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# Model Order Reduction of Single Input and Single Output Discrete Interval Systems

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***Abstract*— This paper presents a new and improved method for finding reduced order model of single-input-single-output (SISO) discrete interval systems using Routh approximation and matching of time moments and Markov parameters of higher order interval systems and reduced order models. In this method, the discrete interval system is first converted to equivalent continuous interval system by simple linear transformation. The denominator of the equivalent continuous interval model is determined using Routh approximation technique by obtaining the Routh table and the coefficients of numerator polynomials are obtained by matching the time moments and Markov parameters. Finally, the obtained reduced order continuous interval model is reconverted into discrete interval model by using inverse linear transformation. The methodology of the proposed technique is illustrated with the help of one test system. The responses of the system and model validate the superiority of the proposed technique.**

***Keywords—Discrete interval systems; Approximation; Routh table; Kharitonov polynomials.***

## I. INTRODUCTION

Modeling of complex dynamic system is one of the prominent research area in engineering problems. Further, the design of controllers for real physical system is complicated due to their higher order dimensions. Therefore, the approximation techniques based on mathematical approaches should be used to construct reduced order models.

Several methods have been developed for the approximation of fixed-coefficient continuous time systems [1-3]. Typical methods are Padé approximation, aggregation method, moment matching technique, Routh approximation [4], Routh stability criterion [5], Hurwitz polynomial approximation [3], stability equation method and evolutionary approach for model order reduction [6, 7]. Among all these techniques, Padé approximation has gained popularity because of its unique features like computational simplicity and matching of time moments. However, this method has a serious disadvantage that it often produces unstable approximants even though the higher order original system is stable. To overcome this limitation, several methods have been proposed [2].

The conventional methods [8, 9] of model order reduction are available for continuous as well as discrete systems. Especially, for order reduction of discrete-time systems, two approaches are available: direct approach and indirect approach. However, direct approaches are not preferred because of their complex computational procedures. In indirect approach, some basic transformation is performed from discrete domain to continuous domain techniques and then, order reduction technique is carried out in the continuous domain. At first, the $z$–domain transfer functions are converted into $\omega$– domain, by the linear transformation and then, after reducing them in $\omega$– domain, approximants are obtained by using suitable methods [10-12] for continuous systems. Again, the reduced order continuous system is converted back into the discrete system using inverse linear transformation.

The interval system can be used as a better choice for modeling uncertain system because of its capability to tackle physical problems like sensor noise, parametric uncertainties, environmental changes, etc. Many methods are proposed in literature [13-18] for analysis of interval systems such as Routh-Padé approximation [13], gamma-delta method [14], time moments and Markov parameter method [15] etc.

The aim of this paper is to develop a new approach for model order reduction of discrete interval systems. In first step, discrete interval transfer function is converted to continuous interval system using linear transformation. The denominator and numerator of the reduced order model are computed by Routh approximation and matching of time moments and Markov parameters, respectively. In the next step, the reduced-order model of continuous interval system is reconverted to the discrete domain using inverse linear transformation.

In this article, section 2, describes the problem statement. Time moments and Markov parameters of interval systems are illustrated in section 3. Routh-Padé approximation is provided in section 4. A SISO test is provided in section 5 to validate the above technique. Finally, the conclusion of this paper is given in section 6.

## II. PROBLEM STATEMENT

Suppose, the single-input-single-output (SISO) of higher order discrete interval system is given as

$$H^n(z) = \frac{N(z)}{D(z)} = \frac{p_0 + p_1 z + \cdots + p_{n-1} z^{n-1}}{q_0 + q_1 z + \cdots + q_n z^n} \tag{1}$$

where $p_i = \left[p_i^-, p_i^+\right]$ for $i = 0,1,\cdots,n-1$ and $q_i = \left[q_i^-, q_i^+\right]$ for $i = 0,1,\cdots,n$ are the interval coefficients of numerators and denominators, respectively, in the discrete domain.

By using linear transformation technique i.e. $z \to \omega + 1$, the discrete transfer function (1) is converted into continuous transfer function. Now, the modified version of equation (1) become

$$G^n(\omega) = \frac{N(\omega)}{D(\omega)} = \frac{c_0 + c_1\omega + \cdots + c_{n-1}\omega^{n-1}}{d_0 + d_1\omega + \cdots + d_n\omega^n} \quad (2)$$

where $\left[c_i^-, c_i^+\right]$ for $i = 0,1,\cdots,(n-1)$ and $\left[d_i^-, d_i^+\right]$ for $i = 0,1,\cdots,n$ are the interval coefficients of numerators and denominators in the continuous domain.

Suppose, it is desired to obtain a reduced order model that has a continuous interval transfer function

$$G_r(\omega) = \frac{\hat{N}(\omega)}{\hat{D}(\omega)} = \frac{\hat{c}_0 + \hat{c}_1\omega + \cdots \hat{c}_{r-1}\omega^{r-1}}{\hat{d}_0 + \hat{d}_1\omega + \cdots + \hat{d}_r\omega^r} \quad (3)$$

The above-reduced order continuous interval model is converted back into discrete interval model using inverse linear transformation i.e. $\omega = z-1$, now the modified form of the equation (3) given by

$$\hat{H}_r(z) = \frac{\hat{N}(z)}{\hat{D}(z)} = \frac{\hat{p}_0 + \hat{p}_1 z + \cdots \hat{p}_{r-1}z^{r-1}}{q_0 + q_1 z + \cdots q_r z^r} \quad (4)$$

## III. TIME MOMENTS AND MARKOV PARAMETERS OF INTERVAL SYSTEM

Let, the transfer function of continuous interval system is given as

$$G^n(\omega) = \frac{N(\omega)}{D(\omega)} = \frac{c_0 + c_1\omega + \cdots + c_{n-1}\omega^{n-1}}{d_0 + d_1\omega + \cdots + d_n\omega^n} \quad (5)$$

where $\left[c_i^-, c_i^+\right]$ for $i = 0,1,\cdots,n-1$ and $d_i = \left[d_i^-, d_i^+\right]$ for $i = 0,1,\cdots,n$ are the interval coefficients of numerators and denominators respectively. For the system, the power series expansion around $\omega = 0$ and $\omega = \infty$ can be written as,

$$H_n(\omega) = T_0 + T_1\omega + \cdots + T_k\omega^k + \cdots \text{ (expansion around } \omega = 0\text{)} \quad (6)$$

$$H_n(\omega) = M_1\omega^{-1} + M_2\omega^{-2} + \cdots + M_k\omega^{-k} + \cdots \text{ (expansion around } \omega = \infty\text{)} \quad (7)$$

where $T_i = \left[T_i^-, T_i^+\right]$ for $i = 0,1,\cdots$ and $M_i = \left[M_i^-, M_i^+\right]$ for $i = 1,2,\cdots$, are the time moments and Markov parameters of the interval system, respectively.

$$T_k = \frac{c_k}{d_0} - \sum_{i=0}^{k-1}\frac{d_{k-i}T_i}{d_0}, \; k = 0,1,2,\cdots, \quad (8)$$

$$M^k = \frac{c_{n-k}}{d_n} - \sum_{i=1}^{k-1}\frac{d_{n-k+i}M_i}{d_n}, \; k = 1,2\cdots \quad (9)$$

## IV. ROUTH-PADÉ APPROXIMANTS OF INTERVAL SYSTEMS

Consider, a stable $r$th-order model described by the transfer function

$$G_r(\omega) = \frac{\hat{N}(\omega)}{\hat{D}(\omega)} = \frac{\hat{c}_0 + \hat{c}_1\omega + \cdots \hat{c}_{r-1}\omega^{r-1}}{\hat{d}_0 + \hat{d}_1\omega + \cdots + \hat{d}_r\omega^r} \quad (10)$$

where $r < n$.

The parameters $\hat{c}_i = \left[\hat{c}_i^-, \hat{c}_i^+\right]$ for $i = 0,1,\ldots,r-1$ and $\hat{d}_i = \left[\hat{d}_i^-, \hat{d}_i^+\right]$ for $i = 0,1,\ldots,r$ denotes the interval coefficients.

The model, (10), in terms of time moments and Markov parameters can be expanded as

$$G_r(\omega) = \hat{T}_0 + \hat{T}_1\omega + \cdots + \hat{T}_k\omega^k + \cdots \text{ (at, } \omega = 0\text{)}, \quad (11)$$

$$G_r(\omega) = \hat{M}_1\omega^{-1} + \hat{M}_2\omega^{-2} + \cdots + \hat{M}_k\omega^{-k} + \cdots \text{ (at, } \omega = \infty\text{)}, \quad (12)$$

where $T_i$ for $i = 0,1,\ldots$ and $M_i$ for $i = 1, 2,\ldots$ denotes, the time moments and Markov parameters respectively. Hence, the expressions become,

$$\hat{T}_k = \frac{\hat{c}_k}{\hat{d}_0} - \sum_{i=0}^{k-1}\frac{\hat{d}_{k-i}\hat{T}_i}{\hat{d}_0}, \quad k = 0,1,2,\ldots \quad (13)$$

$$\hat{M}_k = \frac{\hat{c}_{r-k}}{\hat{d}_r} - \sum_{i=1}^{k-1}\frac{\hat{d}_{r-k+i}\hat{M}_i}{\hat{d}_r}, \quad k = 1,2,\ldots \quad (14)$$

are computed in the same procedure as derived from the system (2)

### A. Determination of the denominator

The Routh table is used to calculate the denominator for the model. The Routh table for continuous interval system is constructed as shown in Table.1

Here, the Routh table elements are calculated by using

$$D_{i,j} = \frac{D_{i-2,j+1}D_{i-1,1} - D_{i-2,1}D_{i-1,j+1}}{D_{i-1,1}}, \quad (15)$$

for $i \geq 3$ and $1 \leq j \leq (n - i + 3) / 2$.

The denominator, $\hat{D}(s)$, of the $r$th-order model is obtained from $(n+1-r)$th and $(n+2-r)$th rows of Table 1 as:

$$\hat{D}(\omega) = D_{n+1-r,1}\omega^r + D_{n+2-r,1}\omega^{r-1} + D_{n+1-r,2}\omega^{r-2} + \cdots. \quad (16)$$

TABLE I. ROUTH TABLE FOR THE DENOMINATOR

| | | | |
|---|---|---|---|
| $D_{1,1} = \left[d_n^-, d_n^+\right]$ | $D_{1,2} = \left[d_{n-2}^-, d_{n-2}^+\right]$ | $D_{1,3} = \left[d_{n-4}^-, d_{n-4}^+\right]$ | $\cdots$ |
| $D_{2,1} = \left[d_{n-1}^-, d_{n-1}^+\right]$ | $D_{2,2} = \left[d_{n-3}^-, d_{n-3}^+\right]$ | $\cdots$ | |
| $D_{3,1}$ | $D_{3,2}$ | $\cdots$ | |
| $\vdots$ | $\vdots$ | $\ddots$ | |
| $D_{n,1}$ | $D_{n,2}$ | | |
| $D_{n+1,1}$ | | | |

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*B. Determination of the numerator*

The denominator of the model $\hat{D}(\omega)$ is derived from Routh table, while the numerator $\hat{N}(\omega)$ is calculated by matching the first $r$ time moments and Markov parameters of the original system and model as given by:

$$\hat{T}_i = T_i, \quad i = 0,1,\ldots,(u-1), \qquad \hat{M}_i = M_i, \quad i = 1,2,\ldots,v; \quad v = r-u. \tag{17}$$

## V. TEST SYSTEM

The following example illustrates the effectiveness of the proposed method.

Consider a third order discrete interval system [19]:

$$H(z) = \frac{[8,10]+[3,4]z+[1,2]z^2}{[0.8,0.85]+[4.9,5]z+[9,9.5]z^2+[6,6]z^3} \equiv \frac{N(z)}{D(z)}. \tag{18}$$

The above discrete system can be converted into the continuous system by replacing $z$ with $1+\omega$

$$H(\omega) = \frac{[12,16]+[5,8]\omega+[1,2]\omega^2}{[20.7,21.35]+[40.9,42]\omega+[27,27.5]\omega^2+[6,6]\omega^3} \equiv \frac{N(\omega)}{D(\omega)}. \tag{19}$$

The Routh table for the above transfer function is shown in Table-2.

$$\hat{D}_1(\omega) = [19.39,22.78]+[35.64,38.03]\omega, \tag{20}$$

$$\hat{D}_2(\omega) = [20.7,21.35]+[35.64,38.03]\omega+[27,27.5]\omega^2. \tag{21}$$

The first two time moments and Markov parameter of the system, calculated by using [15], are given respectively as:

$$T_0 = [0.56,\ 0.77], \tag{22}$$

$$T_1 = [-1.33,\ -0.69], \tag{23}$$

$$M_1 = [0.16,\ 0.33]. \tag{24}$$

By matching the time moments of the system and model, $T_0 = \hat{T}_0$, the first-order model is given by

$$R_1^P(\omega) = \frac{[10.90,17.60]}{[19.39,22.78]+[35.64,38.03]\omega}, \tag{25}$$

In $z$ -domain the above transfer function becomes

$$R_1^P(z) = \frac{[10.90,17.60]}{[-18.64,-12.86]+[35.64,38.03]z}. \tag{26}$$

TABLE II. ROUTH TABLE FOR THE DENOMINATOR (19).

| | |
|---|---|
| [6, 6] | [40.9, 42] |
| [27, 27.5] | [20.7,21.35] |
| [35.64, 38.03] | |
| [19.39, 22.78] | |

By matching the time moments $T_0 = \hat{T}$ and Markov parameters $M_1 = \hat{M}_1$, the second order model becomes

$$R_{TM}^P(\omega) = \frac{[11.63,16.50]+[4.49,9.15]\omega}{[20.7,21.35]+[35.64,38.03]\omega+[27,27.5]\omega^2}, \tag{27}$$

By rewriting the above transfer function in $z$–domain

$$R_{TM}^P(z) = \frac{[2.48,12.01]+[4.49,9.15]z}{[9.67,10.82]+[-19.36,-15.97]z+[27,27.5]z^2}. \tag{28}$$

Similarly, by matching the first and second-time moments, $T_0 = \hat{T}_0$ and $T_1 = \hat{T}_1$, the second order approximants in $\omega$–domain becomes

$$R_T^P(\omega) = \frac{[11.66,16.50]+[1.77,5.29]\omega}{[20.7,21.35]+[35.64,38.03]\omega+[27,27.5]\omega^2}, \tag{29}$$

The second order approximants in $z$–domain ,

$$R_T^P(z) = \frac{[6.37,14.73]+[1.77,5.29]z}{[9.67,10.82]+[-19.36,-15.97]z+[27,27.5]z^2}. \tag{30}$$

The first and second-order approximants are calculated using the technique proposed by [13]

Now, the first order model becomes:

$$R_1^B(\omega) = \frac{[10.91,17.60]}{[19.39,22.78]+[35.64,38.03]\omega}, \tag{31}$$

and its $z$ –domain transfer function is:

$$R_1^P(z) = \frac{[10.91,17.60]}{[-18.64,-12.86]+[35.64,38.03]z}. \tag{32}$$

than, the second order model is formed as:

$$R_2^B(\omega) = \frac{[11.63,16.50]+[-8.69,10.12]\omega}{[20.7,21.35]+[35.64,38.03]\omega+[27,27.5]\omega^2}, \tag{33}$$

and its $z$ –domain transfer function is written as:

$$R_F(z) = \frac{[1.51,25.19]+[-8.69,10.12]z}{[9.67,10.82]+[-19.36,-15.97]z+[27,27.5]z^2}. \tag{34}$$

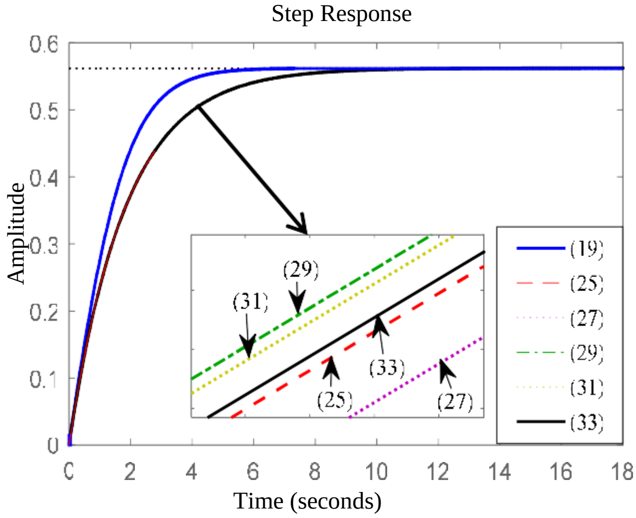


Fig. 1. The step responses of the system and models.

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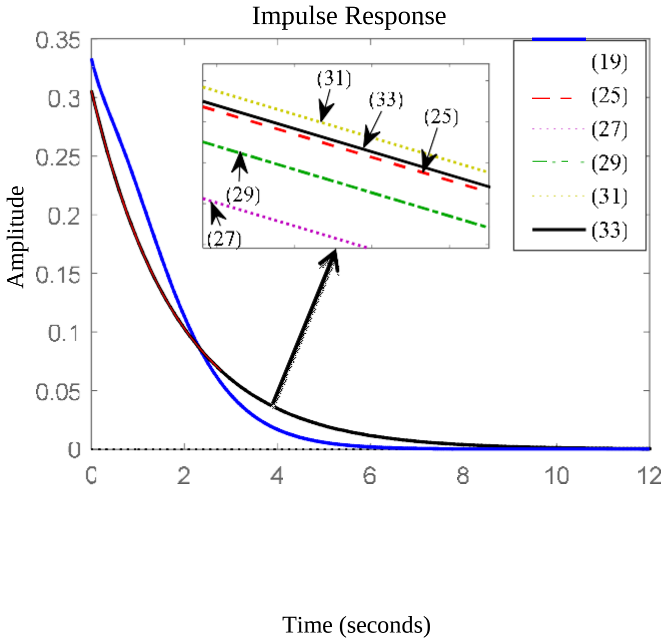


Fig. 2. The impulse responses of the system and models.

Kharitonov's polynomials of numerator and denominator of higher order interval system, (19), is used to obtain step and impulse responses of the rational systems. The responses are shown in Fig. 1 and Fig. 2.

From Fig 1, it is observed that, (27) is better approximant than others. The first order approximants response of (27) is better than (31). From Fig. 2, the conclusion can be extracted that (27) and (29) are better approximants than others in the interval system.

## VI. Conclusion

A time domain order reduction technique for discrete interval system using time moments and the Markov parameters of interval systems is presented in this paper. A simple linear transformation technique is used to convert discrete time interval transfer function into continuous interval system. Then, the denominator of the reduced order model is formed by using Routh table and the numerator is calculated by matching the *r* time moments and Markov parameters in the continuous domain. Further, the obtained reduced-order model of continuous interval system is reconverted to the discrete domain using inverse linear transformation. This method satisfies a sufficient degree of approximation because of Routh-Padé approximants, which signify a stable system.

## Appendix A

For the discrete interval system, sample time $T_s$=0.001Sec can be written as

$$G_n(\omega)=\frac{\frac{c_0}{d_0}+\frac{c_1}{d_0}\omega+\cdots+\frac{c_{n-1}}{d_0}\omega^{n-1}}{1+\frac{c_1}{d_0}\omega+\cdots+\frac{c_n}{d_0}\omega^n},$$

$$G_n(\omega)=T_0+T_1\omega+T_2\omega^2+\cdots,$$

where

$$T_k=\frac{c_k}{d_0}-\sum_{i=0}^{k-1}\frac{d_{k-i}T_i}{d_0},\quad k=0,1,2,\cdots$$

Similarly, the time moments of the model can be written as

$$\hat{T}_k=\frac{\hat{c}_k}{\hat{d}_0}-\sum_{i=0}^{k-1}\frac{\hat{d}_{k-i}\hat{T}_i}{\hat{d}_0},\quad k=0,1,2,\cdots$$

Similarly, the aforementioned procedure is used to obtain around $\omega=\infty$ .

## Appendix B

The interval operations are given as follows:

$$[w,x]+[y,z]=[w+y,x+z],$$
$$[w,x]-[y,z]=[w-z,x-y],$$
$$[w,x]-[w,x]=0,$$
$$[w,x]\times[y,z]=[\min(wy,wz,xy,xz),\max(wy,wz,xy,xz)],$$

$$[w,x]/[y,z]=[w,x][1/z,1/y],\quad y\neq 0, z\neq 0,$$
$$[w,x]/[w,x]=1, w\neq 0, x\neq 0.$$

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