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FUZZY MEAN A-OPEN AND FUZZY MEAN A-CLOSED SETS

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Abstract
This article is to study the concepts of fuzzy mean α-open and fuzzy mean α-closed sets in fuzzy topological spaces. Further, in what way they are similar to those of other. Also we discuss some properties of fuzzy mean α-open and fuzzy mean α-closed with fuzzy α-paraopen and fuzzy α-paraclosed sets in fuzzy topology.

Key words and phrases: Fuzzy mean α-open, Fuzzy mean α-closed, Fuzzy α- paraopen, Fuzzy α -paraclosed.

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1. Introduction

After the introduction of fuzzy sets by the Zadeh[11], the concept of fuzzy topology introduced by Chang[4] in 1968.The notions of minimal open[6] and maximal open[5] sets explored by Nakaoka and Oda. Further the idea of paraopen sets was studied by Ittanagi and Benchalli[**?**]. Consequent of this, the concepts of mean open set and mean closed set investigated by Ajoy Mukherjee and Kallol Bhandhu Bagchiin[1]. The idea of fuzzy minimal open, fuzzy minimal clopen and fuzzy mean open sets studied in [[8, 9, 10]]. The idea of fuzzy minimal and fuzzy maximal α-open sets studied by Sankari in [7].

In this paper we introduce fuzzy mean α-open and fuzzy mean α-closed sets and investigate various properties of the fuzzy mean α-open sets.

2. Preliminaries

Definition 2.1. *A fuzzy set μ in a fts is called a fuzzy α-open [3] if $\mu \leq int\,[cl(int\mu)]$ and a fuzzy closed if $cl[int(cl\mu)]$.*

Definition 2.2. *A fuzzy topological space X is said to be fuzzy α-connected [2] if it has no proper fuzzy α-clopen set.*
A fuzzy topological space X is said to be fuzzy α-disconnected if it is not fuzzy α- connected.

Definition 2.3. *A proper nonempty fuzzy α-open set α of X is said to be a fuzzy minimal α-open[7] set if α and 0_X are only fuzzy α-open sets contained in α.*

Definition 2.4. *A proper nonempty fuzzy α-open set β of X is said to be a fuzzy maximal α-open [7] set if 1_X and β are only fuzzy α-open sets containing β.*

Definition 2.5. *A fuzzy* α*-open set* α *of a topological space X is said to be a fuzzy* α*-paraopen set if it is neither fuzzy minimal* α*-open nor fuzzy maximal* α*-open set. The family of all fuzzy* α*-paraopen sets in a fuzzy topological space X is denoted by FPaO(X).*

Definition 2.6. *A fuzzy α-closed set β of a fuzzy topological space X is said to be α-paraclosed if $1_X - \beta$ is fuzzy α-paraopen.*

Theorem 2.1. Let X be a fuzzy topological space and λ be a nonempty fuzzy α- paraopen subset of X. Then there exists a fuzzy minimal α-open set μ such that $\mu < \lambda$.

Theorem 2.2. Let X be a fuzzy topological space and λ be a nonempty fuzzy α- paraopen subset of X. Then there exists a fuzzy maximal α-open set σ such that $\lambda < \sigma$.

Theorem2.3. If ϑ is a proper fuzzy α-paraopen set, then there exist two proper fuzzy α-open sets $\eta_1, \eta_2\,(\neq\vartheta)$ such that $\eta_1 < \vartheta < \eta_2$.

Proof. Proof follows from definition of α-paraopen set.

Theorem 2.4. Let X be a fuzzy topological space and α be a nonempty fuzzy α- paraclosed subset of X. Then there exists a fuzzy minimal α-closed set γ such that $\gamma < \alpha$.

Theorem 2.5. Let X be a fuzzy topological space and α be a nonempty fuzzy α-paraclosed subset of X. Then there exists a fuzzy maximal α-closed set v such that $\alpha < v$.

Theorem 2.6. If α is a proper fuzzy α -paraclosed set, then there exist two proper fuzzy α -closed sets γ, ν (≠, α) such that $\gamma < \alpha < v$.

3. Fuzzy Mean α-Open and Fuzzy Mean α-Closed Sets

In this section we introduce fuzzy mean α -open and fuzzy mean α -closed sets. **Definition 3.1.** *A fuzzy α -open set μ of a fuzzy topological space X is said to be a fuzzy mean α -open set if there exist two distinct proper fuzzy α -open sets λ1, λ1(≠, μ) such that λ1 < μ < λ2 .*

Remark 3.1. It is observed from the definition that the union and intersection of fuzzy mean α -open sets need not be fuzzy mean α -open sets which is showed in the following example.

Example 3.2. Let X = {a, b, c} . Then fuzzy sets λ_1 = 0.1/ a + 0.2 /b + 0.0 /c ; λ_2 = 0.0 /a + 0.2 /b + 0.1/ c ; λ_3 = 0.0/ a + 0.2 /b + 0.0 /c and λ_4 = 0.1/ a + 0.2 /b + 0.1/ c are defined as follows: Consider the fuzzy topology τ = {0_X, λ_1, λ_2, λ_3, λ_4, 1_X} . Hence λ_1 and λ_2 are fuzzy mean α -open sets but their union $\lambda_1 \vee \lambda_1 = \lambda_4$ and intersection $\lambda_1 \wedge \lambda_1 = \lambda_3$ are not fuzzy mean α -open sets.

Definition 3.2. *A fuzzy α -closed set η of a fuzzy topological space X is said to be a fuzzy mean α -closed set if there exist two distinct proper fuzzy α -closed sets ϑ_1, ϑ_1 (≠, η) such that ϑ1 < η < ϑ2 .*

Proof. Let μ be a fuzzy mean α -open set in X . Then we have fuzzy α -open sets $\lambda_1 \neq 0_X$, μ and $\lambda_2 \neq \mu$, 1_X such that $\lambda_1 < \mu < \lambda_2$ and so $1_X - \lambda_2 < 1_X - \mu < 1_X - \lambda_1$. Since $1_X - \lambda_2 \neq 0_X$, $1_X - \mu$ and $1_X - \lambda_1 \neq 1_X - \mu$, 1_X ; hence $1_X - \mu$ is a fuzzy mean α -closed set.

Conversely, let *μ* be a fuzzy α -open set such that $1_X - \mu$ is a fuzzy mean α -closed set. Hence there exist fuzzy α -closed sets $\vartheta 1 \neq 0_X$, $1_X - \mu$ and ϑ_2 , 1X − μ, 1X such that $\vartheta 1 < 1_X - \mu < \vartheta_2$. It means that $1_X - \vartheta_2 < \mu < 1_X - \vartheta_1$. Since $1_X - \vartheta_2$, 0X, μ and $1_X - \vartheta_1$, μ, 1_X and hence μ is a fuzzy mean α –open set.

Theorem 3.4. A proper fuzzy α -paraopen set is a fuzzy mean α -open set and vice-eversa.

Proof. If α is a proper fuzzy α –paraopen set, then it is easy to follow by Theorem 2.3 that α is a fuzzy mean α -open set.

Conversely, let μ be a fuzzy mean α -open set in 1_X . Then there exist proper fuzzy α -open sets β_1, $\beta_2 \neq \mu$ such that $\beta_1 < \mu < \beta_2$. Since $\beta_1 \neq 0_X$, μ and $\beta_2 \neq 1_X$, μ, μ is neither a fuzzy minimal nor a fuzzy maximal α -open set. As $\mu \neq 0_X$, 1_X, μ is a proper fuzzy α -paraopen set.

Theorem 3.5. A proper fuzzy *α* -paraclosed set is a fuzzy mean *α* -closed set and vice -e-versa.

Proof. If *δ* is a proper fuzzy α -paraclosed set, then it is easy to follow by Theorem 2.6 that *δ* is a fuzzy mean α -open set.

Conversely, let δ be a fuzzy mean α-openset in *X*. Then there exist proper fuzzy α-closed sets η_1, $\eta_2 \neq \delta$ such that $\eta_1 < \delta < \eta_2$. Since $\eta_1 \neq 0_X$, δ and $\eta_2 \neq 1_X$, δ, δ is neither a fuzzy minimal nor a fuzzy maximal α-closed set. As $\delta \neq 0_X$, 1_X, δ is a fuzzy proper fuzzy α -paraclosed set.

Theorem3.6. Let *X* be a fuzzy topological space.

(i) *If β is a fuzzy minimal α-open and ϑ is a fuzzy α-open sets in X, then* $\beta \wedge \vartheta = 0_X$ *or* $\beta < \vartheta$.

(ii) *If β and γ are fuzzy minimal α-open sets, then* $\beta \wedge \vartheta = 0_X$ *or* $\beta = \vartheta$.

Theorem3.7. Let *X* be a fuzzy topological space.

(i) If *β* is a fuzzy maximal α-open and *ϑ* is a fuzzy α-opensets in *X*, then $\beta \vee \vartheta = 1_X$ or $\vartheta < \beta$.

(ii) If *β* and *γ* are fuzzy maximal α-open set, then $\beta \vee \gamma = 1_X$ or $\beta = \gamma$.

Theorem3.8. ([7]) If ϑ_1 is a fuzzy maximal α-openset and ϑ_2 is a fuzzy minimal α-openset of a fuzzy topological space *X*, then either $\vartheta_2 < \vartheta_1$ or the space is fuzzy α-disconnected.

Theorem3.9. Letafuzzyα-connectedtopologicalspace*X*containafuzzymaximal α-openset β_2, a fuzzy minimal α-open set $\beta_1 \neq \beta_2$ and a proper fuzzy α-openset $\vartheta \neq \beta_1$, β_2 .Then only one of the following is true on *X* :

(i) ϑ is a fuzzy mean α-openset such that $\beta_1 < \vartheta < \beta_2$.

(ii) $\beta_1 < 1_X - \vartheta < \beta_2$.

(iii) $\beta_1 < \vartheta$, $\beta_1 \vee \vartheta = 1_X$ and $\beta_2 \wedge \vartheta \neq 0_X$.

(iv) $\vartheta < \beta_2$, $\beta_1 \wedge \beta_2 = 0_X$ and $\beta_1 \vee \beta_2 \neq 1_X$.

Proof. By theorem 3.8, we have $\beta_1 < \beta_2$. Since β_1 is a fuzzy minimal α-open set and β_2 is a fuzzy maximal α-open set, we have $\beta_1 < \vartheta$ or $\beta_1 \wedge \vartheta = 0_X$ and $\vartheta < \beta_2$ or $\beta_2 \vee \vartheta = 1_X$. The feasible combinations are (i) $\beta_1 < \vartheta < \beta_2$, (ii) $\beta_1 \wedge \vartheta = 0_X$ and $\beta_2 \vee \vartheta = 1_X$, (iii) $\beta_1 < \vartheta$ and $\beta_2 \vee \vartheta = 1_X$, (iv) $\beta_1 \wedge \vartheta = 0_X$ and $\vartheta < \beta_2$. $\beta_1 \wedge \vartheta = 0_X$ and $\beta_2 \vee \vartheta = 1_X$ imply that $\beta_1 < 1_X - \vartheta < \beta_2$. If $\beta_1 < \vartheta$ and $\beta_2 \vee \vartheta = 1_X$, then $0_X \neq \beta_1 < \beta_2 \wedge \vartheta$ since $\beta_1 < \beta_2$. If $\beta_1 \wedge \vartheta = 0_X$ and $\vartheta < \beta_2$, then $\beta_1 \vee \vartheta < \beta_2 \neq 1_X$ since $\beta_1 < \beta_2$.

If both (i) and (ii) are true, then we see that $\beta_1 < \vartheta \vee (1_X - \vartheta) < \beta_2$ and $\beta_1 < \vartheta \wedge (1_X - \vartheta) < \beta_2$. $\beta_1 < \vartheta \vee (1_X - \vartheta) < \beta_2$ gives $\beta_1 < 1_X < \beta_2$ and then we get $\beta_2 = 1_X$, an absurd result. Also $\beta_1 < \vartheta \wedge (1_X - \vartheta) < \beta_2$ gives $\beta_1 < 0_X < \beta_2$ and then we get $\beta_1 = 0_X$, again an absurd result.

If both (i) and (iii) are true, then $\vartheta < \beta_2$ and $\beta_2 \vee \vartheta = 1_X$ give $\beta_2 = 1_X$, an absurd result. If both (i) and (iv) are true, then $\beta_1 < \vartheta$ and $\beta_1 \wedge \vartheta = 0_X$ give $\beta_1 = 0_X$, an absurd result. If both (ii) and (iii) are true, then $\beta_1 < 1_X - \vartheta$ and $\beta_1 < \vartheta$ give $\beta_1 < (1_X - \vartheta) \wedge \vartheta = 0_X$ and thus $\beta_1 = 0_X$, an absurd result.

If both (ii) and (iv) are true, then we have$1_X - \vartheta < \beta_2$ and $\vartheta < \beta_2$ give $(1_X - \vartheta) \vee \vartheta = 1_X < \beta_2$ and thus $\beta_2 = 1_X$, an absurd result.

If both (iii) and (iv) are true, then we get $\beta_1 < \vartheta < \beta_2$, $\beta_2 \vee \vartheta = 1_X$ and $\beta_1 \wedge \vartheta = 0_X$. $\vartheta < \beta_2$ and $\beta_2 \vee \vartheta = 1_X$ give $\beta_2 = 1_X$, an absurd result. $\beta_1 < \vartheta$ and $\beta_1 \wedge \vartheta = 0_X$ give $\beta_1 = 0_X$, again an absurd result.

Theorem 3.10. Let a fuzzy α-connected topological space X contain a fuzzy maximal α-closed set η_2, a fuzzy minimal α-closed set η_1 with $\eta_1 \neq \eta_2$ and a proper fuzzy α-closed set $\sigma \neq \eta_1, \eta_2$.Then only one of the following is true on X:

(i) σ is a fuzzy mean α-closed set such that $\eta_1 < \sigma < \eta_2$.

(ii) $\eta_1 < 1_X - \sigma < \eta_2$.

(iii) $\sigma < \eta_2$, $\eta_1 \wedge \sigma = 0_X$ and $\eta_1 \vee \sigma \neq 1_X$.

(iv) $\eta_1 < \sigma$, $\eta_2 \vee \sigma = 1_X$ and $\eta_2 \wedge \sigma \neq 0_X$.

Proof. We see that the fuzzy α-connected topological space X contains a fuzzy maximal α-open set $1_X - \eta_1$, a fuzzy minimal α-open set $1_X - \eta_2$ and a proper fuzzy α-open set $1_X - \sigma$ with $1_X - \eta_1 \neq 1_X - \eta_2$ and $1_X - \sigma \neq 1_X - \eta_1, 1_X - \eta_2$. By Theorem 3.9, only one of the following is true:

(i) $1_X - \sigma$ is a fuzzy mean α-open set such that $1_X - \eta_2 < 1_X - \sigma < 1_X - \eta_1$which in turn implies that $\eta_1 < \sigma < \eta_2$ Theorem, we see that σ is a fuzzy mean α-closed set.

(ii) $1_X - \eta_2 < 1_X - (1_X - \sigma) < 1_X - \eta_1$ i.e.,$\eta_1 < 1_X - \sigma < \eta_2$.

(iii) $1_X - \eta_2 < 1_X - \sigma$, $(1_X - \eta_1) \vee (1_X - \sigma) = 1_X$ and $(1_X - \eta_1) \wedge (1_X - \sigma) \neq 0_X$.i.e., $\sigma < \eta_2, \eta_1 \wedge \sigma = 0_X$ and $\eta_1 \vee \sigma \neq 1_X$.

(iv) $1_X - \sigma < 1_X - \eta_1, (1_X - \eta_2) \wedge (1_X - \sigma) = 0_X$ and $(1_X - \eta_2) \vee (1_X - \sigma) \neq 1_X$.i.e.,$\eta_1 < \sigma$, $\eta_2 \vee \sigma = 1_X$ and $\eta_2 \wedge \sigma \neq 0_X$.

Theorem3.11. If there are two distinct fuzzy maximal α-open sets and a fuzzy mean α-open set in a fuzzy topological space, then the intersection of the two fuzzy maximal α -open sets is nonempty.

Proof. Let γ_1, γ_2 be two distinct fuzzy maximal α-open sets and ω be a fuzzy mean α- open set in a fuzzy topological space X. By Theorem 3.7, $\gamma_1 \vee \gamma_2 = 1_X$.ω being a fuzzy mean α-open set, it is neither fuzzy maximal α-open nor fuzzy minimal α-open which means that, $\omega \neq \gamma_1, \gamma_2$. Also $\omega \neq 1_X$. By Theorem 3.7, we get ω; γ_1 or $\omega \vee \gamma_1 = 1_X$ and ω; γ_2 or $\omega \vee \gamma_2 = 1_X$. The feasible possibilities are (I)ω; γ_1 and ω; γ_2,(II) ω; γ_1 and $\omega \vee \gamma_2 = 1_X$,(III) $\omega \vee \gamma_1 = 1_X$ and ω; γ_2 and (IV) $\omega \vee \gamma_1 = 1_X$ and $\omega \vee \gamma_2 = 1_X$.

Case I: Obviously, $\gamma_1 \wedge \gamma_2 \neq 0_X$ if ω;γ_1 and ω;γ_2.

Case II: If $\omega \wedge \gamma_2 \neq 0_X$, then obviously $\gamma_1 \wedge \gamma_2 \neq 0_X$. Now suppose $\omega \wedge \gamma_2 \neq 0_X$. As ω;$\gamma_1$, then there exists $x_\alpha \in \gamma_1$ such that x_α g γ_2. Since $\omega \vee \gamma_2 = 1_X$, $x \in \gamma_2$. So $\gamma_1 \wedge \gamma_2 \neq 0_X$.

Case III: Similar to Case II.

Case IV: $\omega \vee \gamma_1 = 1_X$ and $\omega \vee \gamma_2 = 1_X$ imply that $\omega \vee (\gamma_1 \wedge \gamma_2) = 1_X$ which in turn imply that $\omega = 1_X$ if $\gamma_1 \wedge \gamma_2 = 0_X$. As $\omega \neq 1_X$, we have $\gamma_1 \wedge \gamma_2 \neq 0_X$.

Theorem 3.12. If there are two distinct fuzzy minimal α-open sets and a fuzzy mean α-open set in a fuzzy topological space, then the union of the two fuzzy minimal α-open sets is not equal to 1_X.

Proof. Let γ_1, γ_2 be two distinct fuzzy minimal α-open sets and ω be a fuzzy mean α-open set in a fuzzy topological space X. By Theorem 3.6, $\gamma_1 \vee \gamma_2 = 0_X$.ω being a fuzzy mean α-open set, it is neither fuzzy maximal α-open nor fuzzy minimal α-open which means that, $\omega \neq \gamma_1, \gamma_2$. Also $\omega \neq 0_X, 1_X$. By Theorem 3.6, we get γ_1; ω or $\omega \wedge \gamma_1 = 0_X$ and γ_2; ω or $\omega \wedge \gamma_2 = 0_X$.The feasible possibilities are (I)γ_1; ω and γ_2; ω,(II)γ_1; ω and $\omega \wedge \gamma_2 = 0_X$,(III) $\omega \wedge \gamma_1 = 0_X$ and γ ; ω and (IV) $\omega \wedge \gamma_1 = 0_X$ and $\omega \wedge \gamma_2 = 0_X$ as $\omega \neq 1_X$.

Case I: Obviously $\gamma_1 \vee \gamma_2 \neq 1_X$ if γ_1;ω and γ_2 ; ω.

Case II : If $\omega \vee \gamma_2 \neq 1_X$, then obviously $\gamma_1 \vee \gamma_2 \neq 1_X$. Now suppose $\omega \vee \gamma_2 \neq 1_X$. Since γ_1; ω, then there exists $x_\alpha \in \omega$ such that x_α g γ_1. As $\omega \wedge \gamma_2 = 0_X$, x_α g γ_2. x_α g γ_1, γ_2 imply that $\gamma_1 \vee \gamma_2 \neq 1_X$.

Case III: Similar to Case II.

Case IV: $\omega \wedge \gamma_1 = 0_X$ and $\omega \wedge \gamma_2 = 0_X$ imply that $\omega \wedge (\gamma_1 \vee \gamma_2) = 0_X$ which in turn imply that $\omega = 0_X$ if $\gamma_1 \vee \gamma_2 = 1_X$. As $\omega \neq 0_X$, we have $\gamma_1 \vee \gamma_2 \neq 1_X$.

On combining Theorems 3.11 and 3.12, we get Theorems 3.13 and 3.14 and the proofs of these two theorems can be proved similar to that of Theorems 3.11 and 3.12.

Theorem3.13. If there are two distinct fuzzy maximal α-closed sets and a fuzzy mean α-closed set in a fuzzy topological space, then the intersection of the two fuzzy maximal α -open sets is nonempty.

Theorem3.14. If there are two distinct fuzzy minimal α-closed sets and a fuzzy mean α-closed set in a fuzzy topological space, then the intersection of the two fuzzy minimal α-closed sets is not equal to 1_X.

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