AN ASSESSMENT OF MATHEMATICS TEACHERS' CONTENT KNOWLEDGE IN CIRCLE THEOREMS USING GEOGEBRA THROUGH PROFESSIONAL DEVELOPMENT SESSIONS IN MANYA KROBO SENIOR HIGH SCHOOLS

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ABSTRACT

The study examined ways of promoting mathematics teachers’ use of existing technological tools in classroom in Ghana through the lens of professional development sessions. A professional development workshop on how mathematics teachers could incorporate GeoGebra in their teaching was organized for Manya Krobo Senior High School Mathematics teachers at Lower Manya Krobo Municipality. An action research design using professional development session was adopted for the study. The study used both quantitative (questionnaire and test) and qualitative (interview and observation) methods. The study used a purposive sampling technique to select ten (10) mathematics teachers who fully participated in the workshop. They were engaged in hands-on activities on proofs of circle theorems in Geometry. Quantitative data were analysed manually into frequencies while the qualitative data were analysed thematically. The results indicated that, about three-fifths (60% - 90%) of the mathematics teachers improved upon their content knowledge in circle theorems. Again, it was found out that mathematics teachers developed their construction, explanatory, assessment and exploratory skills. The study therefore concluded that, GeoGebra software could be used to teach circle theorems in senior high schools when teachers are given Professional Development training. This would also help incorporate ICT into mathematics teaching and learning. The study recommended that factors likely to affect the effective use of GeoGebra in teaching and learning circle theorems for example, Low-level ICT education of some teachers to use GeoGebra effectively and inadequate computers and accessories for teaching and learning circle theorems using GeoGebra software in the school should be addressed.

Keywords: Mathematics, Teachers, Circle theorems, GeoGebra, Professional development

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INTRODUCTION

Mathematics is pivotal in both the formal and informal development of a person in all aspects of life. Through professional development sessions, quality education's successes are identified in the Sustainable Development Goal 4 and the African Union Agenda 2063 goal 2. A more significant number of school-going students could not meet the minimum quality standard in Mathematics (geometry), with Sub-Saharan Africa recording the worst. Eighty-four percent (84%) of students' inability to proficiently work in Mathematics depends hardly on the teacher and teaching methodologies (African Union, 2015; United Nations, 2019).

In recent years, most Professional Development Sessions in Ghana are driven by curriculum changes, the introduction of interventions, and teachers' need to adopt technology-driven pedagogies. Furthermore, it is driven by the necessity to reach an extent and renew teacher practice, skills and beliefs. Stimuli for such desire also call for study change, new classrooms, technology, and advances in pedagogy. However, the call to integrate technological interventions into education and the classroom section has yielded little or no support and publications. Interventions introduced in teaching Mathematical concepts remain yet to influence Professional Development Sessions. These sessions are often detached from the classroom and often done with little practical or hands-on activities and the exit of the donor or sponsor (Asare & Nti, 2014; Doig & Groves, 2011). Hohenwarter et al. (2009) assert that the hands-on nature of sessions and peer teaching seemed to appeal most to participants and helped kept teachers focused and motivated to implement new ideas into their classroom practices. Therefore, allowing teachers to develop themselves among colleagues professionally is much more acceptable and effective than experts in Professional Development Sessions.

This gap in the infusion of technological interventions into mathematics teaching and learning motivated the researcher to introduce professional development sessions to bridge the space. Assessing the chief examiners' reports for a decade on students' performance in mathematics, especially in Geometry and Algebra, necessitated the GeoGebra software application. The application in teaching mathematics has proved helpful in developed countries but is less or not in use in the classrooms in developing countries such as Ghana (WAEC, 2012, 2013, 2014; Ghazali & Zakaria, 2011). The GeoGebra software is created to help teachers and students get a better understanding of mathematics concepts by manipulating variables and stimulating teachers to use and assess technology in the visualisation of mathematics by dragging objects around the drawing's plane or using sliders. GeoGebra software makes learning practical, and it helps both teachers and students learn aspects of Mathematics such as circle theorems, trigonometry, and other geometrical shapes. GeoGebra can also zip students' engagement with mathematics if incorporated into the curriculum for Mathematics at all levels of education (Preiner, 2008).

Masri et al. (2016) studied the results of using GeoGebra as a Teaching Strategy in Circle theorems on Malaysian High school students' performance and attitudes towards this teaching methodology. The students showed significant improvement in using GeoGebra software for Circle theorems. This depicted that the application was significant in the study. Several studies affirm that students taught with GeoGebra performed better than their counterparts who did not use the GeoGebra to learn circle theorems. The researchers noticed

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that GeoGebra usage eased the lessons' tension and aroused the lesson's interest and practicality (Mukiri, 2016; Tay, 2018). The researchers were motivated to do this study because of their experience with the GeoGebra software during their Masters' of Education programme at the University of Education, Winneba in 2015.

The researchers were excited about using the software and realised that the software could be used to address the challenges of teaching geometry topics, especially circle theorems. The researchers, therefore, focused on enlightening the teachers on the use of GeoGebra software to learn circle theorems so that the teachers would intend to teach the students for continuity of the software used in the school.

METHODOLOGY

Research Design

The study is action research using a professional development session. According to Somekh and Zeichner (2009), a significant feature of action research is that it lays claim to teachers' professional development. Bills et al. (2006) opine that action research seeks to identify a particular problem in the educational field, especially in the classroom, and suggests possible rectification of the problem by offering suitable interventional strategies and recommendations for use by other educators. The study used both quantitative and qualitative methods (mixed method) to collect data. In this study, researchers combined action research with a mixed method design to address and answer the research questions.

Population

Population refers to collecting individuals, entities, or elements that fit the criteria (broad or narrow) that the researcher has laid out for research participants (Guven, 2012). The collection of components from which the sample is drawn is known as the study population. The study population embraces individuals in the universe who possess specific characteristics. Therefore, the individual units of analysis chosen represent the population towards which the results will be generalized (Rubin & Babbie, 2009). This study entailed mathematics teachers at Government Senior High schools in the Lower Manya Municipality. It includes fifty-five (55) male and ten (10) female teachers, giving a total population of sixty-five (65) Mathematics teachers.

Sampling Technique

Rubin and Babbie (2001) defined sampling as the process of choosing from a much larger population, a group about which a generalized statement is made so that the selected part represents the entire group. In selecting the sample, researchers used the purposive sampling technique. Purposive sampling was used because researchers were interested in information from only mathematics teachers but no other (subject) teacher in the school. Researchers appealed to all the sixteen (16) mathematics teachers in the school at the time the study was conducted, but only ten (10) fully participated in all the two PD sessions. One out of two female mathematics teachers in the department was convinced to join to take care of gender representation. The mathematics teachers for the workshop needed to have a significant ICT background level since subjects were expected to respond to the GeoGebra

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software application. Five (5) Mathematics teachers were randomly picked for the interview after the professional development session lessons including the only female teacher in the department.

Research Instrument

The main instruments utilized for data collection were questionnaires, quizzes for mathematics teachers, an Interview Guide, and Observation. Other information to guide the research was obtained from sources such as the Researchers' workshop material and Lesson Plan.

Piloting of Research Instruments

Piloting of the research instruments is very critical because it serves among other things, to check the clarity of the statements/questions, give feedback on the validity of the statement/questions, and make sure that the data required would answer the research questions. The pilot study was conducted at Akro Senior High Technical School (Akrosectech) in the Lower Manya Krobo Municipality with five (5) Mathematics teachers. The researcher administered a questionnaire to all the five (5) teachers and interviewed two (2) who willingly agreed to be interviewed using the interview guide as a trial. Akrosectech was used for the pilot study because the mathematics teachers have similar characteristics and features in terms of academic qualifications, the activities, and programmes they undertake as the participants in the senior high school for the actual study. The pilot study was done in Akro Senior High Technical School to avoid giving the respondents the foreknowledge about the information required which will lead to pre-determined responses during the actual study (Kusi, 2012). Another reason for pilot testing the research instruments was to make sure that they (instruments) would be able to elicit the needed response from the respondents during the actual study. Researchers also piloted the interview guide on two of the five mathematics teachers who responded to the pilot questionnaire to check for omissions and make additions where necessary as well as find out how appropriate the responses would be in answering research questions 1, 2, and 3. The recorded response of the respondent was played back to the interviewee for clarification and verification of the research questions 1, 2, and 3.

Reliability and validity

Validity according to Creswell (2009) is the degree to which the study conclusions are authentic. Durrheim, et al. (2006) also mentioned that validity demonstrates that a particular research instrument measures what it purports to measure. There are several approaches to ensuring the validity of an instrument. The researchers adopted face and content validity to test the validity of the instruments. To check for the face validity of the instrument, the researchers gave the questionnaire and the interview guide to the supervisor for scrutiny. The structure, language, depth, and items of the instruments were discussed. Comments and suggestions from the supervisor led to some additions and changes in the structure and wording of some items of the instruments. The supervisor checked whether the items and the questions for the questionnaire and interview guide respectively answered the research questions.

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Data Collection Method

To facilitate the process of data collection from mathematics teachers in Manya Krobo SHS, the researchers asked for an introductory letter from the head of the Mathematics Education Department of the University of Education Winneba to the Headmistress of the school. The researchers first filed the introductory letter through the school's general office to the Headmistress and permission was granted for the PDS involving mathematics teachers. The researchers then appealed to the Academic Head and the Mathematics Department Head, of the school and explained to them the purpose of the research. They then asked for their support to do the intended GeoGebra lessons through PDS with the teachers. The researchers agreed with them that two weeks would be used for the teachers. The researcher did the PDS using GeoGebra application software in a computer laboratory setting at Manya Krobo Senior High School. The test was given to the teachers to answer using GeoGebra followed by a questionnaire which was also distributed to the teachers after the PD sessions. The interview was also conducted later for the teachers through phone calls. In all, ten (10) mathematics teachers were fully involved. Some of the mathematics teachers who were present for the first section were not present for the second session due to their busy schedules. The researchers purposefully selected five mathematics teachers for the interview including the female mathematics teacher. The interview was audio recorded and transcribed. It was a phone call interview because the participants could not wait for their turn. The observation was done throughout the PDS for the teachers.

Data Analysis Procedures

In this study, data were collected using questionnaires, interviews, observations, and tests. Data from the instruments yielded both quantitative and qualitative results and were analysed quantitatively and qualitatively.

Laboratory Setting

The workshop was organized at the school's computer laboratory for teachers. GeoGebra software for windows was installed on all the computers in the laboratory during the PDS. GeoGebra is platform-independent and freely available for online installation from www. Geogebra.org/web start or downloadable for local installation at www.geogebra.org. All the desktop computers were connected to a system unit each. The researchers embarked on the PDS when Ghana was affected by the global pandemic COVD-19, so participants had to wear nose masks, wash and sanitize their hands regularly, and practice social distancing at the workshop to prevent the spread of the COVID-19 or corona various. The researchers organized the workshop in two sections for teachers to ensure that the mathematics teachers benefited from the GeoGebra lessons. Videos and pictures were taken for the research work. Table 1 summarizes the schedule of activities during the period that the study was conducted.

Table 1: Schedule of Activities Duration during Professional Development Sessions using GeoGebra

Date	Type of activity	Main Activity	Duration
23/07/2020	Lesson/PDS	Presentation on circle theorems: 1, 2, and 3 using GeoGebra by the researcher.	2 hours

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30/07/2020	Lesson/PDS	Presentation on circle theorems 4, 5 and 6 using GeoGebra by the researcher.	2 hours
30/07/2020	Assessment	Mathematics teachers answered quizzes and questionnaire.	1 hour

Ethical Considerations

An introductory letter from the University of Education, Winneba Mathematics Department, was given to the researcher to research at Manya Krobo SHS. See Appendix A for the letter. This letter was given to the school's Headmistress, and permission was granted to conduct the research in the school and use the school's computer laboratory for teaching the intended GeoGebra lessons. The Academic Head and the mathematics Head of the department accepted verbal permission. The researchers drew the attention of the ICT head of department to this arrangement, and the necessary adjustments were made to run the GeoGebra lessons with the ICT lessons without clashes of timetable. The researcher had the mathematics teachers' consent that pictures and videos would be taken for the research work. A letter to that effect was written by the researcher and signed by the mathematics teachers concerned. See appendix B for the letter. The research participants were teachers from the mathematics department in the school. The confidentiality of the participants was guaranteed because their identity in answering questions was not required. Pseudo names were used to identify them instead of their real names.

Professional Development Sessions

The researchers introduced the GeoGebra application software to the mathematics teachers in the computer laboratory. The researchers explained and demonstrated to the participants how the software would be used to construct circle theorems using a laptop and a projector. In the beginning lesson, the researchers observed that some participants could work well with the software due to their familiarity with the computer and similar software. Others were struggling to use the software for the first time because of their poor background in computing. The researcher guided the participating mathematics teachers in using the software to construct circle theorems from theorem one to six. Throughout the lesson, the researcher walked around, inspected the participants' work, and gave the necessary guidance to those who needed help. The researcher observed that the participants worked with enthusiasm as far as the software was concerned though a few were having challenges. The researcher observed them calling for assistance on how to use the GeoGebra tools.

RESULTS AND DISCUSSION

Background information of respondents

Background information of the respondents was sought through a questionnaire. Mathematics teachers were asked about their gender, age, academic qualification, professional qualification, present teaching class, number of years of teaching experience, whether they have a personal computer, teachers' level of computer skills, and areas of mathematics difficulty. From Table 2, male and female Mathematics teachers were nine (9) and one (1), respectively. This represented a sample of ten Mathematics teachers who fully

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participated in the research work for the two sections. The participants' ages ranged from 28 years to 56 years, with only one participant aged 28 years, five teachers between the ages of 30-39, forming most of the teachers. Two teachers aged 40 and 2 from 50 years and above. All the participating teachers had their first Degree in Mathematics or Mathematics Education. Every teacher taught SHS2 or SHS3. The research was conducted at a time when SHS1 students were not in school because of covid-19. The mathematics teachers’ number of years of teaching experience also ranged from two years to twenty-seven years. They were all permanent mathematics teachers.

Teachers were asked whether they had personal computers to support the GeoGebra work. Four of the participants had laptops, but six did not have laptops or personal computers. The computer laboratory of the school was used for the workshop. Three of the participants were proficient users of Microsoft Word, Excel and spreadsheet. Two participants could use GeoGebra with Microsoft word. Five participants could use Microsoft word only. All the mathematics teachers were knowledgeable in computing which was an essential requirement to use GeoGebra in circle theorems. Lastly, the mathematics teachers were asked to identify some areas or topics that pose problems during teaching and learning. Geometry was one of the areas majority of the teachers' (N=8) had difficulty in teaching. They explained that they do not have adequate resources to teach the abstract nature of Geometric concepts and students’ inability to visualize 3-Dimensional images of geometric figures. Three of the participants had difficulty in teaching both Geometry and Algebra. One participant had difficulty teaching statistics and finally, one participant indicated he had no problem teaching any of the areas listed in mathematics.

Table 2: Demographic Information of Participants

Participant	Gender	Age	Academic Qualification	Professional Qualification	Present Teaching Class	Years of teaching experience	Do you have a personal computer	level of computer skills	Areas of mathematics difficulty
Yaw	M	40	1st degree	B.Ed. Maths	SHS3	9	Yes	Word, Excel, Spreadsheet.	None
Ogyam	M	56	1st degree	B.Ed. Maths	SHS2	27	No	Word	Geometry
Kuuku	M	39	1st degree	B.Ed. Maths	SHS3	11	No	Word	Geometry
Aba	F	32	1st degree	B.Sc Maths	SHS3	10	Yes	Word	Geometry
Kobina	M	35	1st degree	B.Ed. Maths	SHS2	5	Yes	Word, Excel, Spreadsheet	Geometry
Fiifi	M	50	1st degree	B.Ed. Maths	SHS2	20	Yes	Word, Excel,	Geometry

								spreadsheet	
Ato	M	28	1st degree	B.Ed. Maths	SHS3	2	No	Word	statistics
Kwasi	M	40	1st degree	B.Ed. Maths	SHS2	16	No	Word and Excel	Geometry and algebra
Dabo	M	32	1st degree	BSc Maths Education	SHS2	11	No	Word and GeoGebra	Geometry and algebra
Kwao	M	30	1st degree	BSc Maths Education	SHS2	4	No	Word, Excel and GeoGebra	Geometry and algebra

Source: Fieldwork Data (2020).

Mathematics Teachers' Content Knowledge in Circle Theorems through PDS involving GeoGebra

Research question one sought to determine the extent to which the professional development session involving GeoGebra enhanced Mathematics teachers' content knowledge in the circle theorems. Participants ranked their content knowledge enhancement on a scale of 0 to 4 in each of the six theorems explored with GeoGebra. Very low (0), low (1), moderate (2), high (3), and very high (4) were attached to the various degrees of content knowledge acquired by the mathematics teachers. A questionnaire was used to assess Mathematics teachers' content knowledge in GeoGebra. As shown in Table 4.2, the results indicated that the majority (N=6) of the participants ranked their content knowledge in theorem one, "a chord or arc that subtends at the circumference in the same segment of a circle, the angles formed are equal" high. Then, a few (N=2) ranked themselves moderate, one participant ranked himself low while another one also ranked himself very low.

In theorem two, "angles formed by drawing lines from the ends of the diameter of a circle to its circumference form a right angle, i.e., 90°". The majority (N=6) of the participants ranked their content Knowledge high or very high, followed by four (4) participants ranking themselves as moderate or low. Also, in theorem three, "the angle an arc or a chord subtends at the centre of the circle is twice the angle it subtends at the circumference". Six (6) participants ranked their content knowledge high or very high, three (3) ranked moderate and only one (1) ranked very low. Theorem 4 was, "the opposite angles of a cyclic Quadrilateral equals 180° or are supplementary". In the ranking, three (3) out of ten (10) participants ranked very low, low, or moderately high in their content knowledge. In contrast, seven (7) participants ranked high in their content knowledge.

Theorem 5 was, "the angle between a chord and a tangent at the point of tangency is equal to the chord that subtends in the alternate segment". In the ranking, one, out of ten (10) participants was ranked moderately high in their content knowledge, six (6) were ranked high, and three (3) very high in their content knowledge. Lastly, in theorem six, "tangents from an external point to the circle are equal in length and perpendicular to the radius". In the

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ranking, two (2) of them were moderate, five (5) were high and three (3) were very high in their content knowledge. Cumulatively, at least 3/5 representing 60% of the participants ranked themselves high or very high in their content knowledge enhancement in the various circle theorems. This means that the mathematics teachers' content knowledge improved with the aid of the GeoGebra software through PDS.

Table 3: Distribution of Mathematics teachers ranking of their content knowledge in the circle theorems after PDS

Theorem	Statement	0 Very low	1 Low	2 Moderate	3 High	4 Very high
1.	When a chord or arc subtends at the circumference in the same segment of a circle, the angles formed are equal.	1	1	2	6	0
2.	Angles formed by drawing lines from the ends of the diameter of a circle to its circumference form a right angle, i.e. 90^0	0	2	2	4	2
3.	The angle an arc or a chord subtends at the centre of the circle is twice the angle it subtends at the circumference.	1	0	3	3	3
4.	The opposite angles of a cyclic Quadrilateral equal 180^0 or are supplementary.	1	1	1	3	4
5.	The angle between a chord and a tangent at the point of tangency is equal to the chord that subtends in the alternate segment.	0	0	1	6	3
6.	Tangents from an external point to the circle are equal in length and perpendicular to the radius.	0	0	2	5	3

Mathematics teachers marks in the quiz after using Geogebra to do circle theorems

The researchers again gave participants a trial work to use GeoGebra software to answer. After the researchers had marked the scripts, the scores of the participants out of 12 were as follows:

Averagely, six (6) mathematics teachers scored 8-12 marks representing 67%-100%. The rest scored 3-5 marks representing 25%-50%. This means the majority of the teachers were able to answer the questions using GeoGebra. This also means that the mathematics teachers' content knowledge was improved. To complement the quantitative findings on enhancing Mathematics teachers' content knowledge, interviews were conducted with five Mathematics teachers soliciting information about the improvement on their content knowledge. The

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abbreviation MT followed by their pseudo names is the identity of the mathematics teachers' interviewees. Direct quotes from participants are provided and these quotes have been edited as little as possible. The female mathematics teacher was given MT Aba.

Generally, mathematics teachers' interview responses to how GeoGebra software improved their content knowledge through PDS revealed that their visualisation was improved.

Three of the mathematics teachers had this to say:

MT Kobina said *I could visualize the theorems and diagrams clearly and better than how I taught traditionally in the classroom*.

MT Kuuku also said *the circle theorems were easily understood using the GeoGebra software.*

MT Ato said, the visualisation of the theorems, and how they came about, especially the angles, was explicit and exciting.

The above excerpts show that these Mathematics teachers expressed that, visualizing the theorems using the GeoGebra software was clear and well understood. Again, all the participants interviewed expressed that GeoGebra software made the construction of the circle, diameter, segment, and angle correctly. For example, in theorem two, the angle subtended by diameter to the circumference of a circle was verified to be 90° by most of the participants.

MT Aba, in the interview, explained that *"construction of angles using GeoGebra measures the angles accurately and gives clear diagrams and pictures for the circle theorems to be well understood.*

MT Kuuku also said, '*construction of the circle theorems using GeoGebra is clear and accurate. He then said GeoGebra enabled me to see the construction of the angles perfectly done to prove the theorems instead of the abstract way I learned traditionally'*.

MT Kobina confirmed, *"Practising the circle theorems that have been learned already with GeoGebra makes the construction of the theorems real and well understood".*

MT Ogyam said, *in the construction of theorem two, the position of the angles subtended by the diameter; dragging the apex of the angles along the circumference in the same segment remains 90°, but when moved, to a different segment, the angles change to 270°.*

These excerpts from the mathematics teachers indicated how the GeoGebra software constructs clear and accurate diagrams, addressing preconceptions and misconceptions about the circle theorems.

In theorems, five and six, participants were able to think geometrically. The researchers observed that participants could determine a major and a minor sector, segment, and tangents. Participants were able to construct each theorem severally to verify or prove them. Participants were able to drag the tangent to check the angles that the chord made with the tangent and the one made at the alternate side to be the same for theorem five. For theorem six, they dragged the tangent to check the angle between the tangent and the circumference to be 90°. Participants could construct their knowledge about the theorems systematically using GeoGebra with understanding. Participants were motivated to learn with GeoGebra without

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getting tired because the software made the learning self-centered. Mathematics teachers' geometric reasons for using GeoGebra to teach circle theorems are as follows:

MT Kobina indicated that '*construction of two tangents from an external point (theorem six) was perfectly done using GeoGebra showing clearly, how the tangents touched the circle to produce the 90° angles at the intersection of the circle and the tangent to verify the theorem'.*

MT Kuuku also said, "*GeoGebra enabled me to see the reality of angles measured to verify theorem five (the Alternate angle segment), which sometimes becomes difficult to explain to students.*

MT Ato mentioned that *'working with GeoGebra becomes very easy when participants know how to use the tools like a circle with a centre, segment, and tangents to do the circle theorems correctly'.*

MT Aba revealed, *"GeoGebra software is faster in getting answers when the correct inputs and tools are used".*

MT Ogyam said, "*GeoGebra reveals hidden details of the circle theorems. For example, after practicing the alternate angle segment severally, I verified they are the same".*

The above excerpts show that the mathematics teachers experienced the geometric aspect of using GeoGebra software to learn. It was revealed that GeoGebra software showed hidden details of the theorems than marker board illustrations. Some mathematics teachers elaborated that the GeoGebra software enabled them to solve the questions correctly and faster than not using the GeoGebra. The researchers again observed that some of the teachers were sharing ideas in solving the questions using GeoGebra.

Table 4: Mathematics teachers marks in the quiz after using GeoGebra to do circle theorems

Mathematics Teachers (MT)	*Raw score*	*Percentage (%)*
Yaw	*12*	*100*
Ogyam	*3*	*25*
Kuuku	*11*	*92*
Aba	*8*	*67*
Kobina	*10*	*83*
Fiifi	*6*	*50*
Ato	*11*	*92*
Kwasi	*12*	*100*
Dabo	*10*	*83*
Kwao	*4*	*33*

Discussion of Findings

Programmes are critical to upgrading teachers' pedagogical content knowledge and offering them the opportunity to acquire new knowledge, improve competency, enhance teaching skills and gain confidence in teaching their subject areas effectively. The results from this study showed that three-fifths of the participants ranked themselves high or very high in the various circle theorems. This means that 60% - 90% of the participants improved upon their content knowledge. Mwingirwa, (2015), did research on the uses of GeoGebra as technology and evaluated mathematics teachers' views on training and using GeoGebra as a tool to enhance mathematics learning in secondary schools in Kajiodo County in Kenya. After teaching with the mathematics teachers, the results showed that about 70% of the mathematics teachers enhanced their content knowledge using GeoGebra and wanted to use the software in their classroom. This study corroborates Mwingirwa (2015).

The interview results also revealed that most of the mathematics teachers improved upon their content knowledge during PDS. This study supports the findings by Raymond Duval cited in (Jones, 1998), that, geometrical reasoning involves three kinds of cognitive processes that fulfilled a specific epistemological function. These cognitive processes are visualization, construction, and reasoning. At the PDS, participants were able to visualize how the theorems were constructed. It was revealed in the interview that, some of the mathematics teachers formed a better mental picture of each of the theorems constructed and could at any point answer questions correctly. GeoGebra can build a connection between mathematics concepts and the physical representations within their minds, which would motivate them to learn the circle theorems as seen in Wang et al., (2020).

In the interview again, four (4) participants revealed that they developed their content knowledge also through Construction. One of the participants' said, *'construction of the circle theorems using GeoGebra is clear and accurate. He then said GeoGebra enabled him to see the construction of the angles perfectly done to prove the theorems instead of the abstract way he learned traditionally'.*

Even though the study was about teachers, Moore, (2008) emphasized that, in Mathematics teaching and learning, especially in geometry, it is essential for students to imagine, construct, and understand the construction of shapes and circles to connect them with related facts. This means that teachers should be able to help students to do the construction accurately. Software like GeoGebra allows users to build interactive representations of points, lines, and circles. These geometric objects are interactive and can be resized and moved around the screen by clicking and dragging. The use of GeoGebra in teaching and learning can do accurate constructions. Participants were again able to reason or think geometrically using GeoGebra software in learning circle theorems. Participants were able to determine a major and a minor sector, segment, and tangents. In Caglayan, (2016) study on professional development, mathematics teachers were able to use GeoGebra dynamic software to comprehend circle theorems involving arcs, line segments, chords, tangents, and secant lines.

Conclusion

From the findings, it could be concluded that majority of the mathematics teachers were not familiar with GeoGebra software before the researcher organised the workshop. Majority of mathematics teachers said, the GeoGebra software made the lesson more real. They understood how the GeoGebra software worked and had entirely accepted it and practiced it to teach circle theorems. It was exciting and refreshing that majority of the mathematics teachers could confidently use GeoGebra software to construct circle theorems themselves and lead their students to explore these theorems. It can be concluded also that GeoGebra software can be used to teach circle theorems in senior High Schools when teachers are given Professional Development training.

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