $(U, \tau_R(X), \mathcal{I})$.*

Proof: *Necessary Part: Let W be a nano open set in $(V, \sigma_R(X))$. Since f is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous, $f^{-1}(W^c)$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed set in $(U, \tau_R(X), \mathcal{I})$. But $f^{-1}(W^c) = U - f^{-1}(W)$. Hence $f^{-1}(W)$ $\mathcal{NI}_{\widehat{g}\alpha}$-closed in $(U, \tau_R(X), \mathcal{I})$.*

sufficency: Assum that the inverse image of every nano closed set in $(V, \sigma_R(X))$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed set in $(U, \tau_R(X), \mathcal{I})$. Let W be a nano closed set in $(V, \sigma_R(X))$. By our assumption $f^{-1}(W^c) = U - f^{-1}(W)$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed in $(U, \tau_R(X), \mathcal{I})$, which implies that $f^{-1}(W)$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed set in $(U, \tau_R(X), \mathcal{I})$. Hence f is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous.

Theorem 5.10. *Let $f : (U, \tau_R(X), \mathcal{I}_1) \to (V, \sigma_R(X), \mathcal{I}_2)$ and $g : (V, \sigma_R(X), \mathcal{I}_2) \to (Z, \eta_R(X), \mathcal{I}_3)$ be any two functions. Then the following hold.*

(1) *$g \circ f$ is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous if f is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous and g is nano continuous.*
(2) *$g \circ f$ is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous if f is $\mathcal{NI}_{\widehat{g}\alpha}$-irresolute and g is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous.*
(3) *$g \circ f$ is $\mathcal{NI}_{\widehat{g}\alpha}$-irresolute if f is $\mathcal{NI}_{\widehat{g}\alpha}$-irresolute and g is $\mathcal{NI}_{\widehat{g}\alpha}$-irresolute.*

Proof:

(1) *Let W be a nano closed set in Z. Since g is nano continuous, $g^{-1}(W)$ is closed in V. $\mathcal{NI}_{\widehat{g}\alpha}$-continuous of f implies, $f^{-1}(g^{-1}(W))$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed in U and hence $g \circ f$ is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous.*
(2) *Let W be a nano closed set in Z. Since g is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous, $g^{-1}(W)$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed in V. Since f is $\mathcal{NI}_{\widehat{g}\alpha}$-irresolute, $f^{-1}(g^{-1}(W))$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed in U and hence $g \circ f$ is $\mathcal{NI}_{\widehat{g}\alpha}$-continuous.*
(3) *Let W be a nano closed set in Z. Since g is $\mathcal{NI}_{\widehat{g}\alpha}$-irresolute, $g^{-1}(W)$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed in V. Since f is $\mathcal{NI}_{\widehat{g}\alpha}$-irresolute, $f^{-1}(g^{-1}(W))$ is $\mathcal{NI}_{\widehat{g}\alpha}$-closed in U and hence $g \circ f$ is $\mathcal{NI}_{\widehat{g}\alpha}$-irresolute.*

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