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On $Ng^{*}\alpha$ - closed sets in nano topological spaces

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ABSTRACT

This paper focuses on $Ng^{*}\alpha$ -closed sets and $Ng^{*}\alpha$ -open sets in nano topological spaces and certain properties are investigated. We also investigate and discussed their relationships with other forms of nano sets. Further, we have given an appropriate examples to understand the abstract concept clearly.

Keywords: $Ng^{*}\alpha$ -closed sets, $Ng^{*}\alpha$ -open sets.

1. INTRODUCTION

In 1970, Levine [7] introduced the concept of generalized closed sets in a topological space. This concept was found to be useful and many results in general topology were improved. Lellis Thivagar [6] introduced nano topological spaces with respect to subset X of a universe which is defined interms of lower and upper approximation of X. He study the relationships between some near nano open sets in nano topological spaces. The elements of nano topological space are called nano open sets. The topology is named as nano topology so because of its size. He has also defined nano closed sets, nano interior and nano closure of a set. Since it has almost five elements, Recently bhuvaneshwari [1,2,3,4] introduced nano g-closed, nano gs-closed, nano gp-closed, nano αg-closed, nano $g\alpha$-closed and investigates their properties.

In 2017, punitha Tharani [5] introduced on $g^{*}\alpha$ -closed sets, $g^{*}\alpha$ -open sets In topological space.

In this paper we introduced the concept of $Ng^{*}\alpha$ -closed sets in nano topological spaces and some of its properties are analyzed.

2. PRELIMINARIES

This section is to recall some fundamental definitions and results which are useful to prove our main results.

Definition 2.1. [6] Let U be a non-empty finite set of objects called the universe R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆ U, then

(i) The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_{R(X)}$. *That is,* $L_{R(X)}) = \{\bigcup_{X\in U} \{R(x): R(x) \subseteq X\}\}$,
where R(x) denotes the equivalence class determined by x.

(ii) The Upper approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by UR(X).
That is, $U_{R(X)} = \{\bigcup_{X\in U} \{R(x) : R(x) \cap X \neq \phi\}\}$

(iii) The Boundary region of X with respect to R is the set of all objects which can be classified as neither as X nor as not X and it is denoted by BR(X).
That is, $B_{R(X)} = U_{R(X)} - L_{R(X)}$

Definition 2.2. [6] Let U be the universe, R be an equivalence relation on U and $\tau_{R(X)} = \{U, \phi, L_{R(X)}, U_{R(X)}, B_{R(X)}\}$ where X ⊆ U . $\tau_{R(X)}$ satisfies the following axioms:

(i) U and $\phi \in \tau_{R(X)}$

(ii) The union of elements of any subcollection of $\tau_{R(X)}$ is in $\tau_{R(X)}$.

(iii) The intersection of the elements of any finite subcollection of $\tau_{R(X)}$ is in $\tau_{R(X)}$ That is, $\tau_{R(X)}$ forms a topology on U is called the nano topology on U with respect to X. We call $\{U, \tau_{R(X)}\}$ is called the nano topological space. Elements of the nano topology are known as nano open sets in U. Elements of $[\tau_{R(X)}]^c$ are called nano closed sets.

Remark 2.3. [6] If $[\tau_{R(X)}]$ is the nano topology on U with respect to X. Then the set $B = \{U, \tau_{R(X)}, B_{R(X)}\}$ is the basis for $[\tau_{R(X)}]$.

Definition 2.4. [6] If (U, $\tau_{R(X)}$) is a nano topological space with respect to X. Where X ⊆ U and if *A ⊆ U. Then*

- The nano interior of the set A is defined as the Union of all nano open subsets contained in A and is denoted by Nint(A). Nint(A) is the largest nano open subset of A.
- The nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by Ncl(A). Ncl(A) is the Smallest nano closed set containing A.

Definition 2.5. Let (U, $\tau_{R(X)}$) be a nano topological space and A ⊆ U. Then A is said to be

(i) Nano semi-open [6], if A ⊆ Ncl(Nint(A)).

(ii) Nano pre-open [6], if A ⊆ Nint(Ncl(A)).

(iii) Nano α-open [6], if A ⊆ Nint(Ncl(Nint(A))).

(iv) Nano regular open [6], if A = Nint(Ncl(A)).

Definition 2.6. If (U, $\tau_{R(X)}$) is a nano topological space and if A ⊆ U .Then A is said to be

(i) Ngp-closed [4], if Npcl(A) ⊆ G whenever A ⊆ G and G is nano-open in U.

(ii) Ngpr-closed [4], if Npcl(A) ⊆ G whenever A ⊆ G and G is nano regular open in U.

(iii) Nano sg-closed [10], if Nscl(A) ⊆ G whenever A ⊆ G and G is nano semi open in U.

(iv) Nano gs-closed [10], if Nscl(A) ⊆ G whenever A ⊆ G and G is nano open in U.

(v) Nano gsp-closed [10], if Nspcl(A) ⊆ G whenever A ⊆ G and G is nano open in U.

3. Nano $g^*\alpha$ -closed sets

Definition 3.1. A subset A of (U, $\tau_{R(X)}$) is called Ng*α -closed if Nαcl ⊆ G whenever A ⊆ G and G is Ngα-open set in (U,$\tau_{R(X)}$).

Theorem 3.2. Let (U, τR(X)) be a nano topological space, then each nano closed set is Ng*α - closed set.

Proof: Let A be a nano closed set in a nano topological space (U, $\tau_{R(X)}$). Let G be $Ng\alpha$-open set in (U,$\tau_{R(X)}$) such that $A \subseteq G$. Since A is nano closed, we have $Ncl(A) = A$. But $N\alpha cl \subseteq Ncl(A)$, so $N\alpha cl(A) \subseteq Ncl(A) \subseteq G$. Therefore $N\alpha cl(A) \subseteq G$. Hence A is $Ng^*\alpha$ -closed set in (U, $\tau_{R(X)}$).

The converse of the above theorem need not be true, as proved by the following example.

Example 3.3. Let U = {a, b, c, d}, X = {b, d} and U\R={{a}, {b, c}, {d}}, $\tau_{R(X)}$={U, ϕ, {d}, {b, c}, {b, c, d}} the subset A = {{a, b}, {a, c}, {a, b, d, }, {a, c, d}} is a Ng*α -closed but not a nano closed set in (U, $\tau_{R(X)}$).

Theorem 3.4. Let (U, $\tau_{R(X)}$) be a nano topological space, then each Nα- closed set is Ng*α - closed set.

Proof: Let A be a nano α -closed set in a nano topological space (U, $\tau_{R(X)}$). Let G be any Ngα-open set in (U, $\tau_{R(X)}$) containing A. Since A is nano α-closed, we have Nαcl(A) = A ⊆ G. Therefore Nαcl(A) ⊆ G. Hence A is Ng*α -closed set in (U, $\tau_{R(X)}$).

The converse of the above theorem need not be true, as proved by the following example.

Example 3.5. From the example [3.3], the subset A = {{a, b}, {a, c}, {a, b, d, }, {a, c, d}} is $Ng^*\alpha$ -closed but not a Nα -closed set in $(U, \tau_{R(X)})$.

Theorem 3.6. A subset A of (U, $\tau_{R(X)}$) is $Ng^*\alpha$ -closed then A is a Ngp-closed in (U, $\tau_{R(X)}$).

Proof: Let A be a $Ng^*\alpha$ -closed set of (U, $\tau_{R(X)}$). Let G be any nano open set in (U, $\tau_{R(X)}$), such that A ⊆ G. Since every nano open set is Ngα-open, G is Ngα-open in (U, $\tau_{R(X)}$). Since A is $Ng^*\alpha$-closed, we have Nαcl(A) ⊆ G. But Npcl(A) ⊆ Nαcl(A). So Npcl(A) ⊆ G. Hence A is a *Ngp* -closed set in $(U, \tau_{R(X)})$.

The converse of the above theorem need not be true, as proved by the following example.

Example 3.7. From the example [3.3], the subset A = {{b}, {c}} is Ngp -closed but not a $Ng^*\alpha$ -closed set in (U, $\tau_{R(X)}$).

Theorem 3.8. Let (U, $\tau_{R(X)}$) be a nano topological space, then each $Ng^*\alpha$ -closed set is Ngpr-closed.

Proof: Let A be a $Ng^*\alpha$ -closed set in (U, $\tau_{R(X)}$). Let G be a nano regular open set in (U, $\tau_{R(X)}$), such that A ⊆ G, it is a Ngα –open. Again A is a $Ng^*\alpha$ -closed, Nαcl(A) ⊆ G. But Npcl(A) ⊆ Nαcl(A) ⊆ G. Hence A is a Ngpr -closed set in (U, $\tau_{R(X)}$).

The converse of the above theorem need not be true, as proved by the following example.

Example 3.9. From the example [3.3], the subset A = {{b}, {c}, {b, c}, {b, d}, {c, d}, {b, c, d}} is Ngpr -closed but not a $Ng^*\alpha$ -closed set in (U, $\tau_{R(X)}$).

Theorem 3.10. Let (U, $\tau_{R(X)}$) be a nano topological space, then each $Ng^*\alpha$ -closed set is Nsg -closed.

Proof: Let A be a $Ng^*\alpha$ -closed set in (U, $\tau_{R(X)}$). Let G be a nano open set in (U, $\tau_{R(X)}$), such that A ⊆ G. Since every nano open set is Ngα -open, G is Ngα -open in (U, $\tau_{R(X)}$). Since A is $Ng^*\alpha$ -closed, we have Nαcl(A) ⊆ G. But Nscl(A) ⊆ Nαcl(A). So Nscl(A) ⊆ G. Hence A is a Nsg -closed set in (U, $\tau_{R(X)}$).

The converse of the above theorem need not be true, as proved by the following example.

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Example 3.11. From the example [3.3], the subset A = {{b}, {c}, {d}, {b, c}} is Nsg - closed but not a $Ng^{*}\alpha$ - closed set in (U, $\tau_{R(X)}$).

Theorem 3.12. Let (U, $\tau_{R(X)}$) be a nano topological space, then each $Ng^{*}\alpha$ -closed set is Ngs-closed.

Proof: Let A be a $Ng^{*}\alpha$ -closed set in (U, $\tau_{R(X)}$). Let G be a nano open set in (U, $\tau_{R(X)}$), such that A ⊆ G. Since every nano open set is Ngα-open, G is Ngα-open in (U, $\tau_{R(X)}$). Since A is $Ng^{*}\alpha$ -closed, we have Nαcl(A) ⊆ G. But Nscl(A) ⊆ Nαcl(A). So Nscl(A) ⊆ G. Hence A is a Ngs -closed set in (U, $\tau_{R(X)}$).

The converse of the above theorem need not be true, as proved by the following example.

Example 3.13. From the example [3.3], the subset A = {{b}, {c}, {d}, {b, c}} is Ngs - closed but not a $Ng^{*}\alpha$ -closed set in (U, $\tau_{R(X)}$).

Theorem 3.14. Let (U, $\tau_{R(X)}$) be a nano topological space, then each $Ng^{*}\alpha$-closed set is Ngsp -closed.

Proof: Let (U, $\tau_{R(X)}$) be a nano topological space, then each $Ng^{*}\alpha$ -closed set is Ngsp- closed. If G is a $Ng^{*}\alpha$ -closed subset of (U, $\tau_{R(X)}$) and A is any nano open set containing G, every nano open set is Ngα -open, we have Nspcl(A) ⊆ Nαcl(A) ⊆ G. Hence A is Ngsp closed.

The converse of the above theorem need not be true, as proved by the following example.

Example 3.15. From the example [3.3], the subset A = {{b}, {c}, {d}, {b, c}, {b, d}, {c, d}} is Ngsp -closed but not a $Ng^{*}\alpha$ -closed set in (U, $\tau_{R(X)}$).

Remark 3.16. The concepts of $Ng^{*}\alpha$-closed set is independent of the sets namely semi closed and regular closed sets as seen from the following examples.

Example 3.17. Let U = {a, b, c, d}, X = {b, d} and U \R={{a}, {b, c}, {d}}. Then $\tau_{R(X)}$={U, ϕ, {d}, {b, c}, {b, c, d}} the subset A = {{a, b}} is a $Ng^{*}\alpha$-closed but not a nano semi closed and nano regular closed set in (U, $\tau_{R(X)}$).

4.Properties of Nano $g^*\alpha$ -closed sets

Lemma 4.1. If F is a Nano closed set of (U, $\tau_{R(X)}$). Then the following properties hold.

(i) If A is Nano α-closed in (U, $\tau_{R(X)}$), then A ∩ F is Nα-closed in (U, $\tau_{R(X)}$).

(ii) If A is Nano gα-closed in (U, $\tau_{R(X)}$), then A ∩ F is Ngα-closed in (U, $\tau_{R(X)}$).

Corollary 4.2. If A is a $Ng^*\alpha$ -closed set and F is a Nano closed set, then A ∩ F is a $Ng^*\alpha$ -closed set.

Proof: If G be a Ngα-open set of (U, $\tau_{R(X)}$), such that A ∩ F ⊆ G. By lemma it is shows that A ⊆ G ∪ (U \F) and G ∪ (U \F) is Ngα-open in (U, $\tau_{R(X)}$). Since A is $Ng^*\alpha$ -closed in (U, $\tau_{R(X)}$), we have Nαcl(A) ⊆ G ∪ (U \F) and so Nαcl(A ∩ F) ⊆ Nαcl(A) ∩ Nαcl(F) = Nαcl(A) ∩ F) ⊆ (G ∪ (U \F) ∩ F) = G ∩ F ⊆ G. Therefore, A ∩ F is $Ng^*\alpha$ -closed in (U, $\tau_{R(X)}$).

Example 4.3. Let U = {a, b, c, d}, with U \R={{a}, {b, c}, {d}}. Let X = {b, d} then $\tau_{R(X)}$={U, ϕ, {d}, {b, c}, {b, c, d}} and the nano closed sets are $\tau_{R(X)}{}^C$ = {U, ϕ, {a, b, c}, {a, d}, {a}}. The $Ng^*\alpha$ -closed sets are A ={U, ϕ, {a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}}. Let A= {a, b, d} and B={a, b} and A ∩ B={a, b, d} is also $Ng^*\alpha$ -closed set.

Theorem 4.4. The union of any $Ng^*\alpha$ -closed set in (U, $\tau_{R(X)}$) is also a $Ng^*\alpha$ -closed set in (U, $\tau_{R(X)}$).

Proof: Let A and B be any two $Ng^*\alpha$ -closed set in (U, $\tau_{R(X)}$). Let G be any Ngα nano open set in (U, $\tau_{R(X)}$), such that A ∪ B ⊆ G. Then A ⊆ G and B ⊆ G. Since A and B are $Ng^*\alpha$ -closed set in (U, $\tau_{R(X)}$). Nαcl(A) ⊆ G and Nαcl(B) ⊆ G. Therefore Nαcl(A) ∪ Nαcl(B) = Nαcl(A ∪ B) ⊆ G. Hence A ∪ B is $Ng^*\alpha$ -closed set in (U, $\tau_{R(X)}$).

Example 4.5. Let U = {a, b, c, d}, with U \R={{a}, {b, c}, {d}}. Let X = {b, d} then $\tau_{R(X)}$={U, ϕ, {d}, {b, c}, {b, c, d}} and the nano closed sets are $\tau_{R(X)}{}^c$ = {U, ϕ, {a, b, c}, {a, d}, {a}}. The $Ng^*\alpha$ -closed sets are A ={U, ϕ, {a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}}. Let A= {a, b} and B={a, b, d} and A ∪ B={a, b, d} is also $Ng^*\alpha$ -closed set.

Theorem 4.6. The intersection of any $Ng^*\alpha$ -closed set in (U, $\tau_{R(X)}$) is also a $Ng^*\alpha$ -closed set in (U, $\tau_{R(X)}$).

Proof: Let A and B be any two $Ng^*\alpha$ -closed set in $(U, \tau_{R(X)})$. Let G be any $Ng\alpha$ nano open set in $(U, \tau_{R(X)})$, such that $A \cap B \subseteq G$. Then $A \subseteq G$ and $B \subseteq G$. Since A and B are $Ng^*\alpha$ -closed set in $(U, \tau R(X))$. $N\alpha cl(A) \subseteq G$ and $N\alpha cl(B) \subseteq G$.

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Therefore $N\alpha cl(A) \cap N\alpha cl(B) = N\alpha cl(A \cap B) \subseteq G$. Hence $A \cap B$ is $Ng^*\alpha$ -closed set in $(U, \tau_{R(X)})$.

Example 4.7. *Let* $U = \{a, b, c, d\}$, *with* $U \backslash R = \{\{a\}, \{b, c\}, \{d\}\}$. *Let* $X = \{b, d\}$ *then* $\tau_{R(X)} = \{U, \phi, \{d\}, \{b, c\}, \{b, c, d\}\}$ and the nano closed sets are $\tau_{R(X)}{}^c = \{U, \phi, \{a, b, c\}, \{a, d\}, \{a\}\}$. The $Ng^*\alpha$-closed sets are $A = \{U, \phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$. *Let* $A = \{a, b\}$ *and* $B = \{a, b, d\}$ *and* $A \cap B = \{a, b\}$ is also $Ng^*\alpha$ -closed set.

Proposition 4.8. If a set A is $Ng^*\alpha$ -closed set in $(U, \tau_{R(X)})$, then $N\alpha cl(A) - A$ contains no non empty closed set in $(U, \tau_{R(X)})$.

Proof: Suppose that A is $Ng^*\alpha$ -closed. Let F be a closed subset of $N\alpha cl(A) - A$. Then $A \subseteq F^c$. But $Ng^*\alpha$ -closed, therefore $N\alpha cl(A) \subseteq F^c$. Consequently, $F \subseteq N\alpha cl(A)^c$. We already have $F \subseteq N\alpha cl(A)$. Thus $F \subseteq N\alpha cl(A) \cap (F \subseteq N\alpha cl(A))^c$ and F is empty.

The converse of these need not be true, as proved by the following example.

Example 4.9. Let $U = \{a, b, c, d\}$, with $U \backslash R = \{\{a\}, \{b, c\}, \{d\}\}$. Let $X = \{b, d\}$ then $\tau_{R(X)} = \{U, \phi, \{d\}, \{b, c\}, \{b, c, d\}\}$ and the nano closed sets are $\tau_{R(X)}{}^c = \{U, \phi, \{a, b, c\}, \{a, d\}, \{a\}\}$, then $N\alpha cl(A) = \{\{a, b, c\}, \{a, d\}, \{a\}\}$. If $A = \{\{c\}\}$, then $N\alpha cl(A) - A = \{\{a, b, c\}\} - \{\{c\}\} = \{\{a, b\}\}$ does not contain any non empty closed set. But A is not $Ng^*\alpha$ -closed set in $(U, \tau_{R(X)})$.

Proposition 4.10. A set A is $Ng^*\alpha$ -closed if and only if $N\alpha cl(A) - A$ contains no nonempty $Ng\alpha$- closed set.

Proposition 4.11. If A is $Ng^*\alpha$ -closed set and $A \subseteq B \subseteq N\alpha cl(A)$, then B is $Ng^*\alpha$ -closed in $(U, \tau_{R(X)})$.

Proof: Since $B \subseteq N\alpha cl(A)$, We have $N\alpha cl(B) \subseteq N\alpha cl(A)$. Then $N\alpha cl(B) - B \subseteq N\alpha cl(A) - A$. Since $N\alpha cl(A) - A$ has no nonempty $Ng\alpha$-closed subsets, neither does $N\alpha cl(B) - B$. Then B is $Ng^*\alpha$ -closed.

Proposition 4.12. Let $A \subseteq Y \subseteq U$ and suppose that A is $Ng^*\alpha$ -closed in $(U, \tau_{R(X)})$. Then A is $Ng^*\alpha$ -closed relative to Y.

Proof: Let $A \subseteq Y \cap G$, where G is $Ng\alpha$-open in $(U, \tau_{R(X)})$. Then $A \subseteq G$ and hence $N\alpha cl(A) \subseteq G$. This implies that $Y \cap N\alpha cl(A) \subseteq Y \cap G$. Thus A is $Ng^*\alpha$ -closed relative to Y

Proposition 4.13. If A is a $Ng\alpha$ -open and $Ng^*\alpha$ -closed in $(U, \tau_{R(X)})$. Then A is $N\alpha$-closed in $(U, \tau_{R(X)})$.

Proof: Since A is Ngα-open and Ng*α -closed, Nαcl(A) ⊆ A and hence A is α-closed in $(U, \tau_{R(X)})$.

Proposition 4.14. For each x ∈ U, either {x} is Ngα-closed or $\{x\}^c$ is Ng*α - closed in $(U, \tau_{R(X)})$.

Proof: Suppose that $\{x\}$ is not $Ng\alpha$-closed in $(U, \tau_{R(X)})$. Then $\{x\}^c$ is not $Ng\alpha$-open and the only $Ng\alpha$-open set containing $\{x\}^C$ is the space U itself. Therefore $N\alpha cl(\{x\}^C) \subseteq U$ and so $\{x\}^C$ is $Ng^*\alpha$ -closed in $(U, \tau_{R(X)})$.

Theorem 4.15. Let A be a Ng*α -closed set of a topological space $(U, \tau_{R(X)})$. Then

(i) If A is nano regular open, then Npint(A) and Nαcl(A) are also Ng*α -closed set.

(ii) If A is nano regular closed, then Npcl(A) is also Ng*α -closed set.

Proof: (i) Since A is nano regular open in U, A = Nint(Ncl(A)). Then Nαcl(A) = A ∪ Nint(Ncl(A)) =A. Thus Nαcl(A) is Ng*α in $(U, \tau_{R(X)})$). Since Npint(A) = A∩Nint(Ncl(A)) = A, Npint(A)) is Ng*α -closed.
(ii). Since A is nano regular closed in X, A = Ncl(Nint(A)). Then Npcl(A) = A∪Ncl(A)(Nint(A)) = A. Thus, Npcl(A) is Ng*α in $(U, \tau_{R(X)})$.

The converse of these need not be true, as proved by the following example.

Example 4.16. Let U = {a, b, c, d}, with U \R={{a}, {b, c}, {d}}. Let X = {b, d} then $\tau_{R(X)}$={U, ϕ, {d}, {b, c}, {b, c, d}} and the nano closed sets are $\tau R(x)^c$ = {U, ϕ, {a, b, c}, {a, d}, {a}}. If A= {a} is not a nano regular open. However A is Ng*α -closed and Nαcl(A)= {a} is a Ng*α -closed and Npint(A) = ϕ is also Ng*α -closed.

Example 4.17. Let U = {a, b, c, d}, with U \R={{a}, {b, c}, {d}}. Let X = {b, d} then $\tau_{R(X)}$={U, ϕ, {d}, {b, c}, {b, c, d}}and the nano closed sets are $\tau R(x)^c$ = {U, ϕ, {a, b, c}, {a, d}, {a}} and also

Ng*α = {U, ϕ, {a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}}. Then the set A = {a} is not a nano regular closed. However A is Ng*α -closed and Npint(A) = {a} is also Ng*α -closed.

5.Nano $g^*\alpha$ -open sets

In this section we define and study the concept of nano generalized star alpha open briefly $Ng^*\alpha$ -open sets in nano topological spaces and obtain some of its properties.

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Definition 5.1. A subset A of (U, $\tau_{R(X)}$) is called Ng*α -open in X if A^c is Ng*α-closed. The class of all Ng*α -open set denoted by Ng*α -O(U, $\tau_{R(X)}$).

Proposition 5.2. For any nano topological space (U, $\tau_{R(X)}$)), the following assertions hold.

(i) Every Nano open set is a Ng*α -open set.
(ii) Every Nano α-open set is a Ng*α -open set.
(iii) Every Nano Ng*α -open set is Ngp-open set.
(iv) Every Nano Ng*α -open set is Ngpr-open set.
(v) Every Nano Ng*α -open set is Nsg-open set.
(vi) Every Nano Ng*α -open set is Ngs-open set.
(vii) Every Nano Ng*α -open set is Ngsp-open set.

Proof: The proof follows from 3.2, 3.4, 3.6, 3.8, 3.10, 3.12 and 3.14.

The converse of the above proof need not be true, as proved by the following examples 3.3, 3.5, 3.7, 3.9, 3.11, 3.13 and 3.15.

Theorem 5.3. A set A of U is Ng*α -open if and only if F ⊆ Nαint(A), Whenever F is Ngα-closed and F ⊆ A.

Proof: Assume that A is Ng*α -open in (U, τR(X)) and F be a Ngα-closed set of (U, $\tau_{R(X)}$) such that F ⊆ A. Then U − A is a Ng*α -closed set in (U, $\tau_{R(X)}$). Also U − A ⊆ U − F and U − F is Ngα- open of (U, $\tau_{R(X)}$). This implies Nαcl(U − A) ⊆ (U − F). But Nαcl(U − A) = U − Nαint(A). Thus U − Nαint(A) ⊆ (U − F). So F ⊆ Nαint(A). Conversely, suppose that F ⊆ Nαint(A) whenever F is Ngα-closed and F ⊂ A. To prove that A is Ng*α -open. Let G be a Ngα-open set of (U, $\tau_{R(X)}$), such that U − A ⊆ G. Then U − G ⊆ A. Now U − G is Ngα-closed containing A. So U − G ⊆ Nαint(A), U − Nαint(A) ⊆ G. But Nαcl(U − A) = U − Nαint(A). Thus Nαcl(U − A) ⊆ G. That is $U - A$ is a Ng*α -closed set and hence A is Ng*α -open.

Theorem 5.4. If Nαint(A) ⊆ B ⊆ A and A is a Ng*α-open set of (U, $\tau_{R(X)}$), then B is a Ng*α -open set.

Proof: If Nαint(A) ⊆ B ⊆ A, then U − A ⊆ U − B ⊆ U − Nαint(A). That is U − A ⊆ U − B ⊆ U −Nαcl(U −A). Since U −A is Ng*α-closed and then by proposition (4.10), U −B ia also a Ng*α -closed set. Therefore B is a Ng*α -open set.

Theorem 5.5. If A is Ngα-open and Ng*α -closed set then A is Nα-closed.

Proof: Since A ⊆ A and A is Ngα-open and Ng*α -closed, we have Nαcl(A) ⊆

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A. Thus $N\alpha cl(A) = A$. Hence A is $N\alpha$-closed of $(U, \tau_{R(X)})$.

Theorem 5.6. In a Nano topological space $(U, \tau_{R(X)})$, $Ng\alpha O(U, \tau R(X)) \subseteq \{F \subseteq U, F^{c} \in \tau_{R(X)}\}$ if and only if every subset of U is $Ng^{*}\alpha$-closed.

Proof: Suppose that $Ng\alpha O(U, \tau R(X)) \subseteq \{F \subseteq U, F^{c} \in \tau\}$. let A be a subset of U such that $A \subseteq G$ where G is $Ng\alpha$-open. Then $G \in Ng\alpha O(U, \tau R(X)) \subseteq \{F \subseteq U, F^{c} \in \tau_{R(X)}\}$. That is $G \in Ng\alpha O(U, \tau_{R(X)})$. Thus $N\alpha cl(G) \subseteq G$. Thus $N\alpha cl(A) \subseteq N\alpha cl(G) = G$. Hence A is $Ng^{*}\alpha$ -closed.

Conversely, suppose that every subset U is $Ng^{*}\alpha$ -closed. Let $G \in g\alpha O(U, \tau_{R(X)})$. Since $G \subseteq G$ and G is $Ng^{*}\alpha$ -closed. We have $N\alpha cl(G) \subseteq G$. Thus $N\alpha cl(G) = G$ and $G \in \{F \subseteq U, F^{c} \in \tau_{R(X)}\}$. Thus $Ng\alpha O(U, \tau_{R(X)}) \subseteq \{F \subseteq U, F^{c} \in \tau_{R(X)}\}$.

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