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$\hat{\alpha}$ GENERALIZEDCLOSED SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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Abstract

This paper is devoted to the study of intuitionistic fuzzy topological spaces. In this paper intuitionistic fuzzy $\hat{\alpha}$generalized closed sets and intuitionistic fuzzy$\hat{\alpha}$generalized open sets are introduced. We study some of theirbasic properties.

Key words: Intuitionistic fuzzy topology, intuitionistic fuzzy$\hat{\alpha}$ generalized closed sets, intuitionistic fuzzy $\hat{\alpha}$generalized open sets, intuitionistic fuzzy $\hat{\alpha}T_{1/2}$ space and intuitionistic fuzzy $\hat{\alpha}T^{*}_{1/2}$ space.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [12] and laterAtanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. Krishnakumar andsenthilkumaran[8] introduced - $\hat{\alpha}$generalizedclosed sets in topological spaces. In this paper we introduce intuitionistic fuzzy $\hat{\alpha}$generalizedclosed sets andintuitionistic fuzzy $\hat{\alpha}$generalized open sets and study some of their properties.

2. Preliminaries

Definition 2.1: [1] Let X be a non empty fixed set. An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form

$$A = \{ \langle x, \mu_A (x), \nu_A(x) \rangle / x \in X \}$$

where the functions $\mu_A(x): X \to [0,1]$ and $\nu_A(x): X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. We denote the set of all intuitionistic fuzzy sets in X, by IFS (X).

Definition 2.2: [1] Let A and B be IFSs of the form
$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$.
Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
(d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$
(e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

Forthe sake of simplicity, we shall use the notation$A = \langle x, \mu_A, \nu_A \rangle$ instead of$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. Also forthe sake of simplicity, we shall use the notation$A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_{\sim} = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_{\sim} = \{ \langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3:[2] An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $0_{\sim}, 1_{\sim} \in \tau$

(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [2] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

$int(A) = \cup\{ G / G \text{ is an IFOS in X and } G \subseteq A \}$,

$cl(A) = \cap \{ K / K \text{ is an IFCS in X and } A \subseteq K \}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 2.5:[8]A subset A of a space (X, τ) is called:

(i) regular open if A = int (cl(A)).

(ii) π open if A is the union of regular open sets.

Definition 2.6:[4] An IFS A of an IFTS (X, τ) is an *intuitionistic fuzzy semi pre openset* (IFSPOS in short) if there exists an IFPOS B such that $B \subseteq A \subseteq cl(B)$.

Theorem 2.7:[4] An IFS A of an IFTS (X, τ) is an *intuitionistic fuzzy semi pre closedset* (IFSPCS in short) if there exists an IFPCS B such that $int(B) \subseteq A \subseteq B$.

The family of all IFSPCSs (respectively IFSPOSs) of an IFTS (X, τ) is denoted by IFSPC(X) (respectively IFSPO(X)).

Definition 2.8:[9] An IFS A of an IFTS (X, τ) is an

(i) *intuitionistic fuzzy regular openset* (IFROS in short)if A = int(cl(A)),

(ii) *intuitionistic fuzzy regular closedset* (IFRCS in short)if A = cl(int(A)).

Definition 2.9:[9] An IFS A of an IFTS (X, τ) is an *intuitionistic fuzzy generalized closedset* (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

Definition 2.10:[6] Let an IFS A of an IFTS (X, τ). Then the semi closure of A (scl(A) in short) is defined as $scl(A) = \cap\{ K / K \text{ is an IFSCS in X and } A \subseteq K \}$.

Definition 2.11:[6] Let A be an IFS of an IFTS (X, τ). Then the semi interior of A (sint(A) in short) is defined as $sint(A) = \cup\{ K / K \text{ is an IFSOS in X and } K \subseteq A\}$.

Definition 2.12:[7] An IFS A of an IFTS (X, τ) is an *intuitionistic fuzzy generalized semi closedset* (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

Definition 2.12:[7] An IFS A of an IFTS (X, τ) is an *intuitionistic fuzzy π generalized semi closedset* (IFπGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFπOS in X.

Definition 2.13: [7] Every IFπOSisIFOS in (X, τ).

Definition 2.14: [4] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be a

(i) intuitionistic fuzzy semi closed set (IFSCS for short) if $int(cl(A)) \subseteq A$,

(ii) intuitionistic fuzzy pre-closed set (IFPCS for short) if $cl(int(A)) \subseteq A$,

(iii) intuitionistic fuzzy α-closed set (IFαCS for short) if $cl(int(cl(A))) \subseteq A$,

ISSN- 2394-5125 VOL 7, ISSUE 17, 2020

(iv) intuitionistic fuzzy β-closed set (IFβCS for short) if int(cl(int(A))) ⊆ A.
The respective complements of the above IFCSs are called their respective IFOSs.
The family of all IFSCSs, IFPCSs, IFαCSs and IFβCSs (respectively IFSOSs, IFPOSs, IFαOSs and IFβOSs) of an IFTS (X,τ) are respectively denoted by IFSC(X), IFPC(X), IFαC(X) and IFβC(X) (respectively IFSO(X), IFPO(X), IFαO(X) and IFβO(X)).

Definition 2.15: [4] Two IFSs are said to be q-coincident ($A_q B$ in short) if and only ifthere exists anelement x∈X such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.16: [4] Two IFSs are said to be not q-coincident ($A_q^c B$ in short) if and only ifA ⊆B^c.

Theorem2.17: [11] An IFS A = ⟨x, μ_A, ν_A⟩ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy semi-pre closed set(IFSPCS for short) if there exists an IFPCS B such that int(B) ⊆ A ⊆ B

(ii) intuitionistic fuzzy semi-pre open set(IFSPOS for short) if there exists an intuitionistic fuzzy pre open set (IFPOS for short) B such that B ⊆ A ⊆ cl(B).

The family of all IFSPCSs (respectively IFSPOSs) of an IFTS (X,τ) is denoted by IFSPC(X) (respectively IFSPO(X)).Every IFSCS (respectively IFSOS) and every IFPCS (respectively IFPOS) is an IFSPCS (respectively IFSPOS). But the separate converses need not be true in general

3. α̂generalized closed setsin Intuitionistic fuzzy topological spaces

In this section we have introduced the notion of intuitionistic fuzzy α̂generalized closed sets and studied some of their basic properties. Also we have provided the relation between intuitionistic fuzzy α̂generalized semi-pre closed sets with other intuitionistic fuzzy sets.

Definition 3.1: An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy α̂generalized semi-pre closed set (IFα̂GCS for short) if int (cl (int(A)))⊆ U whenever A ⊆ U and U is an IFOS in (X, τ).The family of all IFα̂GCSs of an IFTS (X, τ) is denoted by IFα̂GC(X).

Example 3.2: Let X = {a, b} and G = ⟨x, (0.4, 0.3), (0.4, 0.5)⟩. Thenτ = {$0_\sim$, G, $1_\sim$} is an IFT on X and the IFS A = ⟨x, (0.3, 0.1), (0.5, 0.6)⟩ is an IFα̂GCS in (X, τ).

Theorem 3.3: Every IFCS is an IFα̂GCS but not conversely.
Proof: Let A be an IFCS in X and let A ⊆ U and U is an IFOS in (X, τ). Since int (cl (int(A)))⊆ cl(A) and A is an IFCS in X, int (cl (int(A)))⊆ cl(A) = A ⊆ U. Therefore A is an IFα̂GCSin X.

Example 3.4: Let X = { a, b } and let τ = {$0_\sim$, G, $1_\sim$} is an IFT on X, where G = ⟨ x, (0.2, 0.2), (0.7, 0.5) ⟩. Then the IFS A = G is an IFα̂GCS but not an IFCS in X.

Theorem 3.5: Every IFGCS is an IFα̂GCS but not conversely.
Proof: Let A is an IFGCS in X and let A ⊆ U and U is an IFOS in (X, τ). int (cl (int(A)))⊆ cl(A) ⊆ U by hypothesis, A is an IFα̂GCSS in X.

Example 3.6: Let X = { a, b } and let τ = {$0_\sim$, G, $1_\sim$} be an IFT on X, where G = ⟨ x, (0.4, 0.3), (0.5, 0.6) ⟩. Then the IFS A = G is an IFα̂GCS but not an IFGCS in X since A ⊆ T but cl(A) = ⟨ x, (0.5, 0.6), (0.4, 0.3) ⟩⊃ G.

Theorem 3.7: Every IFSPCS is an IFα̂GCS but not conversely.

Proof: Let A be an IFSPCS and A $\subseteq$U and U is an IFπOS in (X, τ). Since int (cl (int(A)))= A and A$\subseteq$ U, we have int (cl (int(A)))$\subseteq$ U. Therefore, A is an IFα̂GCS.

Example 3.8: Let X = {a, b} and let τ = {0~, G, 1~} where G = ⟨ x, (0.6, 0.7), (0.3, 0.2) ⟩.Then the IFS A = ⟨ x, (0.7, 0.8), (0.1, 0.2) ⟩is an IFα̂GCS in (X, τ) but not an IFSPCS in (X, τ).

Theorem 3.9: Every IFβCS is an IFα̂GCS but not conversely.
Proof: Let A be an IFβCS and A $\subseteq$ U, U is an IFOS in (X, τ). Since βcl(A) = A and A $\subseteq$ U, we have βcl(A) $\subseteq$ U. Hence A is an IFα̂GCS.

In Example 3.8, the IFS A = ⟨ x, (0.7, 0.8), (0.1, 0.2) ⟩is an IFα̂GCS but not an IFβCS in (X, τ).

Proposition 3.10: Every IFSCS is an IFα̂GCS but not conversely.
Proof: Let A be an IFSCS in X. Since every IFSCS is an IFSPCS, by Theorem 3.7, A is an IFα̂GCS in X.

Example 3.11: Let X = { a, b } and let τ = {0~, G, 1~}, where G = ⟨ x, (0.2, 0.3), (0.5, 0.7) ⟩.Then the IFS A = ⟨x, (0.1, 0.2), (0.6, 0.8) ⟩ is an IFα̂GCS but not an IFSCS in (X, τ).

Proposition 3.12: Every IFPCS is an IFα̂GCS but not conversely.
Proof: Since every IFPCS is an IFSPCS, the proof is obvious from Theorem 3.7.

Example 3.13: Let X = {a, b} and let τ = {0~, G_1, G_2, 1~} where G_1 = ⟨x, (0.4, 0.5), (0.4, 0.3)⟩ and G_2 = ⟨x, (0.3, 0.1), (0.5, 0.6)⟩, then the IFS A = ⟨x, (0.3, 0.1), (0.5, 0.6)⟩ is an IFα̂GCSin (X, τ) but not an IFPCS in (X, τ).

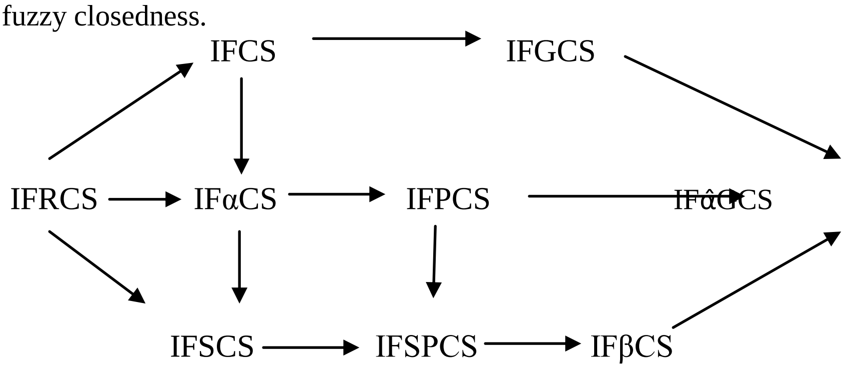
Proposition 3.14: Every IFαCS is an IFα̂GCS but not conversely.
Proof: Let A be an IFαCS. Since every IFαCS is an IFSPCS, by Theorem 3.7, A is an IFα̂GCS.

Example 3.15: Let X = { a, b } and let τ = {0~, G, 1~} be an IFT on X, where G = ⟨ x, (0.7, 0.8), (0.3, 0.1) ⟩. Then the IFS A = ⟨ x, (0.5, 0.4), (0.2, 0.3) ⟩ is anIFα̂GCS but not an IFαCS in X since A $\subseteq$ G but cl(int(cl(A))) = 1~$\supset$ G.

Proposition3.16: Every IFRCS is an IFα̂GCS but not conversely.
Proof: Let A be an IFRCS in X. By definition A = cl(int(A)). This implies cl(A) = cl(int(A)). Therefore, cl(A) = A. Therefore, A is an IFCS in X. By theorem 3.3, A is an IFα̂GCS in X.

Example 3.17: Let X = { a, b } and let τ = {0~, G, 1~} be an IFT on X, where G =⟨ x, (0.3, 0.4), (0.5, 0.6) ⟩. The IFS A = T is an IFα̂GCS but not an IFRCS in X since cl(int(A)) = ⟨ x, (0.3, 0.4), (0.5, 0.6) ⟩$\neq$ A.

In the following diagram, we have provided relations between various types of intuitionistic fuzzy closedness.



ISSN- 2394-5125 VOL 7, ISSUE 17, 2020

The reverse implications are not true in general in the above diagram.

Remark 3.18: The intersection of two IF$\hat{\alpha}$GCSs is not an IF$\hat{\alpha}$GCS in general as seen in the following example.

Example 3.19: Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G_1, G_2, G_3, G_4, 1_\sim\}$ where $G_1 = \langle x, (0.7, 0.8), (0.3, 0.2)\rangle$, $G_2 = \langle x, (0.2, 0.1), (0.8, 0.9)\rangle$, $G_3 = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$ and $G_4 = \langle x, (0.6, 0.7), (0.4, 0.3)\rangle$, then the IFSs $A = \langle x, (0.6, 0.7), (0.4, 0.2)\rangle$ and $B = \langle x, (0.7, 0.6), (0.3, 0.4)\rangle$ are IF$\hat{\alpha}$GCSs in (X, τ) but $A \cap B$ is not an IF$\hat{\alpha}$GCS in (X, τ). Since $A \cap B = \langle x, (0.6, 0.6), (0.4, 0.4)\rangle \subseteq G_4$ but $int(cl(int(A \cap B))) = G_1 \subseteq G_4$.

Theorem 3.20: Let (X, τ) be an IFTS. Then for every $A \in$ IF$\hat{\alpha}$GC(X) and for every $B \in$ IFS(X), $A \subseteq B \subseteq int(cl(int(A))) \Rightarrow B \in$ IF$\hat{\alpha}$GC(X).

Proof: Let $B \subseteq U$ and U be an IFOS. Then since $A \subseteq B$, $A \subseteq U$. By hypothesis, $B \subseteq int(cl(int(A)))$. Therefore $int(cl(int(B))) \subseteq int(cl(int((int(cl(int(A)))) = int(cl(int(A))) \subseteq U$, since A is an IF$\hat{\alpha}$GCS. Hence $B \in$ IF$\hat{\alpha}$GC(X).

Theorem 3.21: An IFS A of an IFTS (X, τ) is an IF$\hat{\alpha}$GCS if and only if $A\ {}_q^c\ F \Rightarrow int(cl(int(A)))\ {}_q^c\ F$ for every IFCS F of X.

Proof:Necessity: Let F be an IFCS and $A\ {}_q^c\ F$, then by Definition 2.16, $A \subseteq F^c$, where F^c is an IFOS. Then $int(cl(int(A))) \subseteq F^c$, by hypothesis. Hence again by Definition 2.16, $int(cl(int(A)))\ {}_q^c\ F$.

Sufficiency: Let U be an IFOS such that $A \subseteq U$. Then U^c is an IFCS and $A \subseteq (U^c)^c$. By hypothesis, $A\ {}_q^c U^c \Rightarrow int(cl(int(A)))\ {}_q^c U^c$. Hence by Definition 2.16 ,$int(cl(int(A))) \subseteq (U^c)^c = U$. Therefore $int(cl(int(A))) \subseteq U$. Hence A is an IF$\hat{\alpha}$GCS.

Theorem 3.22: Let (X, τ) be an IFTS. Then every IFS in (X, τ) is an IF$\hat{\alpha}$GCSS if and only if IFSPO(X) = IFSPC(X).

Proof: Necessity: Suppose that every IFS in (X, τ) is an IF$\hat{\alpha}$GCS. Let $U \in$ IFO(X), then $U \in$ IFSPO(X) and by hypothesis, $int(cl(int(U))) \subseteq U \subseteq int(cl(int(U)))$. This implies $int(cl(int(U))) = U$. Therefore $U \in$ IFSPC(X). Hence IFSPO(X) $\subseteq$ IFSPC(X). Let $A \in$ IFSPC(X), then $A^c \in$ IFSPO(X) $\subseteq$ IFSPC(X). That is $A^c \in$ IFSPC(X). Therefore $A \in$ IFSPO(X). Hence IFSPC(X) $\subseteq$ IFSPO(X). Thus IFSPO(X) = IFSPC(X).

Sufficiency: Suppose that IFSPO(X) = IFSPC(X). Let $A \subseteq U$ and U be an IFOS. Then $U \in$ IFSPO(X) and $int(cl(int(A))) \subseteq int(cl(int(U))) = U$, since $U \in$ IFSPC(X), by hypothesis. Therefore A is an IF$\hat{\alpha}$GCS in X.

Theorem 3.23: If A is an IFOS and an IF$\hat{\alpha}$GCS in (X, τ), then A is an IFSPCS in (X, τ).

Proof: Since $A \subseteq A$ and A is an IFOS, by hypothesis, $int(cl(int(A))) \subseteq A$. But $A \subseteq int(cl(int(A)))$. Therefore $int(cl(int(A))) = A$. Hence A is an IFSPCS.

Theorem 3.24: Let A be an IF$\hat{\alpha}$GCS in (X, τ) and $c(\alpha, \beta)$ be an IFP in X such that $c(\alpha, \beta)\ {}_q int(cl(int(A)))$, then $cl(c(\alpha, \beta))\ {}_q\ A$.

Proof: Let A be an IF$\hat{\alpha}$GCS and let $c(\alpha, \beta)\ {}_q\ spcl(A)$. If $cl(c(\alpha, \beta))\ {}_q^c\ A$, then by Definition 2.16, $A \subseteq [cl(c(\alpha, \beta))]^c$ where $[cl(c(\alpha, \beta))]^c$ is an IFOS. Then by hypothesis,$int(cl(int(A))) \subseteq [cl(c(\alpha, \beta))]^c \subseteq (c(\alpha, \beta))^c$. Therefore by Definition 2.16, $c(\alpha, \beta)\ {}_q^c spcl(A)$, which is a contradiction to the hypothesis. Hence $cl(c(\alpha, \beta))\ {}_q\ A$.

Theorem 3.25: For an IFS A, the following conditions are equivalent.

(i) A is an IFOS and an IF$\hat{\alpha}$GCS

(ii) A is an IFROS

Proof: (i) $\Rightarrow$ (ii) Let A be an IFOS and an IF$\hat{\alpha}$GCS. Then $int(cl(int(A))) \subseteq A$. This implies that $int(cl(int(A))) \subseteq A$. Since A is an IFOS, $int(A) = A$. Therefore $int(cl(A)) \subseteq A$. Since A is an

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IFOS, it is an IFPOS. Hence A $\subseteq$ int(cl(A)). Therefore A = int(cl(A)). Hence A is an IFROS.
(ii) $\Rightarrow$ (i) Let A be an IFROS. Therefore A = int(cl(A)). Since every IFROS in an IFOS, A is an IFOS and A $\subseteq$A. This implies int(cl(A)) $\subseteq$ A. That is int(cl(int(A))) $\subseteq$ A. Therefore A is an IFβCS. Hence by Theorem 3.9, A is an IF$\hat{\alpha}$GCS.

Theorem 3.26: Let F $\subseteq$ A $\subseteq$ X where A is an IFOS and an IF$\hat{\alpha}$GCSS in X. Then F is an IF$\hat{\alpha}$GCS in A if and only if F is an IF$\hat{\alpha}$GCS in X.
Proof: Necessity: Let U be an IFOS in X and F $\subseteq$ U. Also let F be an IF$\hat{\alpha}$GCS in A. Then clearly F $\subseteq$ A $\cap$ U and A $\cap$ U is an IFOS in A. Hence the semi-pre closure of F in A, $spcl_A(F)$ $\subseteq$ A $\cap$ U. By Theorem 3.23, A is an IFSPCS. Therefore spcl(A) = A and the semi-pre closure of F in X, spcl(F) $\subseteq$spcl(F) $\cap$spcl(A) = spcl(F) $\cap$ A = $spcl_A(F)$ $\subseteq$A $\cap$ U $\subseteq$U. That is spcl(F) $\subseteq$ U whenever F $\subseteq$ U. Hence F is an IF$\hat{\alpha}$GCS in X.
Sufficiency: Let V be an IFOS in A such that F $\subseteq$ V. Since A is an IFOS in X, V is an IFOS in X. Therefore spcl(F) $\subseteq$ V, since F is an IF$\hat{\alpha}$GCS in X. Thus $spcl_A(F)$ = spcl(F) $\cap$ A $\subseteq$ V $\cap$ A $\subseteq$ V. Hence F is an IF$\hat{\alpha}$GCS in A.

Theorem 3.27: Let (X, τ) be an IFTS. Then for every A $\in$ IFSPC(X) and for every IFS B in X, int(A) $\subseteq$ B $\subseteq$ A $\Rightarrow$ B $\in$IF$\hat{\alpha}$GC(X).
Proof: Let A be an IFSPCS in X. Then by Definition 2.17, there exists an IFPCS, say C such that int(C) $\subseteq$ A $\subseteq$ C. By hypothesis, B $\subseteq$ A. Therefore B $\subseteq$ C. Since int(C) $\subseteq$ A, int(C) $\subseteq$ int(A) and int(C) $\subseteq$ B. Thus int(C) $\subseteq$ B $\subseteq$ C and by Definition 2.17, B $\in$ IFSPC(X). Hence by Theorem 3.7, B$\in$IF$\hat{\alpha}$GC(X).

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