ISSN- 2394-5125 VOL 08, ISSUE 02, 2021

QUANTUM INFORMATION ENTROPY OF MODIFIED HYLLERAAS PLUS EXPONENTIAL ROSEN MORSE POTENTIAL AND SQUEEZED STATES

Dr. Pawanjit Kaur[1], Dr.Ramesh Kumar Gupta[2]
[1,2]Guru Kashi University, Talwandi Sabo

Abstract

The degree quantum information entropy is the amount of susceptibility that a random variable has that is linked to the amount of localization delocalization. As the radius of confinement increases, the data entropy has local maximums and lowest points. This is true for a few constrained solutions, like an atom and particles in hard, spherical containers. The idea of entropy is important when you learn about information theory. The following are some examples of how information theory is used: solid state physics, nuclear physics; mathematical physics; chemical science; biology; statistical physics; computer science; and more. The information density looks at how much information can be stored in a small space. When we look at this expression, we get an abstract of information in the normal way. This study looks at a significant Hylleraas plus exponential Rosen Morse potential, which has been used in a lot of different fields of physics and chemistry. In physics and math, the Fourier transform is used to get the Eigen functions of momentum spaces in a way that makes sense. The information density and entropy of the potential are investigated, and the BBM inequality is tested using various parameters.

Keywords: quantum, information, entropy, hylleraas, rosen morse, state, etc

1. INTRODUCTION

The degree of susceptibility associated with a random variable that is linked to how localised things are quantum information entropy has been used to describe the process of delocalization. As the radius of confinement increases, the data entropy has local maximums and lowest points. This is true for a few constrained solutions, like an atom and particles in hard, spherical containers. When a control parameter inside a potential changes, the evolution of quantum data hypothesis can also show how a framework changes phase. Using phenomenological ideas, information theoretic measures can show the hidden structure of chemical reactions, which can show whether the reactions are asynchronous or synchronous, and how they work. These metrics can be used to describe the transition state and stationary points that are used to look at the bond building and bond breaking in the study areas.

- **Information at the quantum:**

Quantum information is the information about a quantum system's state. It's the most fundamental concept in quantum information theory, and quantum information processing techniques can be used to control and manipulate quantum information. Both the technical term in terms of Von Neumann entropy and the general term for computing are called quantum information. In this field, quantum mechanics, computer science, information theory, philosophical thought, and cryptography are just some of the things you'll find. It does research in cognitive science, psychology, and neuroscience, as

ISSN- 2394-5125 VOL 08, ISSUE 02, 2021

well as other fields. Its main goal is to get information from things at a microscopic level. In science, observation is one of the most important ways to get data, and measurement is needed to be able to put a value on what you're seeing. This makes measurement important to the scientific method. Because of the uncertainty principle, two non-commuting observables can't be measured at the same time in quantum mechanics. An eigenstate in one basis is not an eigenstate in the other basis. This means that a quantum state can't give you exact information about both variables, because both variables aren't well-defined at the same time.

Quantum entropies

People who study quantum information theory use the term "quantum relative entropy" to show how different two quantum states are from each other. Quantum thing: Relative entropy has a quantum thing that it does. mechanical equivalent.

- **Thermodynamic Entropy**

Thermodynamic entropy, as described in physics and chemistry, is closely connected to Shannon's notion of entropy. Boltzmann and Gibbs worked extensively statistical thermodynamics, which led to the use of the term entropy in the field of information theory. Thermodynamic and informational entropy are linked in certain ways. Thermodynamics, on the other hand, should be thought of as an application of Shannon's information theory, and the thermodynamic entropy should be thought of as an estimate of how much Shannon information (needed to describe the microscopic state of the system) isn't communicated by a description of the system in terms of classical thermodynamics' macro variables. adding heat to a system, for example, increases its thermodynamic entropy because it increases the number of microscopic states that can be in the system at the same time lengthening any entire state description.

1.1 Exponential Rosen Morse Potential with Isospectral Eckart Potential plus Quantum Information Entropy and Entropy Squeezing

The concept of entropy is important in learning about information theory. Solid state physics, nuclear physics, mathematical physics, chemical science, biology, statistical physics, and computer science are some of the fields of study. are all examples of where information theory is used. CE Shannon published "A mathematical theory of communication" in 1948, which examined the measurement and retention of digital data as well as the research of information exchange. Shannon information entropy has recently been linked to a variety of quantum mechanical systems, including cryptography, teleportation, current communication technologies, and quantum entanglement. It's a metric measuring the spatial spread of a system's wave functions across different states. This has been viewed as the degree of localization-delocalization-related uncertainty associated with the particle's position.

2. REVIEW OF THE LITERATURE

Aarti Sharma, Pooja Thakur, Girish Kumar, and Anil Kumar (2021) The study of quantum mechanical systems requires an understanding of information theoretic principles. There has been a lot of research done on the information density of symmetric potentials, and this study has shown that

ISSN- 2394-5125 VOL 08, ISSUE 02, 2021

they are very useful. Information entropy in position and momentum space is calculated, and the Bialynicki–Birula–Mycielski inequality is broken for a number of different factors. There have been a lot of interesting things said about information entropy. The saturation of these entropies for certain values of the parameter is characterized by the variation in these entropies. The symmetry breaking situation has also been investigated. In addition, the entropy squeezing phenomenon has been studied in both position and momentum space. Intriguingly, phase transition predicts entropy compression in both position and momentum space.

Pooja Thakur, Aarti Sharma, Rama Gupta, and Anil Kumar (2017) in this paper, we look at certain information theoretic notions space for the modified Hylleraas plus exponential Rosen Morse possible. For different states, the information density's angular and radial contributions are shown in a visual way. The shape of entropy densities is determined by the values of quantum numbers, so this is how it works. Analysis: The information entropy of the ground state of the potential is calculated. The Bialynicki-Birula and Mycielski inequality is checked for different states with different potential parameters. It has been shown that by carefully choosing certain parameters, the information entropy in both position and momentum space can be reduced. Also, new Hylleraas plus exponential Rosen Morse and Eckart potential eigenstates with information entropy squeezed in are found. It is possible to get squeezed states in both position and momentum space with the Eckart potential. For certain parameter values, the squeezed states are tried to reach saturation.

R. Valencia-Torres, Guo-Hua Sun, and Shihai Dong (2015) with hyperbolic potential, Solutions to the Schrödinger equation are found that are very close to being exact. There are Shannon information entropies for each of the low-lying states &$n=0,1$; that are found. Some interesting things are found out about the information densities and the probability densities, such as that they have interesting properties that aren't found in other places, too. Those parameters have to have values that are at least &$n rm max &$, so we know that those values have to be the right ones. At some points, we also notice that the &$rho (p); is equal to or higher than 1 at some r or p positions. We also notice that some of the &$rho (x)$; is equal to or higher than 1. The Bialynicki–Birula–Mycielski inequality is also true examined in a variety of scenarios and found to be valid in all of them.

Sun, Guo-Hua & Dong, Shihai (2013) the Sun, Guo-Hua, and Shihai look at the symmetrically trigonometric Rosen–Morse potential, they figure out how much information there is about the position and momentum eigenstates for states n = 1–4. In an analytical way, the position information entropies Sx for n = 1, 2, and 3 are given. Graphically, some interesting things about the information entropy densities s(x) and s(p) are shown. As the range of potentials a grows, the Sx gets smaller. Some of the parameters a and D may make Sx negative. We should keep this in mind. The Bialynicki–Birula–Mycielski inequality is also checked out in a lot of different states and found to be true in most cases.

3. OBJECTIVES

- To examine E Rosen Morse Potential with Isospectral Eckart Potential, Quantum Information Entropy, and Entropy Squeezing are some of the things in this.
- To look into the information density of modified hylleraas and rosen morse, as well as the exponential rosen morse potential. information entropy.

ISSN- 2394-5125 VOL 08, ISSUE 02, 2021

4. EXPONENTIAL ROSEN MORSE POTENTIAL PLUS INFORMATION DENSITY OF MODIFIED HYLLERAAS

The information density looks at how much information can be stored in a small space. When we look at this expression, we get an abstract of information in the normal way. One of the most popular models for how two atoms interact in two-atom molecules is the Hylleraas potential, which is also called the Morse potential. As far as I can tell, there's is as follows:

$$V(r) = -\frac{V_0}{b}\left[\frac{a+e^{-2\alpha(r-r_c)}}{1+e^{-2\alpha(r-r_c)}}\right] - \frac{4V_1 e^{-2\alpha(r-r_c)}}{(1+e^{-2\alpha(r-r_c)})^2} + V_2\left(\frac{1-e^{-2\alpha(r-r_c)}}{1+e^{-2\alpha(r-r_c)}}\right) \quad (1)$$

Where rc and α are the distance and adjustable parameter from the equilibrium position, V0, V1, V2 are the potential depths, and a and b are the Hylleraas parameters, respectively. For the potential, the Schrodinger equation is,

$$\frac{dU_{nl}^2}{dr^2} + 2\mu\left[E_{nl} - V(r) - \frac{\ell(\ell+1)}{2\mu r^2}\right]U_{nl} = 0 \quad (2)$$

The potential's radial wave function is given as

$$R_{nl}(x) = N_{nl}(1-x)^{\frac{1+\vartheta}{2}} x^{\frac{\varepsilon}{2}} P_n^{\varepsilon,\vartheta}(1-2x), \quad \textbf{(3)}$$

Using the transformation $s = -e^{-2\alpha(r-rc)}$ and

$$\varepsilon = 2\sqrt{-\frac{\mu E}{2\alpha^2\hbar^2} - C},$$

$$A = \frac{\mu}{2\alpha^2\hbar^2 b}(V_0 + V_2 b) - \frac{\ell(\ell+1)a_1}{4\alpha^2},$$

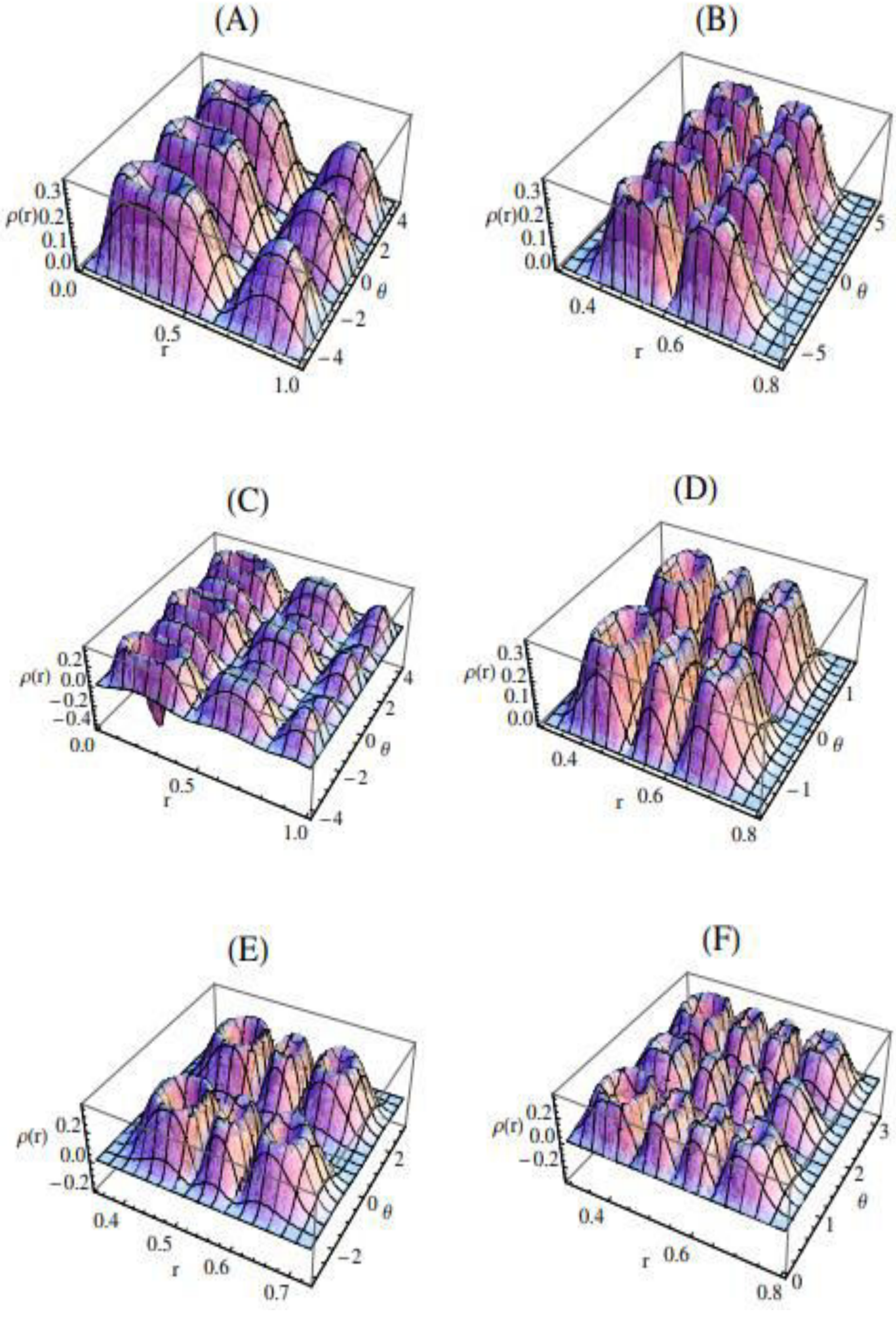
$$B = -\frac{\mu V_0}{2\alpha^2\hbar^2 b}(a+1) - \frac{2\mu V_1}{\alpha^2\hbar^2} + \frac{\ell(\ell+1)}{2\alpha^2}(a_1 + \frac{a_2}{2}),$$

$$C = \frac{\mu}{2\alpha^2\hbar^2 b}(V_0 a - V_2 b) - \frac{\ell(\ell+1)}{4\alpha^2}(a_1 + a_2 + a_3),$$

Where Nnl, $P^{\varepsilon,\vartheta}_n$ (x), ϑ and ε are the Adjustable potential parameters, Jacobi polynomials, and normalisation constant. The spherical harmonics relation connects radial wave function and wave function ψ(r, θ, φ).

$$\psi(r,\theta,\phi) = \frac{R(r)}{r} Y_l^m(\theta,\phi) \quad (4)$$

For the provided potential, Fourier transformations of position space wave functions were employed to obtain momentum space information densities.


(A)
(B)
(C)
(D)
(E)
(F)
ρ(r)
r
θ

ISSN- 2394-5125 VOL 08, ISSUE 02, 2021

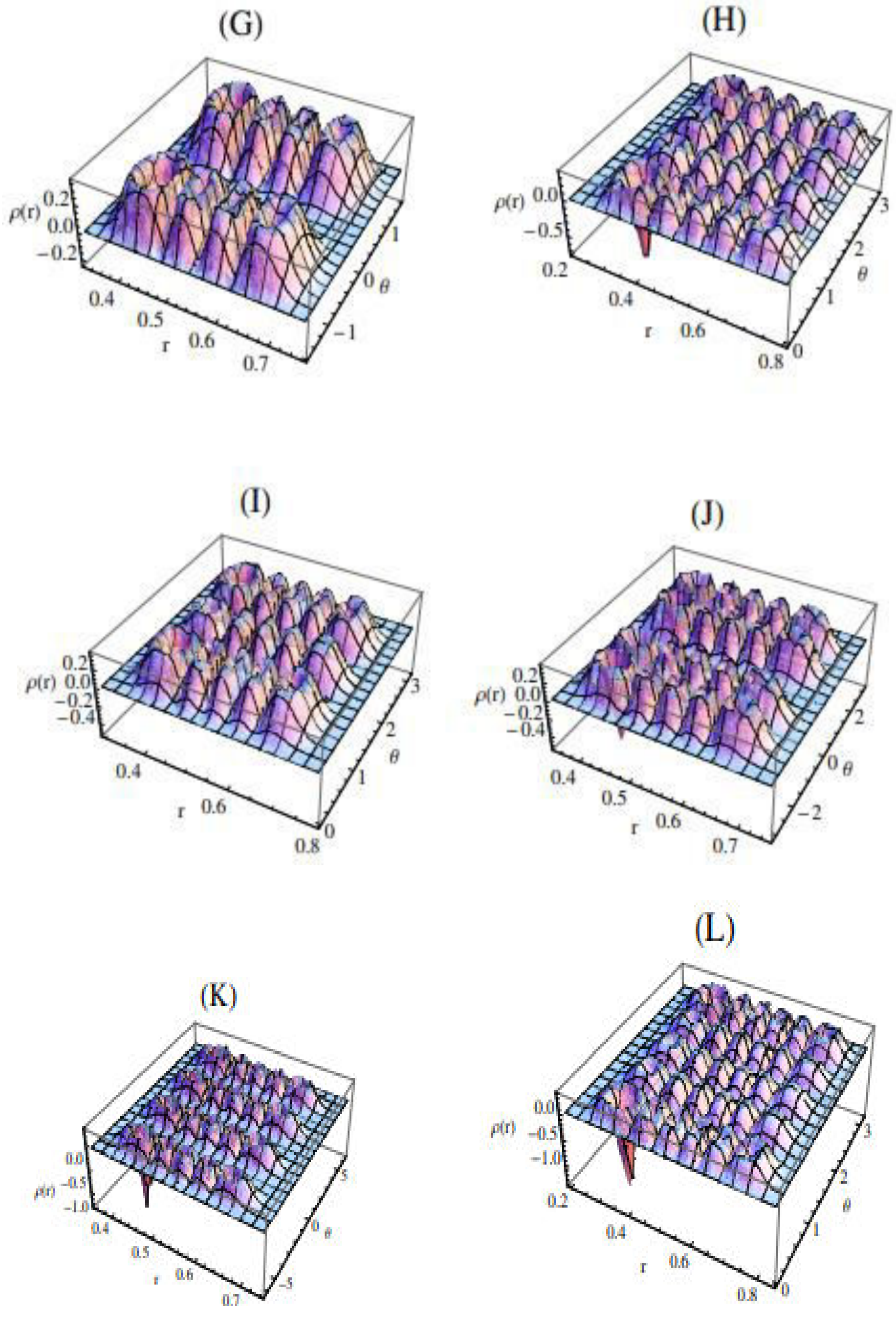


Figure 1: Position space information densities of the potential for α = 0.11, a = 1.1, b = 1.5 and (A) n = 1, l = 0 (B) n = 1, l = 1 (C) n = 2, l = 0 and (D) n = 2, l = 1 (E) n = 2, l = 2 (F) n = 3, l = 1 (G) n = 3, l = 2 (H) n = 4, l = 1 (I) n = 4, l = 2 (J) n = 4, l = 3 (K) n = 4, l = 4 (L) n = 5, l = 1

For more energetic states that the momentum space wave functions become a lot of work and are used for more math and numerical calculations. Fig. 1 shows the angular and radial contributions of information density to different parts of the three-dimensional potential respectively. These distinguishing characteristics are examined for specified values of distinguishing factors. The

ISSN- 2394-5125 VOL 08, ISSUE 02, 2021

potential's momentum probability density function has abrupt distant peaks. With the parameters, the plunge within the momentum density function changes. Based on the values of quantum numbers n and l, Fig. 1 shows the asymmetric structure of position space entropy. The evolution of severe plunges on the crests of density functions is intriguing to watch. In addition, the position density function behaves differently depending on the parameter value.

5. ENTROPY OF INFORMATION

Due to problems generated Integral analysis of information entropy isn't easy, especially in the momentum space, but it's not impossible. An analytical formula for position space information entropy for the ground state of the potential is then found out to be,

$$\begin{aligned} S_{0\ pos} &= -\frac{1}{\beta[1+\varepsilon,2+\vartheta]}(-\varepsilon\beta[1+\varepsilon,2+\vartheta](H_{\varepsilon}-H_{(2+\varepsilon+\vartheta)}) \\ &-(1+\vartheta)\beta[1+\varepsilon,2+\vartheta](H_{(1+\vartheta)}-H_{(2+\varepsilon+\vartheta)}) \\ &+\beta[1+\varepsilon,2+\vartheta]\log[\beta[1+\varepsilon,2+\vartheta]]), \end{aligned} \tag{5}$$

Hn is the harmonic number. However, it is difficult to describe information entropy analytically, especially in the momentum space, due to mathematical difficulties. Thus, numerical results for information entropy in Position space, momentum space, and entropy sum for any set of parameters were obtained using BBM inequality considerations, as shown in Tables 1 and 2.

Table 1: BBM inequality for the ground and excited states of the modified Hylleraas plus exponential Rosen Morse potential for α = 0.1, a = 0.001, b = 0.02, V0 = 0.0007, V1 = 0.0006, V2 = 0.0. The lower bound of information entropy Spos + Smom=(1 + ln(π)) = 2.1447.

n	l	Spos	Smom	Spos + Smom
0	0	0.3269	2.5427	2.8696
1	0	0.4597	3.0164	3.4761
1	1	1.2149	3.8995	5.1144
2	0	0.5306	3.2816	3.8122
2	1	1.0768	4.0782	5.1550
2	2	1.3213	4.3321	5.6534
3	0	0.5726	3.4705	4.0431

ISSN- 2394-5125 VOL 08, ISSUE 02, 2021

3	1	0.9964	4.2090	5.2054
3	2	1.2261	4.4582	5.6843
3	3	1.3834	4.6191	6.0025
4	0	0.6013	3.6208	4.2221
4	1	0.9412	4.3143	5.2555
4	2	1.1561	4.5586	5.7147
4	3	1.3076	4.7172	6.0248
4	4	1.4243	4.8357	6.2600
5	0	0.6227	3.7466	4.3693
5	1	0.9014	4.4039	5.3053
5	2	1.1019	4.6427	5.7446
5	3	1.2476	4.7992	6.0468
5	4	1.3613	4.9163	6.2776
5	5	1.4543	5.0101	6.4644
6	0	0.6396	3.8550	4.4946
6	1	0.8721	4.4836	5.3557
6	2	1.0587	4.7159	5.7746
6	3	1.1986	4.8699	6.0685

Table 2: BBM inequality for the ground and excited state of the modified Hylleraas plus exponential Rosen Morse potential for $\alpha = 0.1$, $a = 1$, $b = 2$, $V0 = 0.0007$, $V1 = -0.0006$, $V2 = 0.006$. The lower bound of information entropy Spos + Smom=$(1 + \ln(\pi)) = 2.1447$

n	l	Spos	Smom	Spos + Smom

ISSN- 2394-5125 VOL 08, ISSUE 02, 2021

0	0	0.2203	2.4799	2.7002
1	0	0.4508	2.9946	3.4454
1	1	1.2142	3.8986	5.1128
2	0	0.5274	3.2524	3.7798
2	1	1.0760	4.0773	5.1533
2	2	1.3210	4.3318	5.6528
3	0	0.5699	3.4348	4.0047
3	1	0.9957	4.2081	5.2038
3	2	1.2259	4.4579	5.6838
3	3	1.3832	4.6189	6.0021
4	0	0.5987	3.5821	4.1808
4	1	0.9405	4.3134	5.2539
4	2	1.1558	4.5582	5.7140
4	3	1.3075	4.7170	6.0245
4	4	1.4243	4.8356	6.2599
5	0	0.6203	3.7081	4.3284
5	1	0.9009	4.4031	5.3040
5	2	1.1016	4.6424	5.7440
5	3	1.2475	4.7990	6.0465
5	4	1.3612	4.9162	6.2774
5	5	1.4542	5.0101	6.4643

ISSN- 2394-5125 VOL 08, ISSUE 02, 2021

6	0	0.6373	3.8189	4.4562
6	1	0.8716	4.4829	5.3545
6	2	1.0584	4.7156	5.7740
6	3	1.1984	4.8698	6.0682

6. THE SQUEEZED STATES

The information entropy is favored to alter as a function of quantum uncertainty, and this variance is responsible for several significant quantum mechanical events such as minimum uncertainty states, or the squeezing of quantum fluctuations. Variance excels in quantitatively describing the spread of a distribution, and entropy cannot totally replace it. Finding state-independent variance-based uncertainty relations is thus a fundamental and crucial problem. However, this problem is rarely discussed, and no nontrivial lower bounds on the sum of variances of two or more operators are known in general, despite the fact that this formulation of uncertainty relations is exactly what people need in some applications. Squeezed states are the states with the least amount of uncertainty in position and momentum observables. The value of (Δx) 2 < 0.5 or (Δp) 2 < 0.5 for a squeezed state because of these uncertainty relations, it is impossible to squeeze both quadratures at the same time. The entropy of a single observable, like variance, can quantum fluctuations will be shown by this.

Information entropy in position and momentum states for ground and excited states with a BBM inequality that has been proven to be true is calculated. For low information entropy, a concentrated wave function and a precise prediction of where each particle will be will lead to a higher value. There are some values of parameters that are interesting for both the ground and excited states of potential. They make information entropy go down and make position space information entropy go down.

7. CONCLUSION

This work examines a significant Hylleraas plus an exponential Rosen Morse potential, which has been used in a lot of different fields of physics and chemistry. In physics and math, the Fourier transform is used to get the Eigen functions of momentum spaces in a way that makes sense. The information density and entropy of the potential are looked at, and the BBM inequality is tested with different parameters. Some things about information density and entropy that were never thought about before are looked into. Modified Hylleraas and an exponential Rosen Morse potential are talked about in this paper. Graphically, the information density of the potential and its properties were looked at for a three-dimensional potential. The number of minimas and their depth were found to be linked to the values of n and l. The entropy of a single observable, like variance, can be a good way to tell if something is changing in a quantum way.

REFERENCES

1. Sharma, Aarti & Thakur, Pooja & Kumar, Girish & Kumar, Anil. (2021). Quantum information entropy and squeezing of $\mathcal{PT}$-symmetric potential. Modern Physics Letters A. 36. 2150065. 10.1142/S0217732321500656.

2. Thakur, Pooja & Sharma, Aarti & Gupta, Rama & Kumar, Anil. (2017). Quantum information entropy of modified Hylleraas plus exponential Rosen Morse potential and squeezed states: SHARMA et al.. International Journal of Quantum Chemistry. 117. e25368. 10.1002/qua.25368.

3. Valencia-Torres, R & Sun, Guo-Hua & Dong, Shihai. (2015). Quantum information entropy for a hyperbolical potential function. Physica Scripta. 90. 10.1088/0031-8949/90/3/035205.

4. Sun, Guo-Hua & Dong, Shihai. (2013). Quantum information entropies of the eigenstates for a symmetrically trigonometric Rosen–Morse potential. Physica Scripta. 87. 045003. 10.1088/0031-8949/87/04/045003.

5. Amadi, P.O., Ikot, A.N., Ngiangia, A.T., Okorie, U.S., Rampho, G.J. & Abdullah, H.Y. (2020a). Shannon entropy and fisher information for screened kratzer potential. International Journal of Quantum Chemistry, 120, e26246. 19

6. Amadi, P.O., Ikot, A.N., Okorie, U.S., Rampho, G.J. & Abdullah, H.Y. (2020b). Information entropies for h2 and scf diatomic molecules with deng- fan-eckart potential. Revista Mexicana de Phisica, 66, 7426748. 20

7. Abdelmonem, M.S., Hady, A.A. & Nasser, I. (2017). Scaling behaviour of fisher and shannon entropies for the exponential-cosine screened coulomb potential. Molecular Physics, 115, 1480. 19

8. Adam, S.K., Antti, N. & Xubiao, P. (2014). Peierls-nabarro barrier and protein loop propagation. Physical Review E, Statistical, nonlinear and soft matter physics, 90, 062717. 81

9. Akhmediev, N., Ankiewicz, A. & Taki, M. (2009b). Waves that appear from nowhere and disappear without a trace. Physics Letters A, 373, 675. 16, 17, 75

10. Angelova, M., Hertz, A. & Hussin, V. (2012). Squeezed coherent states and the one-dimensional morse quantum system. Journal of Physics A: Mathematical and Theoretical, 45, 244007. 54

11. Ankiewicz, A., Akhmediev, N. & Soto-Crespo, J.M. (2010). Discrete rogue waves of the ablowitz-ladik and hirota equations. Physics Review E, 82, 026602. 84, 86

12. Ariadna, J.T.A., Dong, q., Sun, G.H. & Dong, S.H. (2018). Radial position-momentum uncertainties for the infinite circular well and fisher entropy. Physics Letters A, 382, 1752. 20