

WAVELET LIFTING SCHEME FOR IMAGE COMPRESSION BASED ON COMPRESSIVE SENSING

Rehana Farheen¹, S.Bhagya Lakshmi²

Assistant Professor^{1,2}
Department of ECE
MREC

Abstract- In this paper proposes a joint framework where in lifting-based, separable, image-matched wavelets are estimated from compressively sensed images and are used for the reconstruction of the same. Matched wavelet can be easily designed if full image is available. Also compared to standard wavelets as sparsifying bases, matched wavelet may provide better reconstruction results in compressive sensing (CS) application. Since in CS application, we have compressively sensed images instead of full images, and existing methods of designing matched wavelets cannot be used. Thus, we propose a joint framework that estimates matched wavelets from compressively sensed images and also reconstructs full images. This project has three significant contributions. First, lifting-based, image-matched separable wavelet is designed from compressively sensed images and is also used to reconstruct the same. Second, a simple sensing matrix is employed to sample data at sub-Nyquist rate such that sensing and reconstruction time is reduced considerably. Third, a new multi-level L-Pyramid wavelet decomposition strategy is provided for separable wavelet implementation on images that leads to improved reconstruction performance. Compared to CS-based reconstruction using standard wavelets with Gaussian sensing matrix and with existing wavelet decomposition strategy, the proposed methodology provides faster and better image reconstruction in compressive sensing application.

Index Terms- Compressive Sensing, Lifting Theory, L-Pyramid and Wavelet transform

I. INTRODUCTION

Classical signal acquisition technique in the signal processing branch involves sensing the full signal at or above the Nyquist rate. In general, this signal is transformed to a domain where it is compressible. Only some of the largest coefficients of transformed signal having sufficient amount of energy are stored and transmitted to the receiver along with the position information of the transmitted signal. The Associate Editor coordinating the review process was Prof. Yue M. Lu. (Corresponding author: Naushad Ansari.) The transmitted signal is decoded at the receiver to recover the original signal. Thus, this process involves sensing the full signal, although most of the samples in the transformed domain are to be discarded. In [1] and [2], researchers proposed a compressed/ compressive sensing (CS) method that combines sensing and compression in one stage, where instead of sampling a signal sample-wise above Nyquist rate, signal projections are captured via a measurement basis. These samples are very few as compared to those sampled at Nyquist rate. If the signal is sparse in some transform domain and also if the measurement basis is incoherent with the sparsifying basis, then the full signal can be reconstructed with a good probability from a very few projections of the signal. In this context, wavelets are extensively used as a sparsifying basis in compressed sensing problems [2]. One of the advantages with wavelets is that there is no unique basis unlike Fourier transform. One may choose the set of basis depending upon

the type of application. Since the wavelet basis are not unique, it is better to design wavelets that are matched to a given signal in a particular application. The designed wavelet basis are called signal-matched wavelets [3]–[5]. Motivated with the above discussion, this paper proposes to design matched wavelets for compressive sensing application. Unlike previous works on matched wavelets where wavelets are designed from fully sampled signal [3]–[9], this paper proposes a novel method of designing signal matched wavelets from compressively sensed images at the receiver. The proposed method employs a lifting based framework to design image-matched wavelets [10]. Many researchers have designed wavelets using lifting [5], [8], [11]–[17] that requires design of predict and update stage filters.

II. LITERATURE SURVEY

[1] Emmanuel J. In recent years, compressed sensing (CS) has attracted considerable attention in areas of applied mathematics, computer science, and electrical engineering by suggesting that it may be possible to surpass the traditional limits of sampling theory. CS builds upon the fundamental fact that we can represent many signals using only a few non-zero coefficients in a suitable basis or dictionary. [2] Anubha Gupta This paper proposes a new method of estimating both biorthogonal compactly supported as well as semi-orthogonal infinitely/compactly supported wavelet from a given signal. The method is based on maximizing projection of the given signal onto successive scaling subspace. This results in minimization of energy of signal in the wavelet subspace. The idea used to estimate analysis wavelet filter is similar to a sharpening filter used in image enhancement.

[3] Anubha Gupta, Shiv Dutt Joshi This paper presents a new approach for the estimation of wavelets that is matched to a given signal in the statistical sense. Based on this approach, a number of new methods to estimate statistically matched wavelets are proposed. The paper first proposes a new method for the estimation of statistically matched two band compactly supported biorthogonal wavelet system. Second, a new method is proposed to estimate statistically matched semiorthogonal two-band wavelet system that results in compactly supported or infinitely supported wavelet. Next, the proposed method of estimating two-band wavelet system is generalized to n -band wavelet system.

[4] Naushad Ansari This paper proposes design of signal-matched wavelets via lifting. The design is modular owing to lifting framework wherein both predict and update stage polynomials are obtained from the given signal. Successive predict stages are designed using the least squares criterion, while the update stages are designed with total variation minimization on the wavelet approximation coefficients. Different design strategies for compression and denoising are presented. The efficacy of matched-wavelets is illustrated on transform coding gain and signal denoising.

[5] Joseph O Algorithms for designing a mother wavelet (ψ) such that it matches a signal of interest and such that the family of wavelets $2^{-j}\psi(2^{-j}x)$ forms an orthonormal Riesz basis of $L^2(\mathbb{R})$ are developed. The algorithms are based on a closed form solution for finding the scaling function spectrum from the wavelet spectrum.

III. PROPOSED METHOD

We propose to use PCI sensing matrix that, to our understanding, is the simplest sensing matrix proposed so far and “actually” senses the image at sub-Nyquist rate by capturing fewer pixels without sensing information about every pixel [32]. This is explained as below. Consider an image X of dimension $m \times n$. Instead of sampling all

the $N(N = mn)$ pixels of the image using the sensor array of the traditional camera, we capture M samples of the image using the proposed measurement matrix p , where $M \ll N$. The measurement matrix p has the entries shown below:

$$\Phi_{i,j}^p = \begin{cases} 1 & \text{if } i \in \{1, 2, \dots, M\} \text{ and } j \in \Omega \\ 0 & \text{otherwise,} \end{cases} \quad (13)$$

Where, $\Omega \subset \{1, 2, \dots, N\}$ such that $|\Omega| = M$ and $|\Omega|$ denotes the cardinality of the set.

This sensing matrix is known as partial canonical identity (PCI) matrix because it consists of partially selected and permuted rows of the identity matrix.



Fig.1.1(a) Image (with dimension 512×512) captured using PCI sensing matrix with 50% sampling ratio and with zeros filled at positions not sampled (b) Image reconstructed from subsampled image using 'db4' wavelet

The PCI sensing matrix captures only M samples of the actual image; thus, actually sub-samples the original image. This can be accomplished by using existing cameras by switching ON only M sensors of the sensor array. This is unlike the single pixel camera where every captured pixel is the linear combination of the entire image pixel set. Also, in single pixel camera, one has to wait for M units of time to sense M number of samples, whereas all M samples are sensed in one unit of time in the case of PCI sensing matrix. Thus, the PCI sensing matrix reduces the sensing time by a factor of M in comparison to a single pixel camera.

Proposed L-Pyramid wavelet decomposition method for images

In this Section, we propose a new optimized strategy of multi-level wavelet decomposition on images. A separable wavelet transform is implemented on images by first applying 1-D wavelet transform along the columns and then along the rows of an image. This provides 1-level wavelet decomposition that consists of four components labeled as LL, LH, HL and HH, respectively. The same procedure is repeated on the LL part of the

wavelet transform $k - 1$ -times to obtain k -level decomposition of an image (Fig. 6a). We call this decomposition as Regular Pyramid (R-Pyramid) wavelet decomposition. In general, k -level wavelet decomposition of an image consists of the following components:

LL_k, LH_i, HLi and HH_i , where $i=1, 2, \dots, k-1$.

LH_i, HLi and HH_i components are obtained by applying wavelet transform on the columns and rows of LL_{i-1} component. LH_i is obtained by filtering LL_{i-1} column-wise using a lowpass filter and filtering it row wise using a high pass filter. Thus, the current nomenclature of labeling sub bands is: first character represents operation on columns and second character represents operation on rows, where operation implies highpass or lowpass filtering denoted by symbols 'H' and 'L', respectively. In the conventional 2-D wavelet transform (Fig. 6a), wavelet decomposition is applied on LL_i part only to obtain the $(i + 1)$ th level coefficients. Since it is a separable transform, similar to 1-D wavelet transform wherein wavelet is applied repeatedly on lowpass filtered branches, we propose to apply wavelet in the lowpass filtered directions of LH_{i-1} and HL_{i-1} subbands in contrast to the conventional decomposition strategy wherein these subbands are left unaltered.

Thus, the proposed 2nd level wavelet decomposition is as shown in Fig. 6c. Since we apply wavelet in only one direction of LH_{i-1} and HL_{i-1} subbands, we notate these subbands differently compared to the conventional scheme. We assign subscript with both 'L' and 'H' symbols of every subband to denote the no. of times wavelet has been applied in that direction. In order to understand this, let us first consider 1-level wavelet decomposition as shown in Fig. 6b that is similar to the conventional scheme shown in Fig. 6a. However, the subbands are labeled as $L1L1$, $L1H1$, $H1L1$, and $H1H1$. In the 2nd level wavelet decomposition, wavelet is applied in both directions of $L1L1$ subbands leading to $L2L2, L2H2, H2L2$, and $H2H2$ subbands.

But in addition, wavelet is applied on the columns of $L1H1$ yielding two subbands $L2H1$ and $H2H1$. Also, wavelet is applied on the rows of $H1L1$ subband yielding two subbands $H1L2$ and $H1H2$. Applying similar strategy for the 3rd level decomposition, we obtain subbands as shown in Fig. 6d. We name this wavelet decomposition as L-shaped Pyramid (L-Pyramid) wavelet decomposition.

The efficacy of the proposed L-Pyramid wavelet decomposition is shown in CS-based image reconstruction with orthogonal Daubechies wavelet 'db4' and PCI sensing matrix. Fig. 7 shows reconstruction accuracy in terms of PSNR (14) with sampling ratios ranging from 10% to 90% averaged over 10 independent trials. We compare reconstruction accuracy at different sampling ratios with the existing RPyramid wavelet decomposition and with the proposed LPyramid wavelet decomposition on the same three images: 'Beads', 'Lena' and 'House'. From Fig. 7, we note better results with L-Pyramid wavelet decomposition compared to R-Pyramid wavelet decomposition at sampling ratios from 90% to 30%.

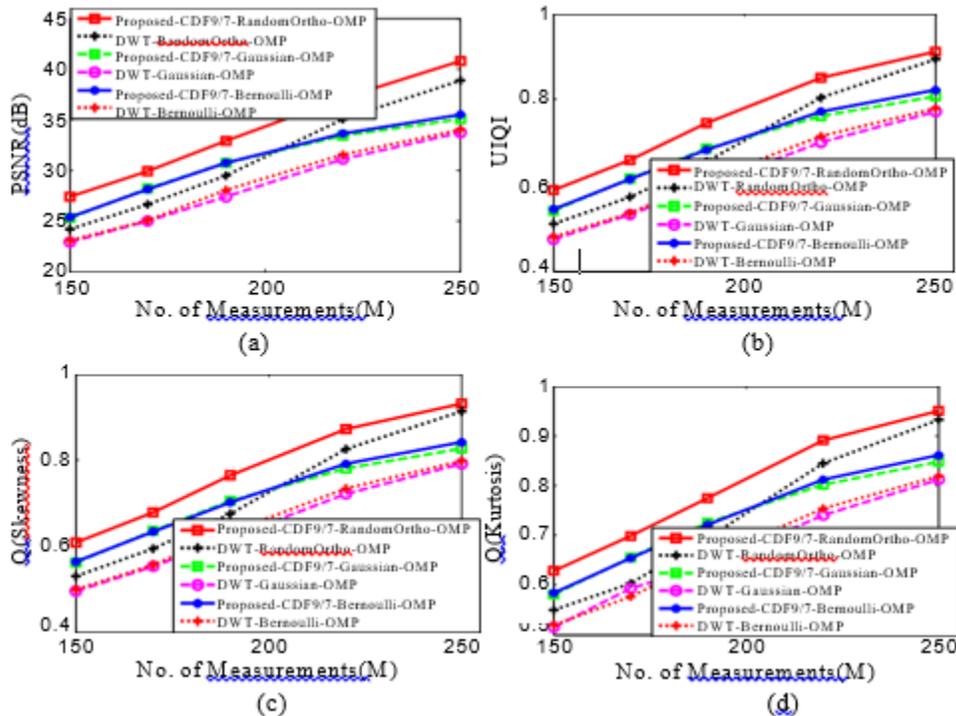
There is considerably less improvement at lower sampling ratios of 20% and 10% (refer to the enlarged view in Fig. 7). This may be due to the reason that the number of samples acquired at such lower sampling ratios do not contain enough information for good image reconstruction. In addition, we note that performance is particularly

improved for image 'House' that is rich in low frequencies. Since the lowpass bands are repeatedly broken in all the subbands in the proposed L-Pyramid unlike the R-Pyramid, images rich in lower frequencies are benefited more. This, further, establishes the significance of the proposed decomposition strategy.

IV. EXPERIMENTAL RESULTS

In this section, we illustrate the reconstruction performance of compressed sensing image; we deploy both the OMP and Basis Pursuit on proposed sparse basis CDF9/7 wavelet transform (WT), DWT and DCT using Gaussian matrix, Bernoulli matrix and random orthogonal matrix measurements. To evaluate the performance of proposed algorithm we used open source databases [18]. The images are for 8-bit 256×256 pixels and 8-bit 64×64 pixels. We used different image quality indexes: PSNR, UIQI, Q(Skewness), Q(Kurtosis), SSIM, SSIM(Skewness), SSIM(Abs-Skewness), SSIM(Kurtosis) [19] [20] to evaluate the qualities of the compressed images. In order to make clearer comparisons, besides our proposed sparse basis CDF9/7 WT with three different measurements matrix's and Basis Pursuit (BP) algorithms or greedy optimization techniques as OMP and are also used. We made experiment on 15 images in our proposed image compression based on CS methods. Comparing the evaluation of image quality (EIQ) indexes for the compressed image at several measurements $M < N$ ($N = 256$), we note that the higher measurements M , the better quality of image compression will be. Figures 3 & 4 show the quality indexes of image compression obtained with proposed sparse

CDF 9/7 wavelet transform (WT) and sparse DWT with OMP algorithm performed on the human body shown in Figure 2. As example, in Figure 3 (a), the PSNR values of proposed sparse basis CDF9/7 with three measurements matrix and OMP in proposed framework is showing higher values than sparse basis DWT with three measurements matrix and OMP in the range [150-250]. Similarly, we also observe that other quality index values including new image quality indexes of proposed sparse basis CDF 9/7 WT are higher values than others. However, the new image quality indexes also show higher value than original UIQI and SSIM. Comparing the evaluation of image quality (EIQ) indexes for the compressed image at several measurements $M < N$ ($N = 256$), we note that the higher the measurements M given the better the quality of image compression will be. Figures 5 & 6 also show the quality indexes of image compression obtained with proposed sparse basis CDF9/7 WT and other techniques of sparse basis DWT or DCT with BP algorithm were also used on the Brain (MR)-1 and Brain (MR)-2 images.



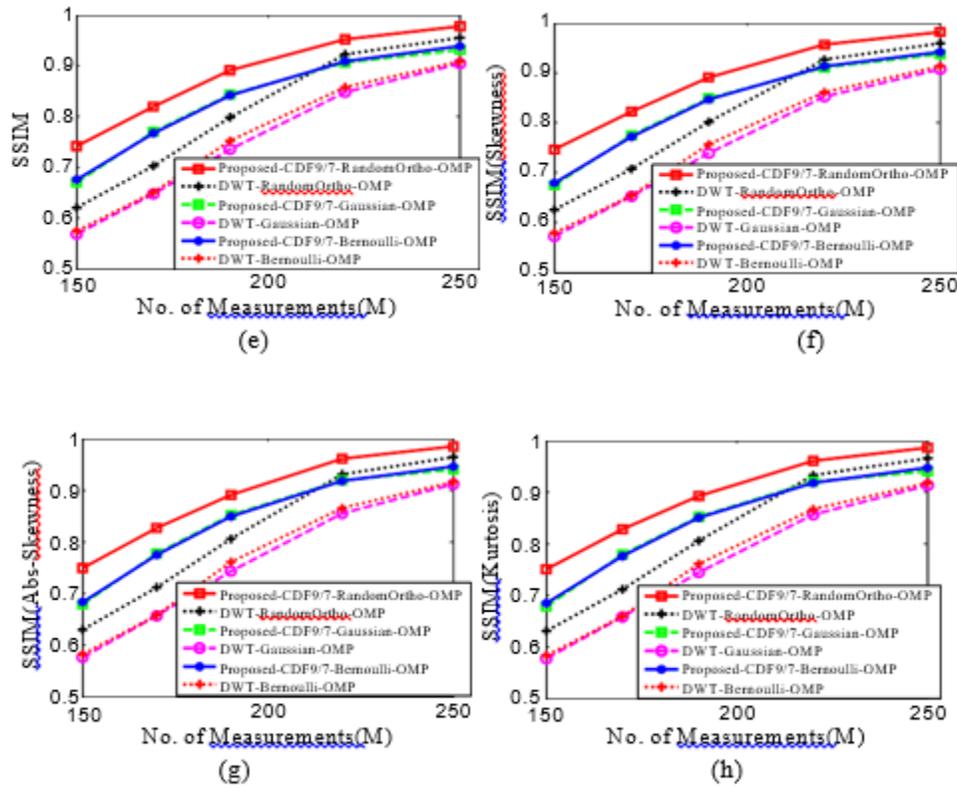


Figure 3: Plots of eight image quality indexes for human body muscle cells image versus number of measurements (M) ($M < N$, $N = 256$) in the range [150-250] for proposed sparse basis CDF9/7 wavelet transform with three measurements matrix and OMP including the sparse basis DWT.

V. CONCLUSION

In this paper, we propose an image compression method based on compressive sensing with a wavelet lifting scheme framework that addresses the best-compressed image components and high-frequency feature preservation in medical images. The proposed method is also compared to three different matrices, namely Gaussian, Bernoulli, and random orthogonal measurement matrices, as well as image reconstruction using convex optimization techniques such as Basis Pursuit (BP) and greedy pursuit algorithms such as Orthogonal Matching Pursuit (OMP). Experiments show that the proposed sparse basis works. CDF9/7A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions. In the suggested framework with CS, wavelet transform compresses images better than sparse DWT or DCT with OMP or Basis Pursuit (BP) method.

REFERENCES

- [1] .Mallat, *A Wavelet Tourof Signal Processing, Third Edition: The Sparse Way*, 3edition. Amsterdam; Boston: Academic Press, 2008.
- [2] S.G.Mallat and Z.Zhang, "Matching pursuits with time-frequency dictionaries," *IEEE Trans.Signal Process.*, vol.41, no.12, pp.3397–3415, Dec.1993.
- [3] S. G. Mallat, "Adaptive time-frequency decompositions," *Opt. Eng.*, vol. 33, no. 7, p. 2183, Jul.1994.
- [4] S. Chen, D. Donoho, and M. Saunders, "Atomic Decomposition by Basis Pursuit," *SIAM J. Sci. Comput.*, vol.20, no.1, pp.33–

61,Jan.1998.

- [5] J. Jin, B. Yang, K. Liang, and X. Wang, "General image denoising framework based on compressivesensingtheory," *Comput. Graph.*, vol.38,pp. 382–391, Feb.2014.
- [6] S. Fischer, G. Cristobal, and R. Redondo, "Sparse overcomplete Gabor wavelet representation basedonlocalcompetitions," *IEEETrans.ImageProcess.*, vol.15,no.2, pp.265–272, Feb.2006.
- [7] B. Wang, Y. Wang, I. Selesnick, and A. Vetro, "Video Coding Using 3D Dual-Tree Wavelet Transform," *EURASIP J. Image Video Process.*, vol. 2007, no.1, p.042761, Mar.2007.
- [8] M. Y. Baig, E. M.-K. Lai, and A. Punchihewa, "Compressed Sensing-Based Distributed Image Compression," *Appl. Sci.*, vol.4,no.2,pp.128–147, Mar.2014.
- [9] M.A.T.Figueiredo, R.D.Nowak, and S.J.Wright, "Gradient Projection for Sparse Reconstruction: Application to Compressed Sensing and Other Inverse Problems," *IEEE J. Sel. Top. Signal Process.*, vol.1,no.4,pp.586–597, Dec.2007.
- [10] R. G. Baraniuk, "Compressive Sensing [Lecture Notes]," *IEEE Signal Process. Mag.*, vol. 24, no. 4, pp.118–121, Jul.2007.
- [11] E. J. Candes and M. B. Wakin, "An Introduction To Compressive Sampling," *IEEE Signal Process. Mag.*, vol.25,no.2,pp.21–30, Mar.2008.