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Performance of elevated isolated liquid storage tank under unidirectional excitation

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Abstract: - Analysis of hydrodynamic structure such as elevated concrete water tank is quite complicated when compared with other structures. As well as dynamic fluid-structure interaction (FSI) plays an important effects in this complexity for which research suggests solution by using different methods. This paper presents the dynamic behaviour of elevated concrete water tank with three degree freedom modelling of liquid i.e. convective, impulsive and rigid mass. Three models of varying capacity with varying staging height are analysed with fixed base and corresponding responses are reduced by applying two types of base isolator. Base isolator used are LRB and Lead LRB. Imperial Valley 1940 El-Centro Earthquake ground motion has been used for generating the various response of the tank. The base shear, displacement of convective mass, displacement of impulsive mass, displacement of base has been calculated for non-isolated tank and compared with non-isolated tank.

***Keywords*— convective, impulsive and rigid mass,LRBand Lead LRB,Imperial Valley 1940 El-Centro Earthquake**

I. Introduction

The elevated liquid storage steel tanks are inherently horizontally flexible their failure against recent devastating earthquakes has attracted considerable attention of researchers [1]. The tanks are very important structure has a multiple uses in industries, nuclear power plants and also other activities connected to public life. The tanks, mostly available in a wide range of capacities, from small to very big size. The earthquake motion excites the liquid contained in the tank. The liquid moves with excitation has been categories into three parts first one is that the amount of liquid which moves independent of the tank wall motion, which is termed, as convective or sloshing while, another part of the liquid, which moves in unison with rigid tank walls, is known as impulsive mass. If the flexibility of the tank wall is considered than the part of the impulsive mass move independently while remaining accelerates back and forth with tank wall known as rigid mass. The accelerating liquid as sloshing, impulsive and rigid masses induces substantial hydrodynamic pressures on the tank wall of liquid storage tanks which in turn generates design forces such as base shear and overturning moment. The base shear is important from insulator design viewpoint, while overturning movement produces significant tower deformation and toppling of the same leads to failure of the tank. The failure of the liquid storage steel tanks is mainly due to buckling of the tank wall, toppling of the tower structure, failure of piping system and uplift of the anchorage system.

In the this chapter, the earthquake response of elevated liquid storage tanks isolated by the elastomeric bearings with and without elastomeric bearing is investigated using recorded earthquake ground motions applied along the X - direction.

The specific objectives of the study are:

(i) To investigate the seismic response of the isolated R.C.C. Elevated liquid storage tanks considering the influence of various capacities of tank, tower and isolation parameters and compare with corresponding response of non-isolated tank to identify the effectiveness of the isolation system.

(ii) To evaluate the influence of isolation on time period and damping of the isolator bearing.

iii) Compare the peak response of liquid storage tank isolated by the elastomeric bearing.

I) Mathematical Modelling Of R.C.C.Elevated Liquid Storage Tank [4]

A structural model of R. C. C. elevated liquid storage tank, which is fixed to the ground, (as shown in Fig. 1 (a)). Supported by R.C.C. beam column framed structure. The continuous liquid mass is lumped as sloshing mass (convective mass), impulsive mass and rigid masses and referred to as m_c,m_i and m_r, respectively. The sloshing and impulsive masses are connected to the tank wall by corresponding equivalent springs having stiffness constants k_c and k_i, respectively. The damping constants of the sloshing and impulsive masses are c_c and c_i, respectively. The tank has three-degrees-of-freedom under uni-directional excitation as u_c, u_i and u_t which denote the absolute displacements of sloshing mass, impulsive mass and tower drift, respectively. The R.C.C.elevated liquid storage tank is isolated by placing the bearings between the base of the frame (column) structure and foundation (shown in Fig. 2.1 (b))

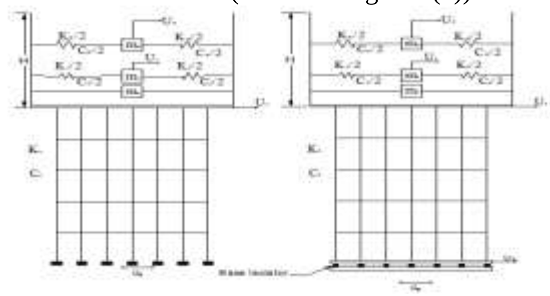

Fig. 2.1. Structural model of elevated liquid storage R.C.C. tank: (a) non-isolated, (b) with isolation at Bottom of column (isolated model-I)

Due to isolation provided at the bottom of the tank which creates the additional degree of freedom, forming the deformation in the isolator which is denoted by U_b. The isolation system considered for the present study is laminated rubber bearings with and without lead with alternate layers of steel and rubber. The presence of steel plates in the isolation bearings makes it rigid in the vertical direction. The force-deformation behaviour of the bearings is considered to be

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linear with viscous damping for LRB and nonlinear for LRB with lead core. The self-weight of R.C.C.framed staging has been calculated. [10].

II) Slender tanks [11][12]

For a cylindrical storage tank of radius R, containing an incompressible liquid of (ρ_w) mass density of the liquid. Filled to depth H, the total mass of liquid is given by

$$m = \pi R^2 H \rho w \quad (1)$$

In the case of tall (slender) tanks, the generation of liquid velocity relative to the tank is restricted to a height of 1.5R from the free surface. Slender tanks are those, whose height to radius ratio is greater than 1.5. The mathematical model corresponding to this case is shown in Fig. 3. The case (i.e. height to radius ratio of 1.5) can be visualized as a fixed rigid membrane separating the tank into two regions. An additional mass known to be rigid mass is used to represent the constrained liquid.

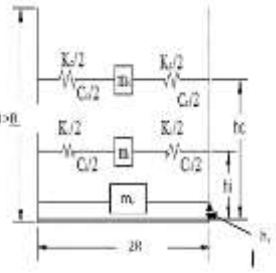

Fig. 3: Slender Tank [4]

This is given by,

$$m_i = \rho_w (1.5R^3)\pi(0.7095) \quad (08)$$

$$m_C = m(0.318).\left(\frac{R}{H}\right).\tanh\left(1.84\left(\frac{H}{R}\right)\right) \quad (09)$$

$$m_r = \rho_w (H - 1.5R)\pi(R^2) \quad (10)$$

The heights are given by,

$$h_i = \frac{3}{8}.(1.5R) + (H - 1.5R) \quad (11)$$

$$h_c = H\left[1 - \frac{\cosh\left(\frac{1.84H}{R}\right) - 1}{1.84\left(\frac{H}{R}\right).\sinh\left(1.84\left(\frac{H}{R}\right)\right)}\right] \quad (13)$$

$$h_r = \frac{H - 1.5R}{2} \quad (14)$$

The period, T, of the oscillating liquid is given by:

$$T_C = \frac{2\pi}{\omega} \quad (15)$$

The value of the natural circular frequency is given by

$$\omega_C^2 = \frac{1.84g}{R}\tanh\left(1.84\frac{H}{R}\right) \quad (16)$$

Time period of impulsive mode,$T_i$in seconds, is given by

$$T_i = 2\pi\sqrt{\frac{m_i + m_s}{k_t}} \quad (17)$$

Where m_s = mass of container and one-third mass of staging, andk_t = lateral stiffness of staging.

The equivalent stiffness and damping of the sloshing and impulsive masses are expressed as

$$k_c = m_c\omega_c^2 \quad (18)$$

$$k_i = m_i\omega_i^2 \quad (19)$$

$$C_c = 2\xi_c m_c \omega_C \quad (20)$$

$$C_i = 2\xi_i m_i \omega_i \quad (21)$$

Where ξ_c and $\xi_i$are damping ratio of sloshing mass and impulsive mass, respectively.

Damping in the convective mode for all types of liquids and for all types of tanks is taken as 0.5of the critical and damping in the impulsive mode is taken as 5% of the critical.

(III) Equations of Motion [13]

The equations of motion for R.C.C.elevated liquid storage tank (non-isolated) subjected to uni-directional earthquake ground motion (along x- direction) are expressed in the matrix form as

$$[m]\{\ddot{x}\} + [C]\{\dot{x}\} + [k]\{x\} = -[m]\{r\}\ddot{u}g \quad (22)$$

Where {x} is the displacement vector; [m], [c] and [k] are the mass, damping and stiffness matrix of the system, respectively; {r} is the influence coefficient vector; and $\ddot{u}_g$is the earthquake acceleration. The displacement vector for non-isolated tank is given by $\{x\} = \{x_c, x_i, x_t\}^t$; $x_c = u_c - u_t$is the relative displacement of the sloshing mass, $x_i = u_i - u_t$ is the displacement of the impulsivemass; $x_s = u_t - u_g$ is the tower displacement relative to ground (i.e. tower drift). The matrices [m], [c], [k] and the vector {r} for non-isolated tank are expressed as.

$$\begin{bmatrix} m_c & 0 & m_c \\ 0 & m_i & m_i \\ m_c & m_i & M + m_b \end{bmatrix}\begin{Bmatrix} \ddot{x}_c \\ \ddot{x}_i \\ \ddot{x}_t \end{Bmatrix} + \begin{bmatrix} C_c & 0 & 0 \\ 0 & C_i & 0 \\ 0 & 0 & C_t \end{bmatrix}\begin{Bmatrix} \dot{x}_c \\ \dot{x}_i \\ \dot{x}_t \end{Bmatrix} + \begin{bmatrix} k_c & 0 & 0 \\ 0 & k_i & 0 \\ 0 & 0 & k_t \end{bmatrix}\begin{Bmatrix} \dot{x}_c \\ \dot{x}_i \\ \dot{x}_t \end{Bmatrix} = -\begin{bmatrix} m_c & 0 & m_c \\ 0 & m_i & m_i \\ m_c & m_i & M + m_b \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}\ddot{u}_g \quad (23)$$

Where $M = (m_c + m_i + m_r)$ -for slender tank is the effective mass of the tank and m_b is mass of base slab.

The stiffness, $k_t$and damping, c_t of the tower structure are based on the assumption of equivalent single degree-of-freedom system which are defined as

$$k_t = \left(\frac{2\pi}{T_t}\right)^2 (M + m_b) \quad (24)$$

$$c_t = 2\xi_t\omega_t(M + m_b) \quad (24)$$

WhereT_t isa time period of the tower structure which is assumed to be 0.5sec. While damping ratio is assumed to be 5% of critical.

(IV) Force Deformation of Laminated Rubber Bearing [14]

The lead rubber bearing consider here consist of alternate layer of rubber and steel sheets without lead core. The bearing are vertically rigid and have horizontal initial stiffness, k_b and viscous damping c_b respectively. The vertical rigidity is derived from the steel plates while parallel layers of the rubber provide the horizontal flexibility.

Since the damping level in the isolator, supporting steel tower and vibrating liquid is very different, the equations of motion of the isolated elevated liquid storage tank cannot be solved using classical modal superposition technique. Alternatively, the equations of motion are solved in the incremental form using Newmark's step-by step method assuming linear variation of acceleration over small time interval, Δt. The equation of motion in incremental form are written as

$$[m][\ddot{x}] + [c]\{\dot{x}\} + [k]\{x\} + \Delta F = -[m]\{r\}\ddot{u}_g \quad (25)$$

And expressed as below for model no-I

$$\begin{bmatrix} m_c & 0 & m_c & m_c \\ 0 & m_i & m_i & m_i \\ m_C & m_i & M+m_b & M+m_b \\ m_C & m_i & M+m_b & M+3m_b \end{bmatrix}\begin{Bmatrix} \ddot{x}_c \\ \ddot{x}_i \\ \ddot{x}_t \\ \ddot{x}_b \end{Bmatrix} + \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_i & 0 & 0 \\ 0 & 0 & k_t & 0 \\ 0 & 0 & 0 & k_b \end{bmatrix}\begin{Bmatrix} \dot{x}_c \\ \dot{x}_i \\ \dot{x}_t \\ \dot{x}_b \end{Bmatrix}$$

$$+\begin{bmatrix} C_c & 0 & 0 & 0 \\ 0 & C_i & 0 & 0 \\ 0 & 0 & C_t & 0 \\ 0 & 0 & 0 & C_b \end{bmatrix} + \Delta F$$

$$= -\begin{bmatrix} m_c & 0 & m_c & m_c \\ 0 & m_i & m_i & m_i \\ m_i & m_i & M+m_b & M+m_b \\ m_C & m_i & M+m_b & M+3m_b \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix}\ddot{u}_g \quad (26)$$

The displacement vector for the isolated model is expressed as $\{x\} = \{x_c, x_i, x_t, x_b\}^T$; $x_c = u_c - u_t$ is therelative displacement of the sloshing mass, $x_i = u_i - u_t$is the relative displacement of the impulsive mass; $x_t = u_t - u_b$is the towerdisplacement (i.e. tower drift) and $x_b = u_b - u_g$ is the relative bearing displacement.$\Delta F$is the incremental restoring force vector of the elastomeric bearing which is given by (for LRB)

$$F = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ (1-\alpha)\Delta z f_y \end{Bmatrix} \quad (28)$$

Fallowing the assumption of linear variation of acceleration over the time interval Δt.

Stiffness k_b and damping C_b of isolation are defined as Δt incremental vectors $\{\Delta\ddot{x}\}$ and$\{\Delta\dot{x}\}$ are given by

$$\{\Delta\ddot{x}\} = a_0\{\Delta x\} + a_1\{\dot{x}\}^t + a_2\{\ddot{x}\}^t \quad (29)$$

$$\{\Delta\dot{x}\} = b_0\{\Delta x\} + b_1\{\dot{x}\}^t + b_2\{\ddot{x}\}^t \quad (30)$$

Where$a_0 = \frac{6}{\Delta t^2}$;$a_1 = \frac{6}{\Delta t}$;$a_2 = -3$;$b_0 = \frac{3}{\Delta t}$; $b_1 = -3$; $b_2 = -\frac{\Delta t}{2}$thesubscrip't' denotes the time.

Substituting the values from equation (29) and (30) in the incremental form of equation of motion, the equation of motion can be written as.

$$\left[a_0[m] + b_0[c] + [k]\right]\{\Delta x\} = \Big([m]\{r\}\ddot{u}_g - [m](a_1\{\dot{x}\}^t + a_2\{\ddot{x}\}^t) - [c](b_1\{\dot{x}\}^t + b_2\{\ddot{x}\}^t)\Big) - (\Delta F) \quad (31)$$

After solving for incremental displacement vector from equation (31) the incremental velocity vector is obtained from the equation (30). The restoring force vector at time t+Δt is obtained by

$$\{F\}^{t+\Delta t} = \{F\}^t + \{\Delta F\} \quad (32)$$

In order to estimate the force vector $\{\Delta F\}$ the incremental hysteretic displacement, ΔZ is determined by incremental solution techniques using equation (33) with the help of third order RungaKutta method. The step by step computational procedure is

$$\frac{dZ}{dt} = -\beta|\dot{x}_b||z||z|^{n-1} - \tau\dot{x}_b|z|^n + A\dot{x}_b \quad (33)$$

Where, β, τ, n and A are dimensionless parameters. The force deformation behavior of the elastomeric bearing can be modeled by properly selecting the parameters Q_y, q, α, β, τ, n and A.

. Assume an initial value of $\Delta Z = 0$ at first iteration value j=1

(i) Substitute the value ΔZ in equation(27) and then putting the value of ΔF in equation (31) to obtain value of Δx

(ii) From equation (30) the velocity vector is determined at time step t+ Δt

(iii) Compute the revised value of ΔZ using the third order Runga Kutta method that is expressed as

$$\Delta Z = \frac{K_0 + 3K_2}{4} \quad (34)$$

Where,K_0, K_2 are Runga Kutta constants.

Repeat Step (ii) & (iv) and iterate till following convergence criteria is satisfied.

$$\frac{\Delta Z_{j+1} - \Delta Z_j}{Z_m} \le tolerence \quad (35)$$

In which subscript j denotes the iteration number;Z_m is the maximum value of hysteretic displacement of the bearing is expressed by

$$Z_m = \sqrt{\frac{A}{\beta + \tau}} \quad (36)$$

$$k_b = \left(\frac{2\pi}{T_b}\right)^2 \left(M + m_{Staging} + m_b\right) \quad (37)$$

$$c_b = 2\xi_b\omega_b\left(M + m_{Staging} + m_b\right) \quad (38)$$

Where T_b is the time period of isolation system which is assumed to be 2sec.and ξ_b damping ratio of isolation system which is assumed to be 0.1.After obtaining the acceleration vector, the base shear is computed, which is directly proportional to earthquake forces transmitted to the tank is expressed as

$$F_b = m_c\ddot{u}_c + m_i\ddot{u}_i + (m_r + m_b)\ddot{u}_t$$
$$\text{(For non − isolated slender tank model)} \quad (40)$$

$$F_b = m_c\ddot{u}_c + m_i\ddot{u}_i + (m_r + m_b)\ddot{u}_t + 2(m_b)\ddot{u}_b$$
$$\text{(For isolated slender tank model − I)} \quad (42)$$

(V)Earthquake Ground Motion

The tank wall is considered of R.C.C with modulus of elasticity, E = 22.36x10^6 (Kn/m^2) and mass density = 2500 kg/m^3. The earthquake response of isolated tank is investigated for50KL, 100KL, 1000KL broad as well as slender tank with varying staging height from 5m, 15m, and 30m. For the present study, the components S90W of Imperial Valley (1940)earthquake ground motion,is used to investigate the response of the tank.The peak ground accelerations of the Imperial Valley (1940) is 0.214.

VI) Numerical Study& conclusion

The tanks considered here are cylindrical in shape of capacities 50KL, 100KL, 1000KL slender tank with varying staging height from 5m, 15m, and 30m.the Properties of tanks are given in table, I.also effective masses and stiffness are computed using Housner's method [4].The earthquake response of isolated tank is investigated for tank model where entire structure is mounted on LRB.The response quantities of interest are base shear (measure at bottom of the foundation level $\frac{F_b}{W}$,(W=Mg),sloshing displacement(xc),impulsive displacement(xi),tower displacement.

The table II&III and fig.1(a,b,c)shows the comparative study of the peak response of isolated elevated slender tanks against corresponding non-isolated tanks for a period of tower structure T_t= 0.5 sec. The isolation parameters considered are T_b=2sec $\xi_b = 0.1$ the tables shows that

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percentage reduction in base shear as a result of isolation, at T_b=2sec,are (83%),(89%),(93%),(90%),(97%),(71%),(84%),(83%),(45%), (74%),(91%),(91%),(86%),(77%),(84%),(84%),(85%), (86%)

Under Imperial Valley (1940) Earthquake. The above results indicates that isolation reduces the base shear component significantly. The comparative results indicates that reduction in base shear is slightly more in tank with stiff framed structure. The percentage reduction in tower displacement for slender tank are as given in Table –II the result indicates that tower drift is reduced due to isolation, significantly and this is more pronounced when tower structure is comparatively rigid.it is also observed that increased flexibility of tower structure transmits less earthquake forces but it increases tower drift significantly. This implies that for stiff tower structures the reduction in the response achieved is more in comparisons to flexible tower structures. The peak sloshing displacement is increased significantly in all the tanks.

Table (I) Properties of tanks (Slender Tanks)

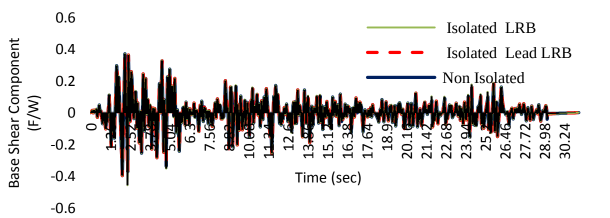
Capacity (KL)	$S = \frac{H}{R}$	Height of container 'H' (m)	Wall Thickness 't' (m)	Base slab thickness (m)	Height of staging'- (m)
50	1.85	3.774	0.17	0.25	5,15,30
100	1.85	4.773	0.20	0.32	5,15,30
1000	1.85	10.286	0.42	0.8	5,15,30

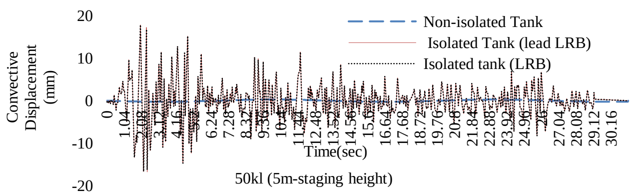
Table II Peak response of elevated Tank

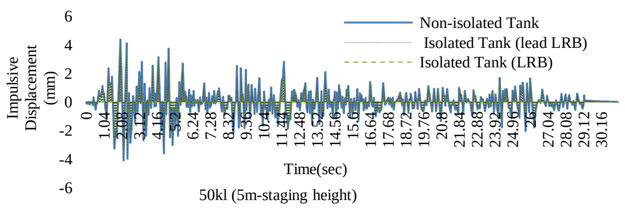
earthquake	Capacity	Support Condition	(F/W)	X_c	X_i	X_b
EL-CENTRO 1940	50kl	Non-isolated	0.372	0.26	4.38	
		Isolated LRB	0.062	17.7	0.06	6.3
		Isolated Lead-LRB	83.26	17.7	0.01	6.3
	50KL	Non-isolated	0.157	0.03	59.5	
		Isolated LRB	0.017	6.24	0.06	6.37
		Isolated Lead-LRB	0.010	17.6	0.06	6.36
	50KL	Non-isolated	0.157	0.03	55.2	
		Isolated LRB	0.015	22.0	0.05	6.37
		Isolated Lead-LRB	0.003	17.6	0.06	6.36

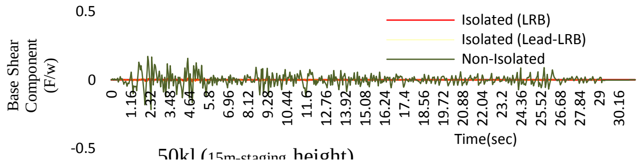
Table III Peak response of elevated Tank

Earthquake	Capacity / Staging height	Support Condition	(F/W)	X_c	X_i	X_b
EL-CENTRO 1940	100kl (5m)	Non-isolated	0.317	29	261	
		Isolated LRB	0.048	258	0.261	113
		Isolated Lead-LRB	0.048	264	0.264	113
	100kl (15m)	Non-isolated	0.042	29	261.6	
		Isolated LRB	0.023	530	0.049	172
		Isolated Lead-LRB	0.010	564	0.052	62
	100kl (30m)	Non-isolated	0.317	29	261.6	
		Isolated LRB	0.026	153	0.049	63.1
		Isolated Lead-LRB	0.026	179	0.049	63.4
	1000kl (5m)	Non-isolated	0.431	43	261.6	
		Isolated LRB	0.059	368	0.031	63.1
		Isolated Lead-LRB	0.095	367	0.054	63.1
	1000kl (15m)	Non-isolated	0.312	43	261	
		Isolated LRB	0.049	368	0.031	63.1
		Isolated Lead-LRB	0.049	367	0.054	63.1
	1000kl (30m)	Non-isolated	0.225	43	261.6	
		Isolated LRB	0.035	368	0.031	63.1
		Isolated Lead-LRB	0.035	367	0.054	63.1

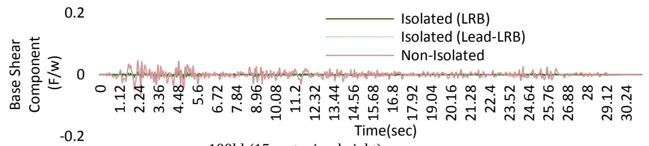




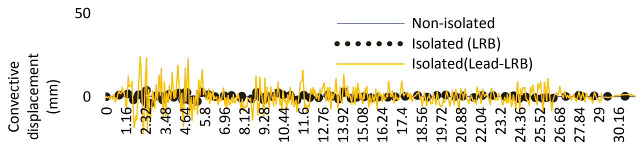




50kl (15m-staging height)



100kl (15m-staging height)



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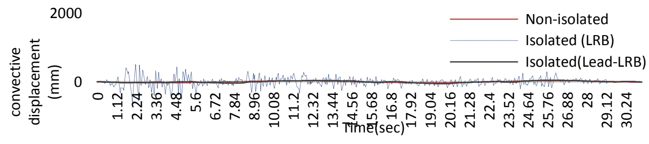

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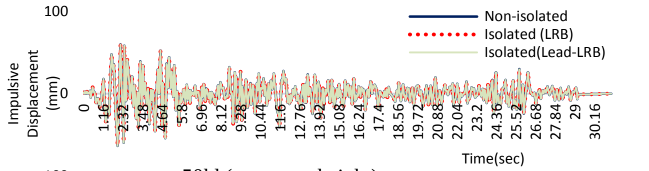

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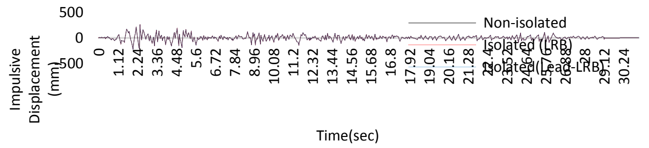

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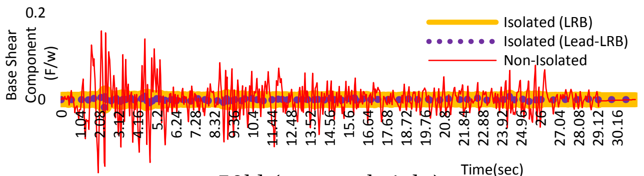
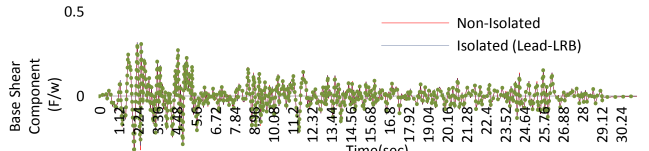
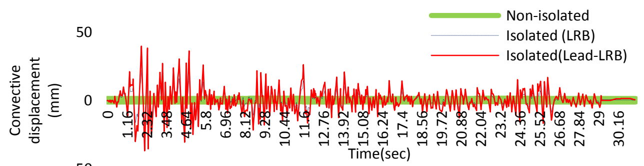


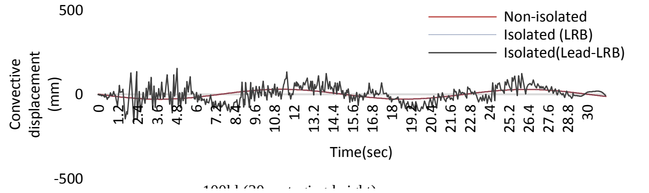
50kl (30m-staging height)

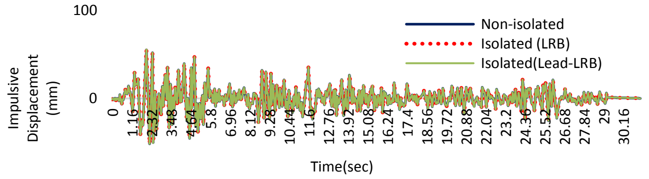


100kl (30m-staging height)

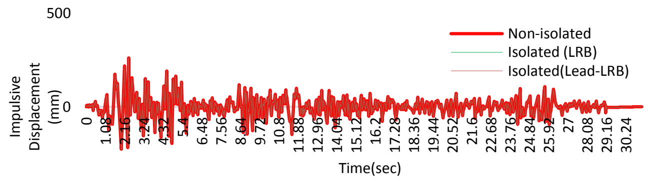


50kl (30m-staging height)

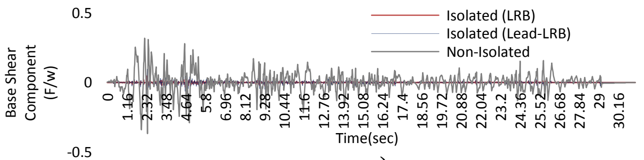




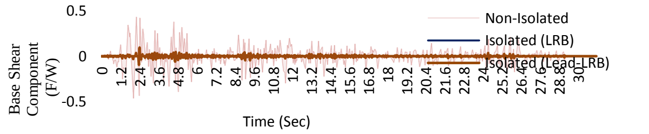
50kl (30m-staging height)



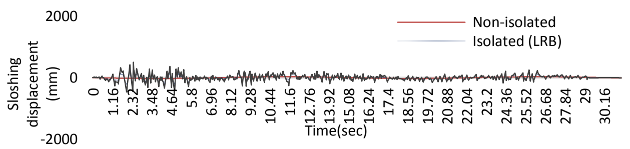
100kl (30m-staging height)



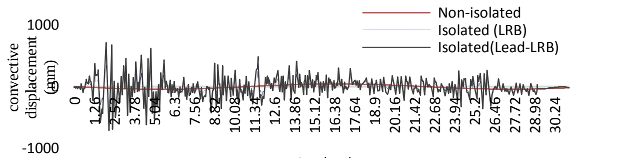
100kl (5m-staging height)



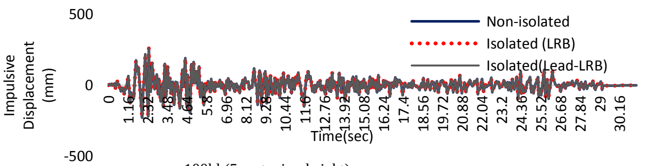
1000kl (5m-staging height)



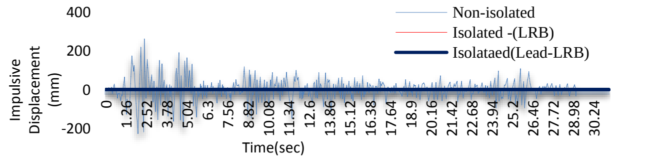
100kl (5m-staging height)



1000kl (5m-staging height)



100kl (5m-staging height)



1000kl (5m-staging height)

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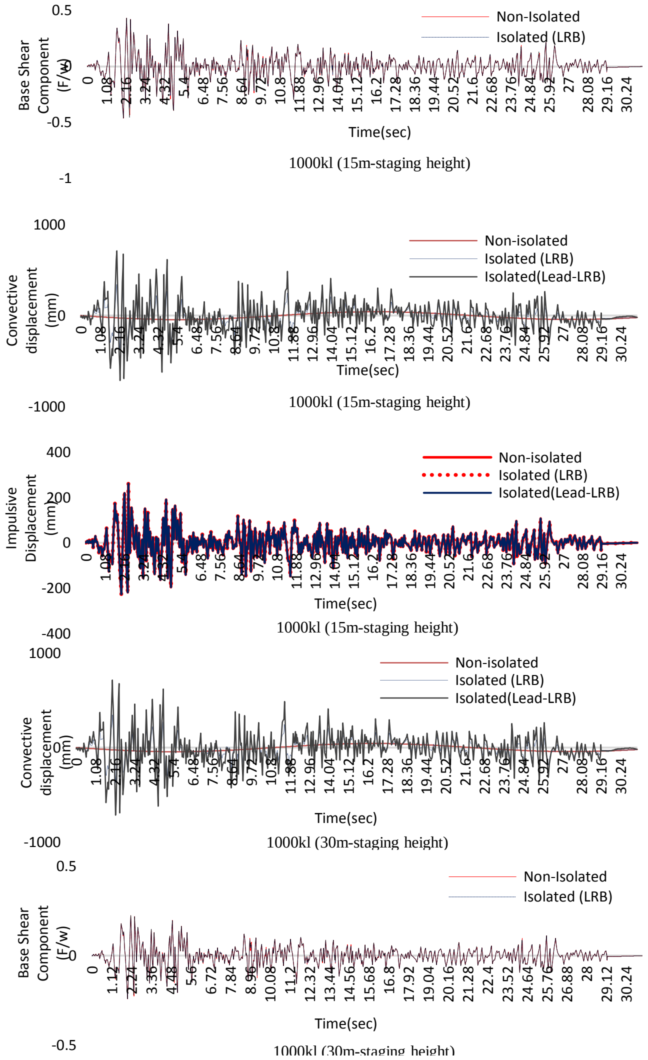

Fig.1 (a)Time Variation of Response quantities of Slender Tank due to Imperial Valley, 1940 Earthquake

(Tb= 2 sec,ξ_b = 0.1,F0=0.05)

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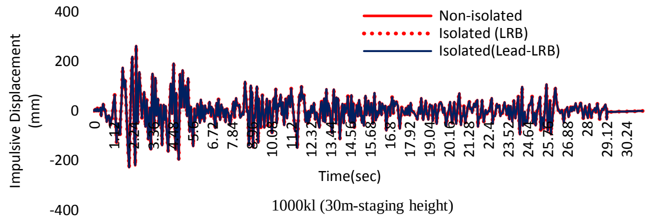

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