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APPLICATIONS OF FRACTIONAL CALCULUS IN SCIENCE AND ENGINEERING

J. Sheela Jacob[1], J. Hellan Priya[2], A. Karthika[3]

[1]Assistant Professor & Head of the Department, Department of Mathematics, Karur Velalar College of Arts & Science for Women, Karur.
2Assistant Professor of Mathematics, Department of Science and Humanities, M.Kumarasamy College of Engineering, Karur.
3Assistant Professor, Department of Mathematics, Karur Velalar College of Arts & Science for Women, Karur.

Email: sheelajocob60@gmail.com[1], helanje14@gmail.com[2], asmithaanjana@gmail.com[3]

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ABSTRACT: The subject of fractional calculus has emerged as a powerful and efficient mathematical instrument during the past six decades, mainly due to its demonstrated applications in numerous seemingly diverse and widespread fields of science and engineering. Although researchers have already reported many excellent results inseveral seminal monographs and review articles, there arestill alarge number of non-local phenomena unexplored and waiting to be discovered. In this perspective, this paper investigates the use of Fractional Calculus in the fields of Physics, Mechanics, Biology, Engineering and Signal Processing. We hope this incomplete, but significant, details will guide young researchers and help new comers to see some of the important applications. We expect this collection of review will also benefit our society.

KEYWORDS: Fractional Calculus, Sinc-Fractional Derivative, Electrical spectroscopy impedance, Newtonian Mechanics, Bio Heat Transfer Equation, HexapodRobot.

I. INTRODUCTION

Fractional calculus was formulated in 1695, shortly after the development of classical calculus. Fractional calculus is deeply related to the dynamics of complicated real-world problems. Many mathematical models are accurately governed by fractional order differential equationsThe earliest systematic studies were attributed to Liouville, Riemann, Leibniz, etc. [45, 56]. For a long time, fractional calculus has been regarded as a pure mathematical realm without real applications. But, in recent decades, such a state of affairs has been changed. It has been found that fractional calculus can be useful and even powerful, and an outline of the simple history about fractional calculus, especially with applications, can be found in Machado et al. [32]. Now, fractional calculus and its applications is undergoing rapid developments with more and more convincing applications in the real world [27,46].Research in fractional differentiation and integration is inherently multi-disciplinary and its application is done in various contexts: continuum mechanics, elasticity, signal analysis, quantum mechanics, bioengineering, biomedicine, financial systems, social systems, pollution control, turbulence, population growth and dispersal, landscape evolution, medical imaging, and complex systems, and some other branches of pure and applied mathematics. This review is organized into 7 sections. We begin with some important results of Fractional Calculus in physics, Mechanics, Biology, Engineering, Control and Signal Processing.

II. APPLICATION OF FRACTIONAL CALCULUS IN PHYSICS

A branch of theoretical physics which has been attracting considerable attention in the last years is quantum gravity. Several independent theories, models and hypotheses are gathered under this broad name, from string theory to asymptotic safety, from non-local to loop quantum gravity, from causal dynamical triangulations to causal sets, and so on [46,15,10, 37]. Since then, eminent mathematicians such as Fourier, Abel, Liouville, Riemann, Weyl, Riesz, and many others contributed to the field, but until later days Fractional Calculus has played a negligible role in physics. However, in recent years, applications of fractional calculus in physics have become more common in fields ranging from classical and quantum mechanics, nuclear physics, hadron spectroscopy, and up to quantum field theory. In theoretical physics we can study the fractional equivalent of many standard physics equations: frictional forces, harmonic oscillator, wave equations, Schr¨odinger and Dirac equations, and several others. In applied physics, various methods of fractional calculus can be used in the description of chaotic systems and random walk problems, in polymer material science, in biophysics, and other fields.

2.1 Fractional calculus technique in random optimal search

One common approach to the animal movement patterns is to use the scheme of optimizing random search [6,65,54]. In a random search model, single or multiple individuals search a landscape to find targets whose locations are not known a priori, which is usually adopted to describe the scenario of animals foraging for food, prey or resources. Many researchers have concentrated on the study of different animals' foraging movements. It is shown that when the environment contains a high density of food items, foragers tend to adopt Brownian walks, characterized by a great number of short step lengths in random directions that maintain foragers in a small portion of the available space [5,21]. In contrast, when the density of food items is low, individuals tend to exhibit Lévy flights, where larger step lengths occasionally occur and relocate the foragers in the environment. Due to the fact that the density of food items is often low, many animals behave a Lévy flight when foraging and their movements have been found to fit closely to a Lévy distribution (power law distribution) with an exponent close to 2 [66,67]. For instance, the foraging behavior of the wandering albatross on the ocean surface was found to obey a power law distribution [68]; the foraging patterns of a free-ranging spider monkey in the forests was also found to be a power law tailed distribution of steps consistent with Lévy walks [9,52].

2.2 Sinc-Fractional Derivative on Shannon wavelets

The sinc-fractional operator will be generalized in order to compute the fractional derivative of the L2(R)-functions belonging to the Hilbert space defined by the Shannon wavelet. In doing so, we will be able to compute the fractional derivative of these functions by knowing only their wavelet coefficients. Moreover, with this approach we will be able to decompose the fractional derivative at different scales, thus showing the influence of a given scale in multi scale physical problems. Sinc function is playing a fundamental role in mathematics and physics. Due to the many properties of this function it deserves a special role in applications. In recent years some authors have proposed [69] a fractional derivative based on this function.

2.3 Linear visco elastic response functions and the Caputo-Fabrizio fractional operator

The deep physics behind the fractional operator with exponential kernel motivated this study and the efforts are oriented to show that the existing knowledge and models, as well as techniques of data treatment, in the framework of linear visco elasticity, lead naturally to formulation of the Caputo-Fabrizio fractional operator. This is in the context of the Sir Isaac Newton quote at the beginning of the article: the steps ahead on the shoulder of existing facts and results on the road to creations of new information are natural ways and actually the exciting moments in the beautiful journey in the world of science. It was demonstrated that in many cases there are viscoelastic materials which experimental behaviors exhibit strong departures from the power-law

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2.4 Fractional Calculus and Electrical spectro scopy impedance

The electrical spectroscopy impedance technique plays an important role from the experimental point of view to obtain information about the electrical properties of many different materials, in particular, of liquids [4]. It has been investigated, from the theoretical point of view, by using the Poisson–Nernst–Planck diffusion model [26] and/or equivalent circuits. In the low frequency limit, these approaches with simple considerations (boundary conditions and/or circuit elements) are not able to describe the experimental behavior. However, by using the well-established features of the fractional calculus and performing suitable changes in the boundary conditions, in order to account the surface effects, it is possible to overcome this issue and describe the experimental behavior in all frequency range[12,25]. Furthermore, this approach can also be used to investigate the ion diffusion in an electrolytic cell through the electrical conductivity, which is directly related to the mean square displacement.

III. APPLICATION OF FRACTIONAL CALCULUS IN MECHANICS

The use of real order derivatives has been found to be very useful in many practical applications. On the other hand, the studies of applications using derivatives of complex number order are still quite rare. However, there are some application areas in which complex order derivatives have been adapted to engineering mechanics, such as continuum mechanics and the modelling of viscoelastic materials [34,2]

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3.1 Fractional Calculus and Newtonian Mechanics

One-dimensional Newtonian mechanics for a point-particle of constant mass m is based upon Newton's second law of motion, a second-order ordinary differential equation:

$$\frac{d^2x(t)}{dt^2} = \frac{F}{m} \qquad (1)$$

We can easily think of at least two possible ways of generalizing Newton's second law using fractional calculus: • Change the order of the time derivative in the left-hand-side of Eq. (1) to an arbitrary number q. • Generalize

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the expression of the force (or force field) F on the right-hand-side of Eq. (1) to include differintegrals of arbitrary order q. This is also routinely done in applications of FC to physics by selecting fractional generalizations of standard electromagnetic potentials, in order to analyze phenomena in nuclear physics, hadron spectroscopy, and other fields

3.2 Fractional calculus for modeling oil pressure

Darcy's law is used to relate fluid motion to pressure and gravitational gradients. The combination of the Continuity Equation and Darcy's Law leads to a heat-conducting differential equation in mathematical physics describing the transfer of the fluid. The application of the fractional calculus can be very useful for the modeling of anomalous diffusion phenomena in which the fractal structure better reflects the real conditions of the medium, as it is the case of the reservoirs in which because of its very nature it is difficult to find a structure Euclidian.

3.3 Micro flows of visco elastic fluids with fractional constitutive relationships

Recently the microflows of viscoelastic fluids have been studied extensively due to their importance in microfluidic systems. However, the application of fractional constitutive models in microchannel flow is still in early stages. Considering the successful applications of fractional constitutive models in the description of viscoelastic materials, the mechanics models to study the electro osmotic slip flows of viscoelastic fluids under the mixed influence of electro osmosis and pressure gradient forcing.

3.4 Unsteady flow towards subsurfacedrains

Glover–Dumm equation (GDE), which is the most practical mathematical model to simulate water table profile between two parallel drainpipes under unsteady flow conditions, was obtained by analytically solving Boussinesq equation (BE). Fractional derivatives, because of having non-locality property, can reduce the scale effects on the parameters and, consequently, better simulate the hydro-geological processes. Hereby a fractional BE (FBE) was proposed and analytically solved for one-dimensional unsteady flow towards parallel subsurface drains. The applicability and accuracy of the resultant solution, called fractional Glover–Dumm equation (FGDE), were examined using both laboratory and field data measured at an experimental farm in Abadan, Iran.

IV. APPLICATION OF FRACTIONAL CALCULUS IN BIOLOGY

Fractional calculus provides novel mathematical tools for modeling physical and biological processes. The bioheat equation is often used as a first order model of heat transfer in biological systems. Formulation of bio heat transfer in one dimension in terms of fractional order differentiation with respect to time can be described. In the future we hope to interpret the physical basis of fractional derivatives using Constructal Theory, according to which, the geometry biological structures evolve as a result of the optimization process.

4.1 Bio Heat Transfer Equation

The methods of fractional calculus, reviewed recently by Magin [33], are developed as the basis for formulation and solution of the bio heat transfer problem in peripheral tissue regions. Investigators have studied bio heat transfer using mathematical models for more than 50 years [70,59,16]. In these models tissue cooling (or warming) is approximated by coupling tissue perfusion to the bulk tissue temperature through Newton's law of cooling (or heating). In addition to full body models, there are numerous models in literature Design and Nature II, M. W. Collins & C. A. Brebbia (Editors) © 2004 WIT Press, www.witpress.com, ISBN 1-85312-721-3 that describe heat transfer mechanisms in a single organ or a portion of the body. In this regard, an analytical model developed by Keller and Seiler examines bioheat transport phenomena with heat generation (metabolism) occurring in the peripheral tissue regions. The Keller and Seiler [20] model was solved numerically using parallel computers to simulate all possible modes of bioheat transfer by Boregowda et al. [8]. Recently a number of investigators [14,1,29] have applied the bioheat transfer model to periodic diffusion problems in localized tissue regions such as that which occurs in the skin when laser heating and/or cryogen cooling is applied. Fractional calculus is ideally suited to address this kind of periodic heating or cooling, but to our knowledge has not been used in modeling bio heat transfer either at the tissue, organ or whole body level. The study demonstrates that fractional calculus can provide a unified approach to examine periodic heat transfer in peripheral tissue regions. For example, in an experimental study conducted by Pikkula et al. [48], cryogen spray cooling is utilized to cool the skin surface during the laser skin surgery. A generalized fractional calculus approach developed by Kulish and Lage [23,24] is adopted to model the localized periodic bioheat transfer problems similar to the one posed by Pikkula et al. [48]. The one-dimensional heat flow problem can be completely solved for well defined surface temperature or thermal flux boundary conditions by applying

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Laplace transforms [7,49]. The solution can also be expressed as a fractional differential equation for the semi-infinite peripheral tissue region [23]. Further, the fractional differential equation can be solved to compute the heat flux at the boundary for different periodic or on-off boundary conditions that closely represent the heating and cooling of skin surface during laser surgery. The approach offered by fractional calculus models a large class of biomedical problems that involve localized pulse heating and/or cooling. One advantage of this approach is that there is no need to solve first for the temperature in the entire domain.

4.2 Models of bone remodeling and bone tumors using variable order derivatives

Bone tissue is not static. Like every other part of our body, its cells are always dying and being replaced. The main actors of this process are the cells destroying bone tissue, called osteoclasts, and the cells that build bone back, called osteoblasts. The presence of osteoblasts influences the rate of increase of osteoclasts and the number of osteoclasts also influences their own evolution. The changes in dynamic behavior when there is a tumor can be modeled by tuning the parameters of au- tocrine and paracrine effects. Models found in the literature include intricate mathematical expressions for such variations.Our research has shown that the same effect can be obtained merely changing the order of the time derivative in the partial differential equations that model the involved diffusion phenomena. We studied the dynamic behavior of the resulting vari- able order partial differential equations and found in accord with the known qualitative behavior of healthy and tumorousbone remodeling.

V. APPLICATION OF FRACTIONAL CALCULUS IN ENGINEERING

Recently Fractional Calculus has been a fruitful field of research in science and engineering and many scientific areas are currently paying wider attention to the Fractional Calculus concepts. In the field of dynamical systems theory, some work has been carried out but the proposed models and algorithms are still in a preliminary stage of establishment. This leads several case studies on the implementation of Fractional Calculus-based models, being demonstrated the advantages of using the Fractional Calculus theory in different areas of science and engineering

5.1 Tuning of PID Controllers Using Fractional Calculus Concepts

The PID controllers are the most commonly used control algorithms in industry. Among the various existent schemes for tuning PID controllers, the Ziegler-Nichols (Z-N) method is the most popular and is still extensively used for the determination of the PID parameters. It is well known that the compensated systems, with controllers tuned by this method, have generally a step response with a high percent overshoot. Moreover, the Z-N heuristics are only suitable for plants with monotonic stepresponse.

PIDcontroller Plant

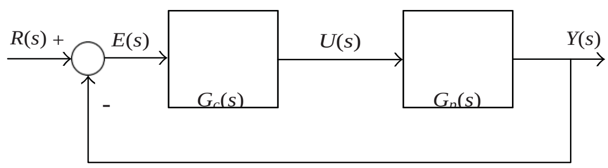

Figure 1: Closed-loop control system with PID controller $G_c(s)$.

5.2 Fractional PD$^\alpha$Control of a HexapodRobot

Figure 2 presents the dynamic model for the hexapod body and foot-ground interaction. It is considered robot body compliance because walking animals have a spine that allows supporting the locomotion with improved stability. The robot body is divided in n identical segments (each with mass $M_b n^{-1}$) and a linear spring-damper system is adopted to implement the intra body compliance [58]. The contact of the i th robot feet with the ground is modelled through a nonlinear system [57], being the values for the parameters based on the studies of soil mechanics[57].

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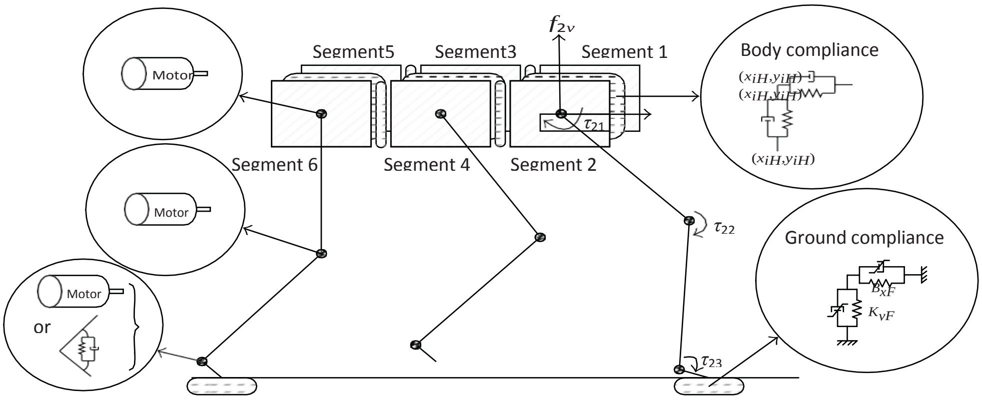

Figure 2: Model of the robot body and foot-ground interaction.

It is analysed the system performance of the different PD^{α}tuning, during a periodic wave gait at a constant forward velocity V_F, for two cases: two leg joints are motor actuated and the ankle joint is mechanical actuated and the three leg joints are fully motor actuated [58].

5.3 Heat Diffusion

The heat diffusion is governed by a linear one-dimensional partial differential equation (PDE) of the form:

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \quad (2)$$

where k is the diffusivity, t is the time, u is the temperature, and x is the space coordinate. However, (2) involves the solution of a PDE of parabolic type for which the standard theory guarantees the existence of a unique solution[31].For the case of a planar perfectly isolated surface we usually apply a constant temperature U_0atx= 0 and analyzes the heat diffusion along the horizontal co ordinate x.

5.4 Circuit Synthesis Using Evolutionary Algorithms

In recent decades evolutionary computation (EC) techniques have been applied to the design of electronic circuits and systems, leading to a novel area of research called Evolutionary Electronics (EE) or Evolvable Hardware. Several papers proposed designing combinational logic circuits using evolutionary algorithms and, in particular, genetic algorithms (GAs) [30, 18] and hybrid schemes such as the memetic algorithms (MAs)[18].

VI. APPLICATION OF FRACTIONAL CALCULUS IN CONTROL

6.1 Application of D-decomposition technique in solving some controlproblems

The basic idea of D-decomposition technique, conceived by the Russian scientist Neimark during the 1950s, is now extended for the case of linear fractional order systems and gives powerful tool for the analysis of systems stability and performance. In order to control as many different processes as possible, a fractional order proportional-integral-derivative (PID) controller is introduced, as a generalization of classical PID controller. Another useful application of this technique is control of underactuated systems. The D-decomposition method can be successfully used to solve a problem of asymptotic stability of inverted pendulum systems controlled by a fractional order controller[35,36].

6.2 The application of fractional order control for anair-based precision positioning system

Precision, bandwidth (speed) and stability of motion are the most important performance indexes of any motion system. Fractional order PID has proven to be very effective to improve the performance. A recent work at TU Delft [55], utilizes the fractional order calculus to control a precision positioning stage. In this work, a contactless precision positioning system is designed by floating a silicon wafer on a thin film of air (see Fig. 3(a)). The system has been controlled as shown in Fig. 3(b) in which two cascade single-input/single-output (SISO) controllers are designed. By using only the fractionality, the bandwidths are extended by 14.6% and 62%, for the inner and outer loops, respectively. Furthermore, a closed-loop positioning bandwidth of the wafer of 60 Hz is achieved, resulting in a positioning error of 104 nm, which is limited by sensor noise and pressure disturbances. (ContributedbyS.HassanHosseinNia,Fractionalordercontrol).

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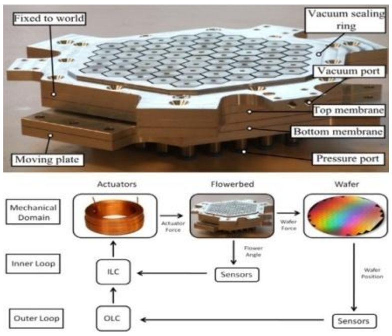

Fig. 3. (a) Overview of the air based precision positioning stage (so called the Flowerbed) designed at TU Delft. (b) Proposed Control Strategy, with an InnerLoopController(ILC)andanOuterLoopController(OLC).

VII. APPLICATION OF FRACTIONAL CALCULUS IN SIGNAL AND IMAGE PROCESSING

7.1 Astudyonfractionalcalculusapplicationsinimageprocessing

Employing fractional differential to image processing is a prospering subject branch under discourse [55,19,72,73]. Recently, fractional calculus has been significantly examined in computer vision [11,74]. The principle purpose behind this advancement is the desire that the utilization of this theory will prompt a considerably more exquisite and viable method to treat problems of blocky effect and detail information protection. The fractional- order derivative operator has a non-local behavior because the fractional-order derivative at a point relies upon the characteristics of the entire function and not just the values in the vicinity of the point, which is helpful to enhance the performance of texture preservation. The numerical outcomes in published works show that the fractional-order derivative performs we eliminating the stair case effect and preserving textures[74].It has been demonstrated in [50] that the fractional-order derivative fulfills the lateral inhibition principle of thebiological visual system better than the integer-order derivative. Pu et al. [51] considered the kinetic physical meaning ofthe fractional-order derivative and demonstrated that fractional differential-based methods can protect the low-frequency contour features in those smooth areas, and non-linearly keep high-frequency marginal feature in those regions where gray-level changes significantly, and furthermore preserve texture details in those areas that gray-level does not change obviously.It is noted in [75] that for low-frequency signal, fractional differential lessens the signal not as much as the integerone and for high-frequency one, fractional differential improves signal not as much as the integer one. Hence, we get the conclusion that fractional differential can upgrade the high-frequency signals, and reinforce the medium frequency one,while non-linear retain the low-frequency one.

7.2 Application of the GPCF and DGIs for improving the resolution and quality of nano images

We apply the generalized Pearson correlation function (GPCF) [38] POLS [41] and discrete geometrical invariants (DGI) for improving the quality and sharpness of nanoimages in the range of resolution (10–1000) nm. The GPCF helps to compare one piece of image with another one and the procedure of reduction to three incident points [42] allows finding "hidden" self-similar objects. The DGI based on the generalization of the Pythagoras theorem obtained by Babenko[3] allows comparing two randomly taken parts of images with each other and finding distinct differences expressed in terms of the integer moments. The quantitative parameters determined by the DGIs of the second and fourth orders, correspondingly allow monitoring the dynamics/changings of the chosen image in time. It can be applied for a wide set of random curves (experimental measurements) that are needed to be compared in terms of a limited number of the integer moments.

7.3 NAFAS Sinaction: intermediate fractal model for the fitting of complex systems data

We essentially modernize the NAFASS (Non-orthogonal Amplitude Frequency Analysis of the Smoothed Signals) approach suggested earlier [43,44]. The NAFASS opens an alternative way for creationofnewfluctuationspectroscopywhenthesegment of the Fourier series can fit any random signal with trend. However, the dispersion spectrum of the Fourier series

$$\omega 0 \cdot k(\omega 0 = 2\pi/T) \Rightarrow \Delta_k(k = 0,1,2,\ldots,K-1)$$

is replaced by the specific dispersion law Δ_k calculated by the original algorithm. It implies that any finite signal will have a compact amplitude-frequency response (AFR), where the number of the modes is much less in comparison with the number of data points (K N). The NAFASS approach can be applicable for quantitative description of a wide set of random signals/fluctuations and allows one to compare them with each other based on one general platform. We combine also the NAFASS with generalized Pearson correlation function [38, 40] that allows to apply this combination for analysis of signals having self-similar origin with their subsequent fitting.

VIII. CONCLUSION

In recent years Fractional Calculus has been a fruitful field of research in science and engineering. In fact, many scientific areas are currently paying attention to the Fractional Calculus concepts and we can refer its adoption in viscoelasticity and damping, diffusion and wave propagation, electromagnetism, chaos and fractals, heat transfer, biology, electronics, signal processing, robotics, system identification, traffic systems, genetic algorithms, percolation, modeling and identification, telecommunications, chemistry, irreversibility, physics, control systems as well as economy, and finance.The applications of fractional derivatives in condition monitoring and some of the above fields were discussed in this paper. We try our best to organize this Theme Issue in order to offer fresh stimuli for the fractional calculus community to further promote and develop cutting-edge research on fractional calculus and its applications. This survey cannot be considered as a complete one, but as a collection of sample applications, which can be used for further developments using analogies in the mathematical description of real problems arising in different fields of science.

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